# AC 2008-2283: A STRUCTURED APPROACH TO PROBLEM SOLVING IN STATICS AND DYNAMICS: ASSESSMENT AND EVOLUTION

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# A Structured Approach to Problem Solving in Statics and Dynamics: Assessment and Evolution

## Introduction

It has been the authors' experience that, in spite of even the most careful presentation, students often perceive the solutions to problems in dynamics to be a hodgepodge of techniques and "tricks". Interestingly, to address this perception problem, only limited resources can be found in textbooks published during the 50 years since the first editions of Meriam in 1951, Shames in 1959, and Beer and Johnston in 1962 that initiated a complete change in the way engineering mechanics was taught. Specifically, up until two years ago, little could be found in textbooks that one could use to teach a systematic approach to problem solving in Statics and Dynamics. As a result, we wondered why the approach used in more advanced mechanics courses (and often in sophomore-level Strength/Mechanics of Materials) is not used in Statics and Dynamics. This approach is much more structured and it is based on the idea that the equations needed to solve problems derive from three areas:

1. balance laws (e.g., force, moment, momentum, angular momemtum, energy, etc.);
2. constitutive equations (e.g., friction laws, drag laws, etc.); and
3. kinematics or constraints.

Since we didn't see any reason why this approach can't and shouldn't be applied to problems in Statics and Dynamics, we developed a structured approach to problems in these courses based on the classes of equations listed above and this approach was presented at the 2005 ASEE Annual Conference.[1] At the time, a similar approach had just appeared for the first time in Statics and Dynamics textbooks,[2,3] though we were not aware of it when we developed ours. Since then, we have taught Dynamics using our structured approach to problem solving and have discovered a number of interesting aspects of it that we will discuss in this paper. In particular, we will:

- discuss our original approach, the reasoning behind its structure, and present an example of how we implemented it in the classroom,
- describe our experience using it in teaching Dynamics in the spring 2007 semester,
- present feedback from students who took our Dynamics course during the spring 2007 semester,
- discuss how and why we modified our approach based on our experience teaching with it, and
- discuss why we think this is the future of teaching problem solving in introductory mechanics courses.

## Our Original Structured Problem Solving Approach

In our original approach to a structured problem solving procedure (see[1] for extensive details), we meant to emphasize the *modeling process*, by which a "real system" is turned into a *mathematically tractable system* whose behavior can be predicted via the application of simplifying assumptions and fundamental laws of nature. Generality and consistency were the main qualities we wished to have in our procedure to counter the perception among many students that every problem in mechanics is different from every other problem. In addition, we wished to have a procedure able to counter what we perceive as the dominant problem solving strategy among students, namely "pattern matching" coupled with "coming up with *any* $n$ equations in $n$ unknowns" (whether or not the $n$ equations are relevant to the problem). By contrast, our approach was intended to reinforce the idea that the equations governing the solution of a problem are always based on the following three basic elements:

(i) Newton-Euler equations and/or balance laws,

(ii) material or constitutive equations, and

(iii) kinematic equations,

where, by Newton-Euler equations and/or balance laws, we mean Newton's second law for particles, Euler's first and second laws for rigid bodies (which provide the translational and rotational equations for rigid bodies), and balance laws for energy and momentum that are derived from them. This solution paradigm is universally practiced in graduate courses as well as in real-life engineering modeling. Our approach emphasizes to the students that exhausting each of the three items mentioned above results in a *complete system of independent equations* (i.e., not just any $n$ equations in $n$ unknowns) leading to the solution of the problem. This approach removes some of the mystery as to where to begin to write the equations in Dynamics since students often just keep writing equations hoping that they will come up with enough of them. In addition, it gives the teaching of Statics and Dynamics the same mathematical and conceptual foundation as other mechanics courses that the students encounter (e.g., Strength of Materials, Continuum Mechanics, Elasticity).

## Our Five Steps of Problem Solving

Previously, the proposed problem solving procedure[1] consisted of the following five steps:

**Road Map** This is a summary of the *given* pieces of information, an extremely concise statement of what needs to be found, and an outline of the overall solution strategy.

**Modeling** This is a discussion of the assumptions and idealizations necessary to make the problem tractable. For example, are we including or neglecting effects such as friction, air drag, and nonlinearities? Whether or not we are including these effects, we make it very clear how sophomore-level Statics and Dynamics deals with them and are careful to discuss the fact that our solution is restricted to the particular model system that has been analyzed. The free-body diagram (FBD), a visual sketch of the forces acting on a body, is the central element of the modeling feature and is included here.

**Governing Equations** The governing equations are *all* the equations needed for the solution of the problem. These equations are organized according to the paradigm discussed earlier, that is, ($i$) Newton-Euler/balance equations, ($ii$) material equations/models, and ($iii$) kinematic equations. In Statics, the Newton-Euler/balance equations are called *equilibrium equations*. At this point in our approach we encourage students to verify that the number of unknowns they have previously identified equals the number of equations they have written in the Governing Equations section.

**Computation** The manipulation and solution of the governing equations.

**Discussion & Verification** A verification of the apparent correctness of the solution and a discussion of the solution's physical meaning with an emphasis on the role played by the assumptions stated under the *Modeling* heading.

We viewed (and presented to the students) the structured problem solving approach described above as a *universal* problem solving procedure to be applied to any problem concerning forces and motion both in undergraduate and graduate courses, as well as in research and development. When we first proposed it, we felt that this approach to problem solving was quite different from what could actually be found in current textbooks (again, we note that a similar approach had just appeared in the Statics and Dynamics textbooks authored by Sheppard and Tongue[2,3]), though we strongly suspected that many engineering faculty may have already been teaching problem solving using this structure.

## Class Test of the Proposed Structured Problem Solving Approach

The five-step procedure described above, was class tested in all of the sections of the sophomore-level Dynamics course we offered at Penn State during the spring 2007 semester. The total number of students affected by the class-testing was about 450. The class test consisted of:

1. using the proposed procedure in the solution of *every single example* done in class,
2. *requiring* the students to use the proposed procedure in every homework problem, and
3. requiring the students to use the proposed procedure in solving every problem on the midterm and final exams.

After a short "grace period" at the beginning of the semester, students who did not use the required procedure were penalized by lowering the assignment or exam score by roughly 5%.

In an effort to provide the same examples in all sections and in an effort to provide the students with uniform study material, the lectures offered during the class test were delivered as presentations projected to the class via computer. The same presentations were delivered to all sections. These presentations were sometimes complemented by *ad hoc* derivations done on the chalkboard or on a electronic document camera so as to engage the students in discussion and to respond to specific student questions. As an example of the material presented in class and shown to the students, in Figs. 1 and 2 we present the slides used in class to demonstrate our problem solving approach. The example in question concerns a simple one-dimensional kinetics problem. While we are only showing a single example, again, we wish to emphasize that every single example

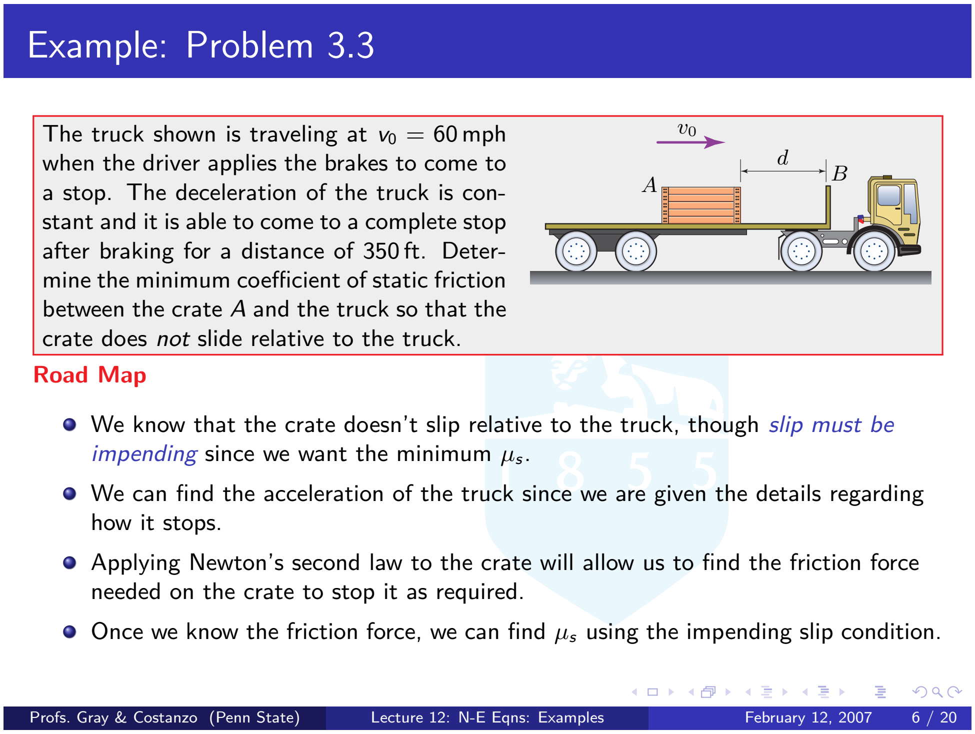



**Figure 1.** Slide 1 (of 3) of a simple particle kinetics problem demonstrated in class.

presented in class, whether on particle kinetics of rigid body kinetics, adhered to the format described earlier.

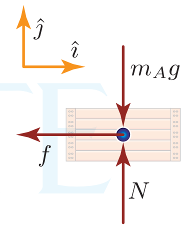
## Our Experience and Student Feedback

From the instructor's viewpoint, enforcing a fixed problem-solving procedure gave us a powerful tool to reinforce the idea that the entire course is about the application of the same few principles rooted in Newton's laws. This is a message that the authors have always emphasized while teaching sophomore-level Dynamics and the use of a fixed problem-solving procedure made the delivery of this important message easy, thoughtful, and always in the proper context in every lecture. Specifically, the consistent labeling of problem-solving steps using keywords like "Newton-Euler/Balance Equations" as headings in example after example, whether dealing with particle or rigid body problems, or whether dealing with impact or the work-energy principle, gave us the perception that we were able to broadcast our "mantra" (i.e., everything stems out of Newton's laws of motion) more effectively than in any previous semester.

From a "public relations" viewpoint, the enforcement of a fixed problem solving routine was initially met by significant frustration on the part of the students. Based on conversations with students before and after class, as well as during office hours, we became aware that the students perceived the new requirement as unfamiliar, unnecessary, and as a time-wasting complication added to their workload. However, as the semester progressed, the attitude toward the required problem solving procedure changed and became more tolerant. After the first half of the semester,

## Example: Problem 3.3 (continued)

**Modeling** Treating the crate as a particle, its FBD is as shown, where $f$ is the friction force *on the crate* due to its interaction with the truck bed.




**Governing Equations**

Newton-Euler/Balance Equations

$$\sum F_x = ma_x : \quad -f = m_A(a_A)_x,$$

$$\sum F_y = ma_y : \quad N - m_A g = m_A(a_A)_y.$$

Material Models Since we want the *minimum* coefficient of static friction between the crate and truck required for no sliding, that means that slip must be impending and so

$$f = \mu_s N \quad \text{(impending slip)}$$

Kinematic Equations Since the crate doesn't slip relative to the truck, we have

$$(a_A)_x = (a_B)_x \quad \text{and} \quad (a_A)_y = (a_B)_y = 0.$$

Now, using the constant acceleration eqns, we can find the acceleration of the truck, that is,

$$(v_B)_x^2 = (v_{B0})_x^2 + 2(a_B)_x(x_B - x_{B0})$$



## Example: Problem 3.3 (continued)

Plugging in numbers:

$$0 = (88)^2 + 2(a_B)_x(350) \quad \Rightarrow \quad (a_B)_x = -11.06\,\text{ft/s}^2.$$

where I have used the fact that $(v_{B0})_x = 60\,\text{mph} = 88\,\text{ft/s}$.

**Computation** Plugging kinematic and material equations into the Newton-Euler equations, we obtain the following *equation of motion* and *constraint force eqn*:

$$\left.\begin{aligned} -\mu_s N &= m_A(a_B)_x, \\ N &= m_A g. \end{aligned}\right\} \quad -\mu_s m_A g = m_A(a_B)_x \quad \Rightarrow \quad \mu_s = -\frac{(a_B)_x}{g} = -\frac{-11.06\,\text{ft/s}^2}{32.2\,\text{ft/s}^2}.$$

So that the minimum required coefficient of static friction is $\mu_s$ is $\boxed{\mu_s = 0.343.}$

**Discussion & Verification**

- $\mu_s$ is of reasonable size.
- $(a_B)_x$ is negative as expected.



**Figure 2.** Slides 2 and 3 (of 3) of a simple particle kinetics problem demonstrated in class.

a number of students *reluctantly* admitted to us that they thought that our required problem solving procedure had made a significant difference in their problem solving effectiveness.

An important realization concerning our class testing of the proposed procedure was that, in answering student questions about the choice of solution strategy, which we placed in the "Road Map" step (the very first step), we would often find ourselves pointing to a feature of the FBD, which, from a procedural viewpoint, was actually placed in the "Modeling Step". This observation, along with feedback received from the students led us to an important modification of our problem solving strategy, which we will describe in the next section.

Additional student feedback on the satisfaction with the enforced problem solving approach was obtained in two ways. Specifically, we carefully reviewed the end-of-semester course evaluation surveys that every student has the opportunity to fill out during a 15–20 minute period taken from the regularly scheduled class time. In addition, two focus groups of selected students were organized, in which students were queried about a variety of issues, including their typical problem solving strategies and the effectiveness of the required procedure.

As far as the end-of-semester surveys are concerned, we found comments concerning problem solving that were offered in response to the following two questions:

1. What aspect(s) of this course most enhanced you learning? Why?
2. What aspect(s) of this course least enhanced you learning? Why?

The answers to these questions indicated that the enforced problem solving approach had been a useful tool that helped students focus their efforts during the solution of problems. In addition, some students seemed to see a connection between the problem solving style and the understanding and application of conceptual knowledge. Here is a list of the positive comments provided by the students via the end-of-semester assessment survey (the negative comments are listed after the positive comments).

- "Examples in lectures did a good job showing how to apply concepts"
- "The problem-solving format helped me to solve problems more easily and to understand the material better. The "road map" forced me to collect my thoughts and the "discussion & verify" [sic] helped me make sure I got the right answer."
- "I think that the problem solving steps and integrating all of the knowledge that I have gathered from other classes into one class has really made me better problem solver."
- "Method of doing work (ROAD MAP, FBD, . . . ), organized my thoughts [sic]"
- "The lecture notes and homework because they went through examples problems very systematically"
- "I believe the solving steps most enhanced my learning because it made it clear why we did each step and how they contributed to produce answers."
- "problem solving with 5 steps helped me solve it efficiently."
- "[the instructor] outlined a uniform problem solving method for all problems, this helped a lot."

- “very good and thorough approach to problem solving.”
- “the method of solving a problem, Because the method that we using [sic] now is quiet [sic] effective of solving a problem, such as the road map, Governing eqns [sic] and discussion . . . ”

Some students had negative comments to make, such as

- “I did not like the style of solving problems. I can’t see everything I’m going to need right off the bat.”
- “The “5 steps” half the homework time was wasted Roadmap and discussion rather than learning to problem sole [sic] . . . ”

A student, in response to the question “What, if anything, would you change about this course? Please explain.”, stated “Difficult to understand certain problem solving methods [sic].”

Despite the presence of negative comments, we wish to remark that the positive and negative answers reported above accurately reflect the proportion of positive vs. negative comments we obtained. Therefore, we conclude that the use of a structured problem solving approach was beneficial to the students’ perception that they were learning the subject matter. Regarding what we observed in the students’ exams and homework problem solutions, we can say that there was some moderate improvement in student performance relative to previous semesters although at the present this is purely anecdotal and we are not in a position of properly quantifying such improvement. What we saw as being a dramatic improvement with respect to previous semesters is a significant reduction of what can be called “meandering”, “false starts”, and “ill-fated attempts”. In other words, we saw that the students derived the equations necessary to solve a problem in a much more direct and focussed manner. While some students would still make mistakes in writing equations, the type and number of equations provided by the students was much more closely correlated with the “right equations” with respect to what we observed in the past.

As far as feedback via student focus groups is concerned, we selected fourteen students to participate in two distinct focus groups. The students selected were among those with the best class attendance record, but otherwise having a wide range of final expected grade. Note that the questions the students answered during the focus groups interviews were diverse, i.e., not limited to the topic of problem solving. The focus group interviews were carried out by two people not involved with the course, who also queried the students about how students normally used the course textbook. The answers gathered from the students confirmed that when they attempt to solve a problem in a homework assignment, by and large, they try and identify an example in the textbook that is perceived as having the same features as the problem to be solved. As far as problem solving in particular is concerned, the students indicated that the structured problem solving strategy was very useful in following the solutions of the examples demonstrated in class. However, they felt that the procedure was more difficult to implement when they had to create it on their own. Interestingly, the step that they found to be the most difficult was the very first step, namely the “Road Map”. Students indicated that, in trying to create the “Road Map” they would end up being confused on how to proceed. These responses indicated to us that the creation of the “Road Map” required more experience and hindsight than is possessed by the average “novice problem solver” in a sophomore-level Dynamics class.

## Updated structured problem solving approach

As a result of the feedback received from the students, as well as a more careful analysis of how we were actually presenting the solution of a problem (with the understanding that problem solving in complex situations is a highly iterative process), we concluded that the "Road Map" step in the solution procedure *cannot* be disjoint from the "Modeling" step, in which modeling assumptions are stated and the system's FBD sketched.

To explain the rationale for this conclusion, consider a problem in which one wants to determine an object's change in speed due to a corresponding change in position, while the object is under the action of a given force system. A problem of this type can always be solved by applying Newton's second law to relate forces to accelerations, and then integrating the accelerations with respect to position or time to establish the required speed-position relation. However, in Dynamics it is often the case that the type of problem in question can be solved more directly by an application of the work-energy principle. According to our original structured problem solving procedure, the decision of what solution strategy to adopt is made in the "Road Map" step, before the system's FBD is sketched and therefore before one has the opportunity to carefully review the character of each force that acts on the body. In Dynamics problems are usually simple enough that an experienced problem-solver can immediately assess whether or not the application of the work-energy principle leads more directly to the solution than the application of Newton's second law. However, a novice may not be able to make a decision without assessing whether or not each of the forces acting on the system allows him/her to carry out a straightforward computation of the force's work. For example, if the forces acting on the system are conservative forces and constraint forces that do no work, then the application of the work-energy principle is indeed the faster solution method. However, if the body is subject to a velocity dependent drag force or, say, if the body moves along a curved rough surface and is therefore subject to a path dependent variable friction force, then the application of the work-energy principle results in a solution that is often more mathematically complex than that due to a straightforward application of Newton's second law.

The situation just discussed leads to the conclusion that not only does the FBD play a central role in allowing one to go from a physical model of reality to a corresponding mathematical model, but it plays a crucial role in deciding what solution strategy to adopt for a particular problem. In making decisions about a problem solving strategy, the FBD is used as a "list" of the forces acting on the system. In going over the list of forces, one can make value judgements about how each force is related to the problem's given data and unknowns and how the character of each force would affect the complexity of the calculations resulting from a particular choice of solution strategy. Consequently, *we have modified our structured problem solving approach by combining the Road Map and Modeling step is a single step*. While this change may appear as a minor revision to our structured problem solving approach, in reality it reflects an important shift in understanding the relation between modeling assumptions, which rely mostly on abstraction and conceptual understanding, and solution strategy decision making, which is based on procedural knowledge and experience.

We have implemented additional changes to the problem solving procedure, primarily in the vocabulary used in identifying two of the sub-steps in the Governing Equations. Specifically, we have decided to rename the "Newton-Euler/Balance Equations" sub-step as "Balance Principles"; and to rename the "Material Models" sub-step as "Force Laws". The reason for these minor revisions was to use a vocabulary that would be more easily recognized outside the Engineering Dynamics community.

The revised problem solving procedure is being used during the spring semester 2008 in an honors sophomore-level Dynamics course. As with the non-honors course taught in spring 2007, all the examples in class are solved using this procedure and the students will be required to use the procedure in homework and on exams. In addition, the authors are in the process of developing a comprehensive assessment program with the collaboration of colleagues in the College of Education at Penn State and the Penn State Leonhard Center for the Enhancement of Engineering Education.

## Summary and Conclusion

The class testing of our proposed problem solving procedure yielded some interesting clues regarding the effectiveness of a structured approach to problem solving in a typical sophomore-level Dynamics course. We feel that both the students and the instructors benefit from using a consistent approach to problem solving. One of the major benefits for the instructor is the opportunity to constantly and consistently point the students' attention to the relation between a particular topic or solution technique and the underlying fundamental physical principles rooted in Newton's laws of motion. Overall, we feel rather confident in affirming that the students greatly benefit from a structured approach because it helps them focus their attention and prevents, to a significant extent, the "meandering" in search for "$n$ equations in $n$ unknowns". From a more specific pedagogical viewpoint, the class implementation of a structured problem-solving approach made us realize how the FBD is at the center of a problem's solution because the creation of the FBD can be made a place where both conceptual as well as procedural knowledge must be used and coordinated. Traditionally, in sketching an FBD one makes important choices as to what to include and what to neglect in the model. However, the creation of the FBD also provides an invaluable opportunity to discuss how the forces included in the model affect the various solution strategies available to the student and, in this way, provides an opportunity for the student to make an informed decision as to what solution strategy to adopt.

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