A STUDY OF EFFECTIVENESS OF MULTI-STAGE HEAT EXCHANGERS

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SUMMARY

The level of effectiveness of heat transfer in heat exchangers is always a challenging problem. One of the very effective methods of intensifying heat transfer in so called pneumatic type heat exchangers is use gaseous suspension (gas-solid particulates). However, in such heat exchangers the process takes place most effectively in the so called "acceleration" section where relative velocity (between gas and solid particulates) and the heat transfer coefficient are maximum. Thus, one more way of intensifying the heat transfer process is to increase the number of acceleration sections; hence leading to the use of the multi-stage heat exchanger.

In this paper, based on analysis of heat transfer in the heat exchanger, the equation is developed to determine the number of stages. These number of stages is sufficient enough to achieve the design final temperatures of the heat carriers. However, analysis shows that intensity of heat transfer decreasing with increasing the number of stages. This has lead to develop additional method which allows to determine the number of stages when the final temperature of heat transfer carriers are less than design but as close to final temperature as desired. For example, 2%, 5%, or 10% less than design final temperature.

NOMENCLATURE

C_p Specific heat, kJ/kg-K

m Mass flow rate, kg/s

M $= f(\beta)$

n Number of stages in a multi-stage apparatus

t Temperature of the gaseous heat carrier

α Dimensionless difference in temperatures

β Ratio of dimensionless temperatures

υ Temperature of the solids

Θ_s Dimensionless temperature of solid $= (\upsilon - \upsilon_i)/(t_i - \upsilon_i)$

Θ_g the dimensionless temperature of gaseous heat carrier $= (t - \upsilon_i)/(t_i - \upsilon_i)$

Subscripts

g Gas

i Initial value

f Final value

s Solid

Superscripts

1 First stage

n Last stage

MULTI-STAGE HEAT EXCHANGERS

Under "heat exchanger," it is assumed any apparatus which is used to utilize heat and heat-and-mass transfer apparatus such as thermal treatment of materials (solids), drying, chemical reactions, warming or cooling of solids or gaseous substances, etc. Figure 1 shows a multiple hearth furnace. It is constructed of a number of circular hearths enclosed in a refractory lines steel shell. In the center of the cylindrical shell is a vertical rotating shaft with radial arms and rabbling teeth which move the feed material in a spiral path across each hearth. In its travel across each hearth, the material is constantly agitated and exposed by the rabbling teeth before it falls through drop holes from level to level. Process air can be supplied in a regulated quantities by combustion air blowers through burners or ports, or by induction through pair ports.

Multiple process steps such as drying, calcining, incinerating, and cooling can be carried out simultaneously in the same furnace. Processing time is controlled by the speed of the rotating shaft. The shaft is driven by a variable speed drive which can be automated to change speeds with variations in feed moisture or weight. Heat transfer occurs through material-gas contact and some radiation. Hot gases from combustion rise counter-current to the flow of the material. Contact between the gas and feed occurs across every hearth and during drops from hearth to hearth.

Another example is presented in Figure 2. Here also the gas flows counter-current to the motion of the materials (solids). In each stage of this apparatus the flow of gaseous heat carrier and material is parallel, but for the apparatus itself, the flow of these two heat carriers is counter flow. In both of these examples, the feed materials (solids) enter the apparatus at a stage which serves as exit for the gaseous heat carrier. On the other hand, the entrance stage of the gas is the exit stage for the solids.

THEORETICAL ANALYSIS

In our analysis, we assumed that the gaseous heat carrier enters the first stage of the apparatus (1) and exits the last one (n). On the other hand, the solids enter the last stage of the apparatus (n) and exit the first stage (1). Thus,

$t_i = t^{(1)}$, and $t_f = t^{(n)}$

$\upsilon_i = \upsilon^{(n)}$, and $\upsilon_f = \upsilon_f^{(1)}$.

It is further assumed that the initial temperature of gaseous carrier is greater than that of solids. This assumption does not have any effect on analysis or development of equations. These conditions are only used to clarify the terminologies used in this work.

The following equation determines the final temperature of the solids [1]:

$$\upsilon_f^{(1)} = \Theta_s t_i^{(1)} \left[\sum_{m=1}^{n} \{(1 - \Theta_s)/\Theta_g\}^{(n-m)} \right] / \left[\{(1 - \Theta_s)/\Theta_g\}^{(n-1)} + \Theta_s \sum_{m=2}^{n} \{(1 - \Theta_s)/\Theta_g\}^{(n-m)} \right] \quad \ldots \quad \ldots \quad (1)$$

From equation (1), the final temperature of solids, $\upsilon_f^{(1)}$ may be determined. In a number of cases, the final temperature of solids is given. In those cases, it becomes necessary to obtain the number of sections, n, required to achieve this final temperature of the solids.

To simplify this conversion further, let us denote

$$\beta = (1 - \Theta_s)/\Theta_g \qquad (2)$$

Then equation (1) can be presented as

$$\upsilon_f^{(1)} = \Theta_s t_i^{(1)} \left[\sum_{m=1}^{n} \beta^{(n-m)} \right] / \left[\beta^{(n-1)} + \Theta_s \sum_{m=2}^{n} \beta^{(n-m)} \right]$$

$$= \Theta_s t_i^{(1)} [\sum_{m=1}^{n} \beta^{(n-m)}]/[\beta^{(n-1)}$$

$$+ \Theta_s \sum_{m=1}^{n} \beta^{(n-m)} - \Theta_s \beta^{(n-1)}] \qquad (3)$$

Equation (3) divided by

$$\sum_{m=1}^{n} \beta^{1-m}$$

can be further simplified into the following identity:

$$\sum_{m=1}^{n} \beta^{1-m} = \{\upsilon_f^{(1)}/[t_i^{(1)} - \upsilon_f^{(1)}]\}[(1 - \Theta_s) / \Theta_s] \qquad (4)$$

Thus with a known value of final temperature of the solid particulates, $\upsilon_f^{(1)}$, a sum of the series on the left hand side of equation (4) is determined. On the other hand, a sum of the finite geometric series is defined as

$$\sum_{m=1}^{n} \beta^{(1-m)} = [1 - \beta^n]/\{\beta^{(n-1)} (1 - \beta)\} \qquad (5)$$

Equating the right hand sides of equations (4) and (5) and after some manipulation, one determines the number of stages, n, as

$$n = - \mathrm{Log} \{1 + [(1 - \beta) \upsilon_f^{(1)} (1-\Theta_s) / \beta (t_i^{(1)} - \upsilon_f^{(1)}) \Theta_s]\} / \mathrm{Log}\, \beta \qquad (6)$$

It follows from equation (3) that the final temperature of the solids, $\upsilon_f^{(1)}$, rises with increase in the number of sections, n. Hence, the temperature difference between the gaseous heat carrier (with decreasing temperature) and solids with increasing number of sections decreases. As a result, the effectiveness of heat removal, all other parameters being equal, is reduced. The latter may be easily shown analytically.

It should be noted that similar effects may be observed in any heat exchanger regardless of mutual flow direction of both heat carriers [3]. Thus, due to the exponential nature of the heat carrier's temperature change, beginning at some stages the increase of the final temperature of heat carrier is diminished. Calculations show that a rapid rise in temperature of the solids and corresponding rapid fall in gaseous heat carrier temperature occurs within comparatively small number of sections. In case when initial temperature of the gaseous heat carrier is greater than that of solids, the solids are warmed up. For example, for $\Theta_s = 0.5$, $(C_{ps} m_s)/ (C_{pg} m_g) = 0.8$, and the initial temperature of gaseous heat carrier, $t_i^{(1)} = 1000$ C, the final temperature of the solids in a six section heat exchanger reacher $\upsilon_f^{(1)} = 900$ C. For an apparatus of ten sections, $\upsilon_f^{(1)} = 960$ C. This means in the additional four sections, the solids are warmed up by only 60 C which is less than 7%. Thus, it is possible to assume small reduction of the final temperature of the solids by properly choosing the number of sections where the heat transfer is still intensive. Hence, in addition to equation (6), there should be another equation or another method to determine the reasonable number of sections in the heat exchanger.

Analysis of equation (3) shows that the temperature of the solids, $\upsilon_f^{(1)}$ depends on parameter β and number of stages, n. Hence, a sufficiently acceptable rise in solids temperature, according to equation (3), will take place until the following equality is fulfilled:

$$\beta/\beta^n = M \qquad (7)$$

where M is some number to be determined. Thus, the value of M determines the number of terms of the series in equation (3) to achieve the solids' temperature of υ_f^* which is different from $\upsilon_f^{(1)}$ according to equation (3).

Let the dimensionless difference between these two temperatures be given as

$$\alpha = 100\ (\upsilon_f^{(1)} - \upsilon_f^*)\ /\ \upsilon_f^{(1)}\ \% \qquad (8)$$

Then α, with given value of β, is clearly governed by the value of M. Figure 1 graphically represents the relationship $M = f(\beta)$ for various values of α. Calculations show that the final temperature $\upsilon_f^{(1)}$ in all cases where $n \geq 10$ remains almost constant. Therefore, it is advisable to consider number of stages, $n < 10$.

From equation (7), it follows that

$$n = 1 - \{Ln(M)/Ln(\beta)\} \qquad (9)$$

for $\beta \neq 1$. Thus, using equation (9) and Figure 3, one can easily determine the number of stages in the multistage heat exchanger to achieve the desired final temperatures of the heat carriers with a designed percentage offset α.

Table 1 provides some examples of design parameters. Of course, if one wishes to obtain exactly the design final temperatures of the heat carriers, use equation (6); however in any case, the number of stages, n must not be larger than 10.

Table 1. Examples of Design Parameters Using Figure 3

	β	α, %	M	n
1	0.8	10	3	≈ 6
2	0.8	5	4	≈ 7
3	1.4	10	0.245	≈ 5

CONCLUSIONS

It should be especially emphasized that the final temperature of solids $\upsilon_f^{(1)}$ according to equation (1) is only determined for the entire heat exchanger having n sections. However, it is possible to develop equations for determining the temperature of solids and gaseous heat carrier of any stage.

In this paper, we considered analysis of a particular type of heat exchanger. However, we believe that the principal approach presented in the paper my be used to analyze any type of multistage apparatus regardless of hydrodynamics regime. It also may be used to analyze the performance of multistage apparatus having either parallel or counter flow as in case under consideration in this project.

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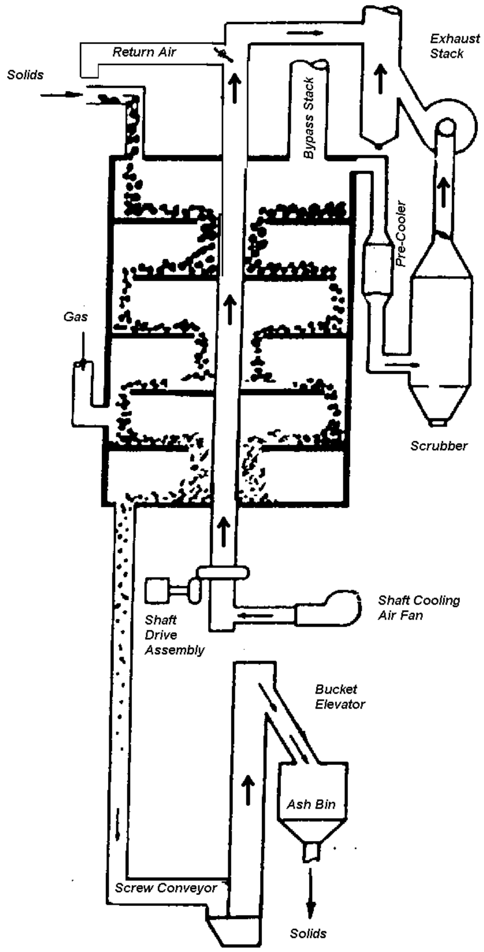

Figure 1. Vertical Multistage Apparatus

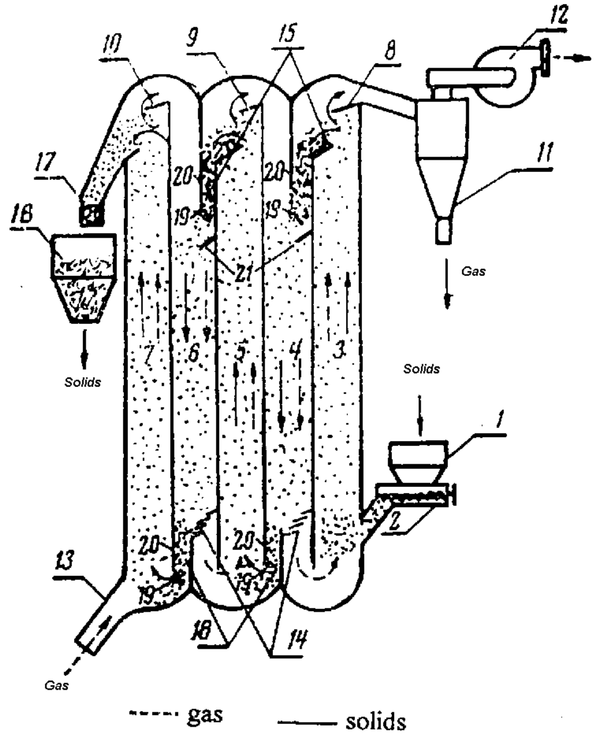

Figure 2. Horizontal Multistage Apparatus

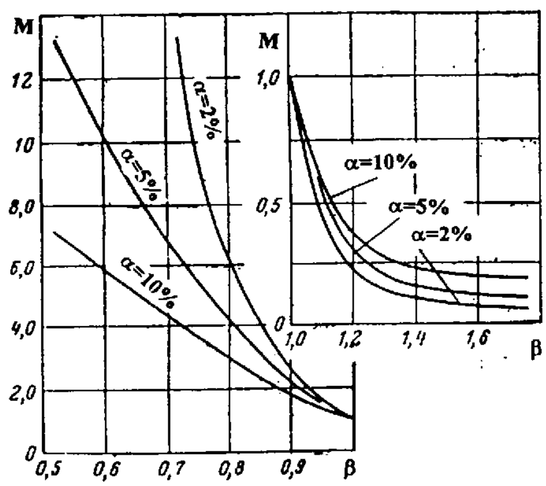

Figure 3. Values of M as a Function of β