A Proposal for Unifying some of the Fundamental Concepts of Engineering

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Abstract

Mathematics is the descriptive language of engineering while physics provides the foundation for engineering. At many engineering institutions, mathematics and physics are frequently taught by departments other than the engineering department. This tradition often has the result that undergraduate students experience considerable difficulty in applying their mathematics skills in physics and engineering. Additionally, students infrequently learn the relevance and significance of several of the common fundamental mathematical relationships that underscore all technical fields of study.

Two of the many important results from mathematics that are essential for all technically oriented students are Taylor's theorem and Fourier analysis. A working knowledge of the implications and consequences of these theorems serves as a unifying theme that underscores many aspects of the foundation of engineering. Students skilled in the use of these theorems develop deeper insights into many different fields of study and are able to quickly comprehend fundamental concepts in many seemingly unrelated technologies.

The implications and application of Taylor's theorem and Fourier analysis as foundational concepts has been successfully incorporated into several engineering and physics courses. In this paper the fundamental importance of these two theorems is discussed. A method that has been used to incorporate fundamental concepts into existing courses is reviewed. And finally, the foundation for a new course based on this approach, titled "Introduction to the Physics of Engineering," is discussed.

Introduction

Students typically begin taking core undergraduate science and technology classes during their junior and senior years. By this time, the students have usually taken the prerequisite mathematics and physics. A typical mathematics curriculum for physics and engineering students usually includes calculus, differential equations, linear algebra, multivariable calculus, and occasionally complex variables. The introductory physics curriculum usually includes general calculus-based-physics with an introduction to classical and modern physics. Based on observations made at several institutions, the unfortunate reality is that even students who have excelled in these introductory classes often have difficulty in applying the basic principles to upper level science and technology courses. This observed deficiency can be partially attributed to the slow maturation process that often accompanies the process of developing a command of any field of study. However, another important source of this deficiency is the lack of understanding, attention to, or appreciation by the students of the general underlying concepts that unify all scientific fields of study. This deficiency is starting to be addressed by several institutions[1, 2]. However, a significant amount of planning and intradepartmental coordination is required to affectively achieve a complete integration of the appropriate knowledge. Furthermore, the ability of these integrated approaches to adapt-to-change remains to be seen. Discrete modules that can be integrated into existing courses or new curriculum that focuses on the desired level of concept integration, such as the subsequent course proposal, may serve as an attractive alternative in many instances.

One method of visualizing the general dependence of engineering on physics and mathematics is illustrated in figure 1. In general, physics provides the theoretical foundation for all engineering fields of study while mathematics serves as the descriptive language of engineering. Although there are many possible methods of integrating the general unifying concepts that underlie all engineering fields of study, as illustrated in the figure, two of the many possible fundamental mathematical 'pillars' of engineering are Fourier analysis and Taylor's theorem. Both Fourier analysis and Taylor's theorem have an intrinsic beauty that is worth studying and both play a fundamental role in all fields of science and technology.

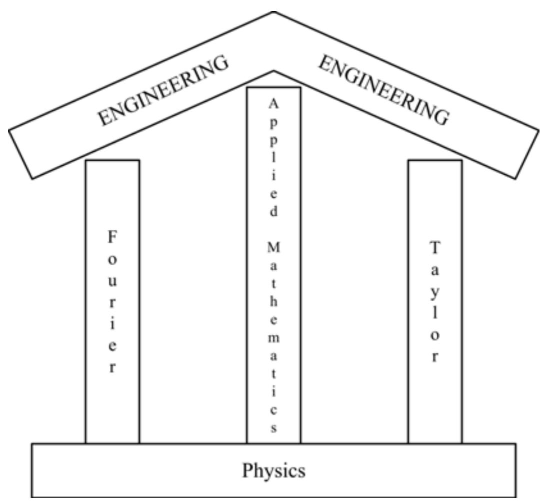

Figure 1. Theoretical foundation of engineering

Brief Review

Taylor's Theorem often does not receive the emphasis that it truly deserves. However, this theorem underlies many important aspects of engineering and science in general. Taylor's Theorem can be represented as[3]

$$f(x) = f(a) + f^{(1)}(a)(x-a) + \frac{f^{(2)}(a)(x-a)^2}{2!} + \ldots + \frac{f^{(m)}(a)(x-a)^m}{m!} + R_m(x) \qquad (1)$$

where $R_m(x)$ is the remainder of the truncated Taylor series and $f^{(n)}$ represents the n^{th} derivative of the function f. In the limit as $m \to \infty$, $R_m(x) \to 0$. $R_m(x)$ allows the error in approximating f(x) by a finite series to be bounded. $R_m(x)$ is given by

$$| \; R_m(x) \; | \le \frac{Q}{(m+1)!} | \; x - a \; |^{(m+1)} \qquad (2)$$

where $| f^{(m+1)}(x) | < Q$ for $| x\text{-}a | < d$. Taylor's theorem can also be generalized for multivariable functions[4].

Taylor's theorem has several practical implications. First of all, a reasonable function can be approximated to arbitrary accuracy by a finite polynomial series. Thus the concept of a polynomial orthogonal-basis-set (e.g. Legendre functions) can be associated with a Taylor polynomial. Furthermore, via $R_m(x)$, the error in this approximation can be bounded. Thus $R_m(x)$ is also useful for estimating errors in many practical situations. In most cases, the series is truncated after the linear term, $f^{(1)}(a)(x\text{-}a)$. Hence one of the more important implications of Taylor's theorem is that if $| x\text{-}a |$ is small enough and if the first derivative of $f \neq 0$, then variations in the function about x = a can be linearized. This permits the application of the immense power of linear algebra to problems that are not intrinsically linear. Another implication of Taylor's theorem is that to first order, relative changes in a function are always linear if $f^{(1)} \neq 0$. Taylor's theorem also paves the way for the discretization of differential equations. With the modern PC and software this opens up a host of applications that were unavailable several years ago. Finally, Taylor's theorem also provides the foundation for at least one method of teaching calculus or calculus-based physics[5].

The ubiquitous Fourier Transform (FT) and its cousin, the Fourier Series (FS), provide the foundation for an incredibly diverse range of subjects and applications. An understanding of both the FT and the FS is essential to a solid foundation in physics and engineering. The FT pair (FTP) can be expressed in several equivalent forms. The one-dimensional symmetric FTP can be expressed as[6]

$$F(f_x) = \int_{-\infty}^{\infty} f(x)\exp(-j2\pi f_x x)dx \qquad (3)$$

$$f(x) = \int_{-\infty}^{\infty} F(f_x)\exp(j2\pi f_x x)df_x \; . \qquad (4)$$

$F(f_x)$ is the FT of the spatial function f(x). The complex form of the FS is given as

$$f(x) = \sum_{n=-\infty}^{\infty} c_n \exp(j2\pi nx/P) \qquad (5)$$

where f(x) is periodic with spatial period P. In addition, equations 3 through 5 can be expressed in terms of time instead of the spatial variable x. These equations can also be extended to higher dimensions.

The implications of Fourier analysis are far reaching and the following brief discussion is by no means exhaustive. Historically the FS was developed to solve the heat-flow partial differential equation[7]. This naturally led to the development of using the FS (and subsequently the FT) as a means of solving differential equations. The remarkable discovery that periodic functions could be used to analyze non-periodic phenomena was initially very controversial but subsequently the approach was placed on a very solid mathematical foundation[8]. Later developments by many prominent researchers paved the way for such concepts as reciprocal spaces, orthogonal functions, alternative representations, and many additional integral transforms; e.g. the Wigner Transform. Even today, this field of study continues to develop. Hence an understanding of the basic concepts of Fourier analysis cannot be over emphasized.

Applications

Many practical examples exist for demonstrating the usefulness of Taylor's Theorem and Fourier analysis. Physical insights can be obtained from practical applications of these concepts. In this section several illustrative examples are given which demonstrate the power of applying Taylor's theorem and Fourier analysis. Many of these examples have been incorporated into the type of modules that are described later and should reinforce the importance of integrating and applying these concepts in the undergraduate engineering and science curriculums.

Example Applications of Taylor's Theorem in Physics and Engineering

Perhaps the most common example of a practical application of Taylor's theorem is found in the approximation, where for $|x|$ sufficiently small,

$$\sin(x) \approx \tan(x) \approx x \quad . \qquad (6)$$

This approximation occurs in optics, periodic motion, communication theory, Fourier analysis, and quantum mechanics (to name a few) and results in the linearization of several intrinsically non-linear phenomena. For example, geometric optics is based on this approximation. In physical optics, the image plane of an imaging system is often divided into spatial regions where the system is approximately linear; i.e. space-invariant. Such regions are termed isoplanatic patches[9]. These applications follow as an immediate consequence of Taylor's theorem. Each of these areas-of-study has associated equations that describe the general phenomena and simplifications that result when the systems are linearized. Several common functions are also readily approximated using Taylor's theorem. For example,

$$\begin{aligned} &\exp(\pm x) \approx 1 \pm x \\ &(1 \pm x)^{-n} \approx 1 \mp nx \qquad (7) \\ &\log_e(1+x) \approx x \end{aligned}$$

The error in these approximations can be bounded via equation 2. Mental approximations can be performed using approximations of the kind noted in equation 7. Integrals can be approximated with the judicious application of Taylor's theorem as well as asymptotic forms developed for complicated functions. Hence Taylor's theorem is useful in many situations where an approximation is needed or desired.

The incremental form of Taylor's theorem where h = (x-a) can be obtained by manipulating equation 1 with the result

$$f(x+h) = f(x) + hf^{(1)}(x) + \frac{h^2}{2!} f^{(2)}(x) + \frac{h^3}{3!} f^{(3)}(x) + \ldots \quad . \qquad (8)$$

In this form, Taylor's theorem can help students learn to think in a differential sense. Truncating this series after the linear term and rearranging results in the discrete form of the first derivative. Generalizing this result to higher-order and partial derivatives is the heart of the finite-difference method for solving partial differential equations[10] as well as the Runge-Kutta method of numerically solving differential equations[11]. The incremental form of Taylor's Theorem is also the starting point for many root finding algorithms with one of the most common being Newton's method [11].

Another common example of the linearization of an intrinsically non-linear system is found in electronics. The analysis of many analog circuits is routinely separated into a DC (Direct Current) analysis followed by a small-signal AC (Alternating Current) analysis[12]. The details of this process are readily determined by inspecting equation 8. For example, if $I_{out}(t)$ represents the circuit response to an input voltage $V_{in}(t) = V_{DC} + V_s\sin(\omega t)$ where $I(t) = f(V_{in}(t))$, then the output response of the circuit can be approximated as $I_{out,DC} + I_s\sin(\omega t)\bullet f^{(1)}(V_{DC})$. $I_{out,DC}$ and $I_s\sin(\omega t)\bullet f^{(1)}(V_{DC})$ respectively correspond to $f(x)$ and $h\bullet f^{(1)}(x)$. The results of this reduction are the linearization of the circuit output response, segmentation of circuit analysis into manageable parts (i.e. separate DC and AC analysis), and the corresponding simplification of the analysis.

Finally, there has been considerable discussion on various methods of teaching calculus and introducing calculus into the physics curriculum. Zvonko[5] makes a strong case for using the finite-difference method as a means of teaching calculus and introducing calculus into the introductory and intermediate physics curriculum. This approach is clearly based on Taylor's theorem and readily implemented using popular software. Ortega[13] shows several examples of how Taylor's theorem is used as a tool for integrating calculus into dynamics.

These examples hopefully illustrate several of the important ways in which Taylor's Theorem provides an important foundation for the sciences and serve as one possible vehicle for integrating Taylor into the physics and engineering curriculum.

Example Applications of Fourier analysis in Physics and Engineering

There are many applications of Fourier analysis. Any attempt to give 'typical' illustrative examples runs the risk of trivializing the far-reaching consequences of this method. Figure 2 is offered in an attempt to illustrate some of the applicable areas where Fourier analysis is used. Clearly the impact of Fourier analysis is significant. This is why the maturation process of learning Fourier analysis needs to begin early in the undergraduate mathematics, engineering and science curriculum.

Two fundamental examples of how Fourier analysis has changed our understanding of the universe are found in the Heisenberg uncertainty principle (HUP) and the concept of the frequency domain. The HUP is one of the core principles of quantum mechanics and expresses the concept that "uncertainties arise from the quantum structure of matter"[14]. Basically the HUP implies that it is fundamentally impossible to simultaneously measure the exact momentum p_x and position x of a particle. This can be expressed as

$$\Delta p_x \Delta x \geq \hbar / 2 \quad . \tag{9}$$

The HUP arises in a natural manner from the wave-nature of matter and is a direct result of the FTP relationship between momentum and position. Hence what is possibly a very mysterious aspect of matter under any circumstance arises as a natural consequence of Fourier analysis. Indeed, the HUP in-and-of-itself is fundamentally a consequence of the FTP relationship between position and momentum space.

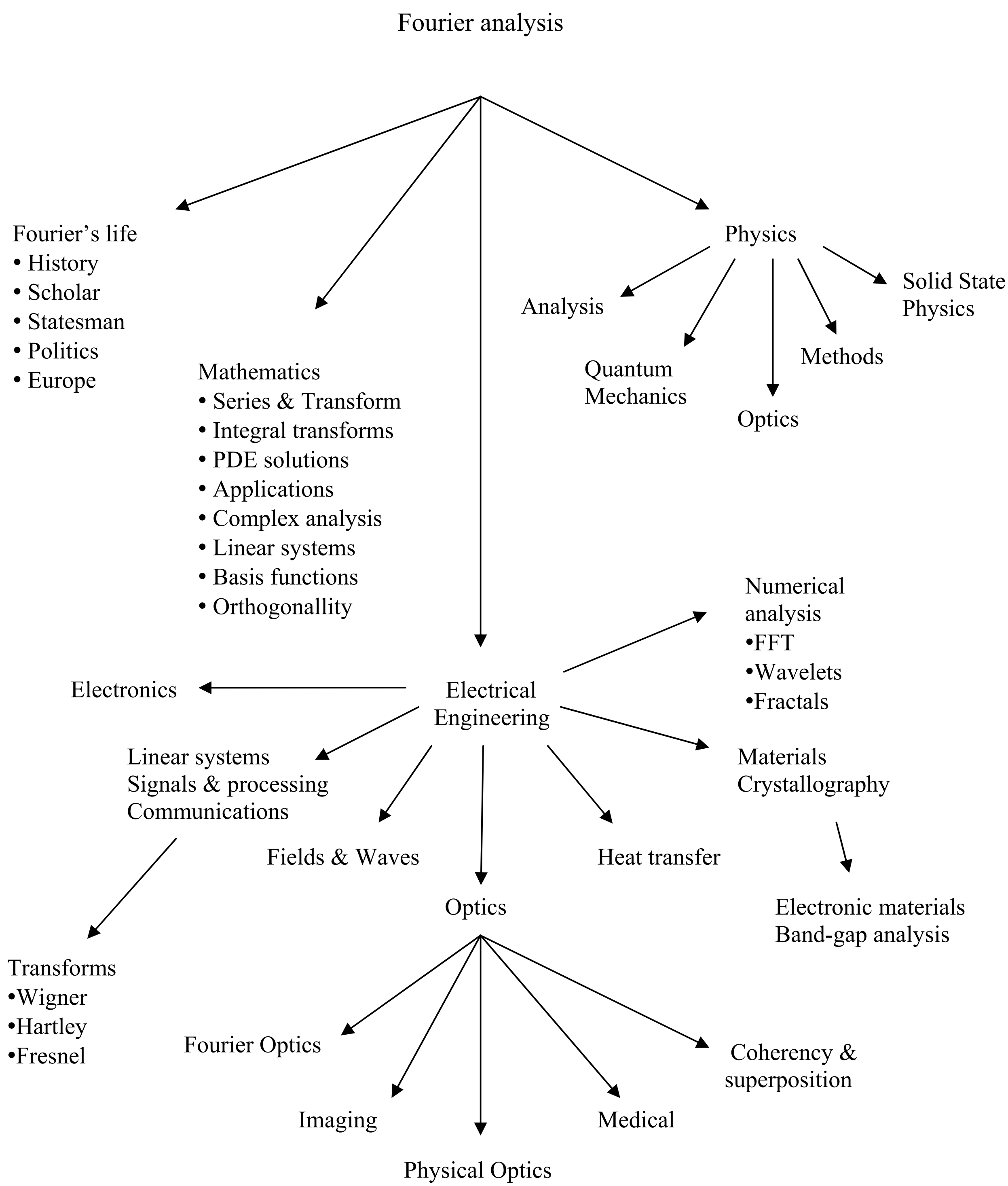

Figure 2 Application examples of Fourier analysis

Another example of Fourier analysis that is closely related to the HUP is found in the Fourier analysis of optics[9]. The introduction and application of spatial frequency domain concepts via

Fourier analysis to optics, resolution, and image processing in general can be directly linked back to physics via Maxwell's equations and the analogous relation between position and momentum in quantum mechanics. Although students rarely encounter difficulty in comprehending the reciprocal space relations between time and temporal frequency, the reality of analogous distance and spatial frequency concepts often eludes many students. Hence the application of Fourier analysis to spatial functions often results in a deeper appreciation and understanding of the transform. Optics readily provides a means of visualizing these relations. Finally, since the Fourier transform is a linear transform, Fourier analysis also provides an excellent format for reinforcing the fundamental concepts of linearity and linear algebra.

Tools

Many software packages exist that are useful for integrating and applying Taylor's Theorem and Fourier analysis into a technical curriculum. Several of the more popular software packages include Matlab®, Mathematica®, and EXCEL® [15, 16, 17]. A key-word web search on any of these software packages will lead to many additional resources. In particular, the text by Bloch[18] is an example of a resource that can serve as a vehicle for helping students integrate and apply the foundational principles of mathematics and physics.

Outline of "Introduction to the Physics of Engineering"

As noted earlier, a course specifically design to address the integration of information from various fields of study is an alternative to integrating and coordinating the curriculum of several departments. The initial target groups for this class proposal are undergraduate senior-level physics and engineering students. Seniors will typically have completed the majority of their core curriculum requirements in their particular field of study as well as the associated mathematical requirements with the consequent expectation of appropriate technical maturity. The topical format for the class is based on the model illustrated in figure 1 where physics provides the foundation of the particular topic-module while mathematics provides the support for the particular application. During the course, topics will cover multiple technical disciplines with the intent of broadening the background of the students as well as providing insight into the interdependencies of multiple fields of study. Several examples of modules illustrating this approach follow. These examples and others have been successfully integrated into existing curricula.

The conceptual algorithm for developing modules that reinforce fundamental concepts is outlined in table 1. The algorithm is generic and clearly not limited to Taylor's theorem or Fourier analysis applications. Consequently this algorithm can be used to develop modules that illustrate the integration of fundamental concepts in almost any fields-of-study. The examples that follow are intended to illustrate how several of the fundamental concepts underlying the various technologies can be integrated into existing curricula. The selected examples arbitrarily address applications found in electrical and mechanical engineering.

Table 1. Module Development
a. Select an application emphasis
b. Develop a description of the application
c. Prepare data appropriate to the application
d. Review the underlying physics and equations of the application
e. Review the applicable background mathematics
f. Apply the mathematics to the physics underlying the application with software tools to model and present the application data
g. Compare theory with experiment and analyze the results and implications

Example 1

a. Electrical Engineering
b. Lumped Circuit DC and AC Analysis
c. Ohm' law and results from simple electrical network analysis
d. Underlying physics
   - Newton's laws
   - Elementary statistical mechanics
   - Maxwell's equations
   - Linear and nonlinear system approximations with no time delays
e. Taylor's Theorem review; 'Poor-man' review of Fourier analysis via complex-variable phasors
f. Reduction of equations to appropriate circuit models and small signal equations
g. Comparison of theoretical and experimental data
   - Circuit analysis
   - Small signal diode response

Example 2

a. Thermal-Mechanical Engineering
b. Heat flow
c. Spatial dependencies of various steady-state heat flow
d. Underlying physics
   - Energy conservation
   - Elementary thermodynamics
   - Heat equation
e. Taylor's Theorem review for finite difference solution of heat equation; Review of Fourier analysis solution of differential equations
f. Reduction of heat equation to appropriate heat flow model; Laplace equation analysis
g. Comparison of theoretical and experimental data

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Conclusion

An example representing the manner in which engineering depends on physics and mathematics is illustrated in figure 1. In a practical sense, Taylor's theorem and Fourier analysis play a fundamental role in engineering and therefore common to all engineering disciplines. The application of these mathematical results enables students to appreciate and learn these relations as well as providing a foundation for integrating and applying physics and mathematics to engineering. An outline for developing instructional modules based on these concepts was developed along with several examples. Clearly there is always the need for improving the ability with which students integrate and apply their knowledge. This proposal hopefully serves as a useful step toward this goal.

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