



## A "Trick and Think" Approach to a Second-Order Circuit Lab

### **Dr. Ilan Gravé, Elizabethtown College**

Ilan Gravé received B.Sc. in Physics and Electrical Engineering and M.Sc. in Physics from Tel-Aviv University in Israel, and a PhD in Applied Physics from Caltech, in Pasadena, California (1993). In the past he has lead high-tech R&D avionics projects at the Israeli Aircraft Industries; has been a senior researcher and adviser at the Fondazione Ugo Bordoni, in the Ministry of Post and Communications in Rome, Italy; and has been on the faculty of the Department of Electrical Engineering at the University of Pittsburgh. He is currently an Associate Professor of Physics and Engineering at Elizabethtown College in Pennsylvania. He has on his record numerous publications in a number of fields in Applied Physics and Engineering, including superconductivity, semiconductor quantum devices, nonlinear optics, semiconductor lasers, infrared detectors and signal processing of medical signals.

### **Dr. Tomas Estrada, Elizabethtown College**

Dr. Tomas Estrada is an Assistant Professor in the Department of Engineering and Physics at Elizabethtown College, in Elizabethtown, PA. He received his B.S. in Electrical Engineering from Universidad de Costa Rica in 2002 and his M.S. and Ph.D. (both in Electrical Engineering) from the University of Notre Dame in 2005 and 2009, respectively. His research interests include control systems, engineering education, technology-related entrepreneurship, and sustainable engineering applications.

## A “Trick and Think” Approach to a Second-Order Circuit Lab

As instructors, we always look to engage students in a way that keeps them alert, stimulates their attention and interest, and adds some elements or insights to their skills. This is important in a first circuit lab course<sup>[1]</sup>, where students are struggling with many obstacles, such as understanding and performing correct circuit analysis, building the circuit on a breadboard, simulating it with appropriate software, and measuring relevant circuit parameters. One demanding task in a first circuit analysis lab course is the study of second-order circuits<sup>[2-6]</sup>. The analysis of an RLC circuit, involving second order differential equations and different regimes with under-damped or over-damped solutions is very demanding to sophomores, and the subtleties of understanding and then designing circuit performances as needed by tuning values of the different components in the circuit might be lost or overlooked.

In this paper we present one way to make a second-order circuit lab more lively, interesting and stimulating. In what might be called a “trick and think” approach, the students are provided with a pre-lab task to theoretically calculate and simulate the behavior of an RLC circuit. More specifically, the students are provided with a lab handout that includes the following guidelines for prelab calculations/simulations and for lab measurements:

1. In the lab you will build the circuit shown below using the solderless breadboards, resistors, jumper wires, and DC power supply.

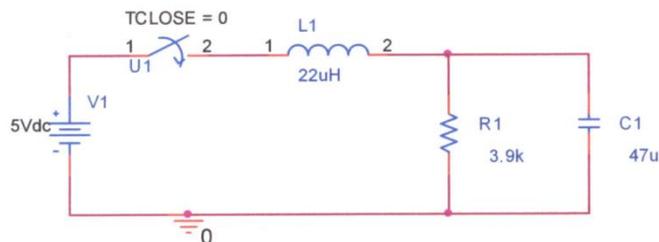


Figure 1: The RLC circuit considered

2. In the prelab, calculate the voltage across the capacitor and the current through the inductor, showing all of your work. Estimate the nature of the response and all the parameters of the transient response.
3. Simulate the circuit with PSpice.
4. Using the oscilloscope and the trigger single function measure and record the transient phenomenon of voltage build-up across the capacitor. Be sure to include a printout of the data captured from the scope in your laboratory notebook. Use cursors and scope measurements and displays to experimentally extract all relevant parameters.
5. Compare your experimental results with your calculations and discuss errors or discrepancies.

Most students performing these tasks for the prelab come with an analytic solution that matches their PSpice simulation; both indicate that the system response is heavily underdamped. Shown below in Figures 2a and 2b is, for example, such a PSpice simulation for the RLC circuit in

Figure 1.

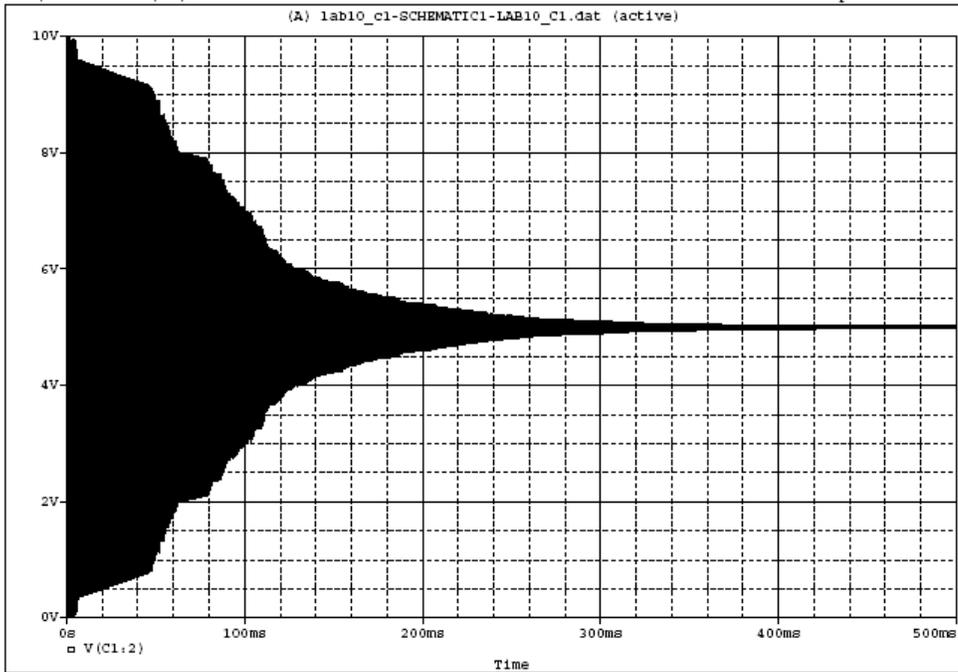


Figure 2a: PSpice simulation for the circuit in Figure 1, showing a heavily underdamped response for the voltage on the capacitor from 0 to 500 milliseconds.

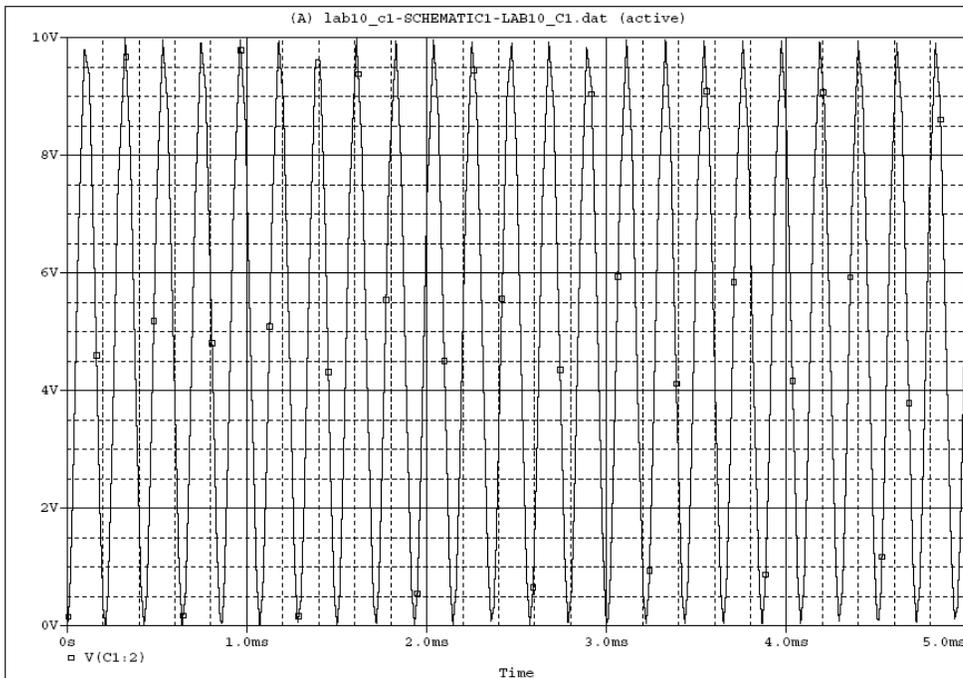


Figure 2b: Details of the heavily underdamped response obtained by expanding the time axis in Figure 2a and showing the first 5 milliseconds of the response.

To their surprise, upon monitoring the circuit response by measurement of the voltage on the capacitor, the result does not match their prelab calculations and/or simulations. The response appears overdamped. Figure 3 shows the response as captured on the scope by measuring the voltage across the capacitor.



Figure 3: The obviously overdamped response of the RLC circuit shown in Figure 1 as measured and captured on the oscilloscope in the lab.

Most lab teams are puzzled and usually their first step is to review their analytical solutions as well as their simulation files. Next, upon confirming their original results, they discuss them with other lab groups. Upon noting similar contrast between calculations/simulations and measurements they refer their puzzlement to the instructor.

At this time we elicit a class discussion asking for ideas what can be the cause of the mismatch between theoretical/simulation results and measured ones.

Usually the first suggestions from the students address possible tolerance of the RLC components since all elements are usually specified within a 5% tolerance. We then proceed to make a precise measurements of the value of each component and run solutions and simulations with the corrected values; this however does not change the general outlook of a very heavily underdamped calculated/simulated response.

At this stage the instructor usually suggests to review some theoretical understanding and consider the role of the resistor and its damping effect in both parallel and serial RLC circuits. This triggers some thoughts and discussions and the students realize that for a parallel circuit, a “large” resistor brings the circuit behavior close to an ideal LC circuit, so one should expect the response to be heavily underdamped. (“Large” resistor with respect to relevant impedances of the storage elements will result in little current flowing through the parallel resistive branch, so that

little energy will be dissipated at each cycle.) A serial RLC circuit is another story, and even a moderate resistance will be felt much more significantly in the energy balance of every cycle of the eventual damped oscillations. At this point some bright student sometimes jumps in with a suggestion in the right direction, stating that perhaps we have neglected something in describing the real circuit layout – maybe the resistance of the connecting wires and/or the internal resistance of the voltage source? (In fact the connections are made with long wires set on purpose to carry a resistive loads around 0.5 to 1 Ohms, and some fractions of Ohms up to a possible few Ohms are added by the internal resistance of the power source.) In any case, we eventually suggest the lab groups to consider simulating the following circuit in Figure 4, where a second resistance is added, thus mixing the parallel nature of the original RLC circuit with a serial resistive addition<sup>[7]</sup>.

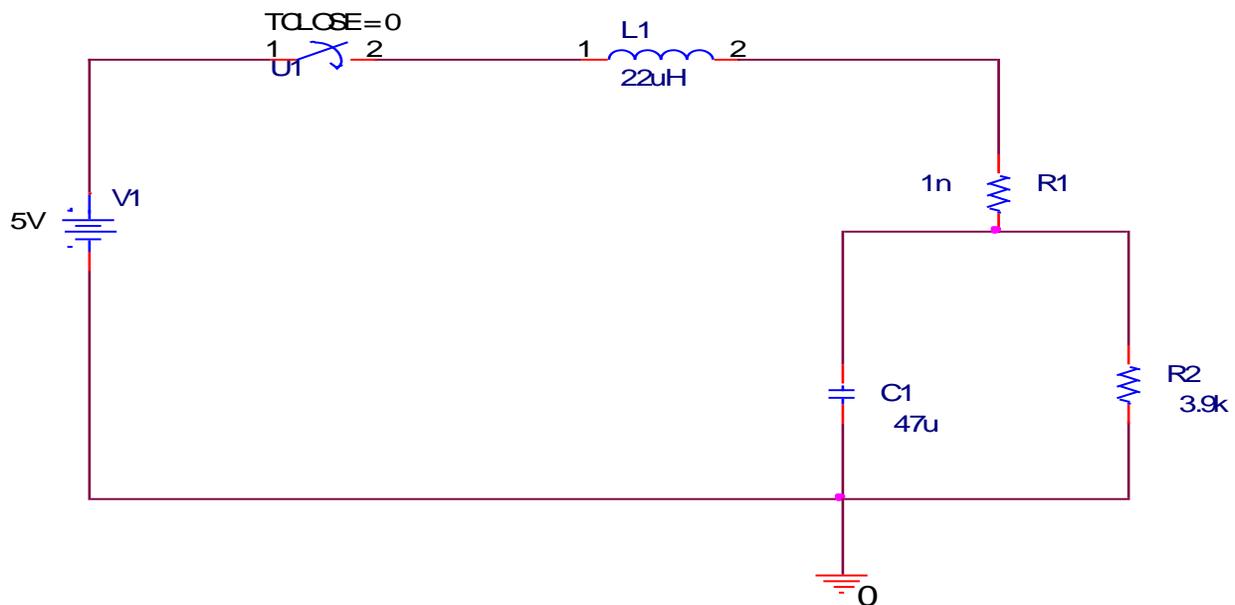
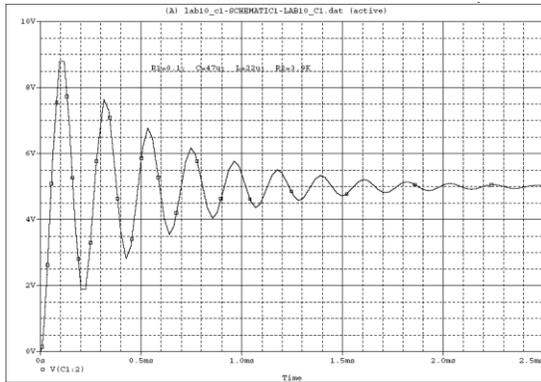


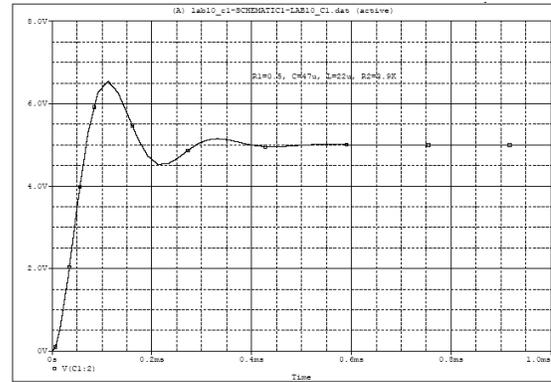
Figure 4: The RLC circuit with the added serial resistor R1 is initially set to 1 nano-ohm to mimic the circuit with parallel only nature, and is later varied to try and match the measured response. (Notice the relabeling of resistors R1 and R2 with respect to Figure 1.)

The students go back to calculate and simulate the RLC circuit with the addition of the serial resistance R1, while varying its value from basically zero (at 1 nano-ohm) to fractions of Ohms, a few Ohms and tens of Ohms.

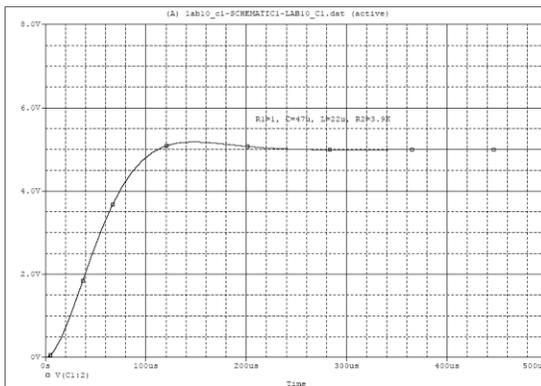
Figure 4 displays results of some of these PSpice simulations, for different values of the serial resistor, now labeled R1. A simulation with the value of R1= 1 nano-Ω (as laid out in Figure 4) basically recovers the heavily underdamped response shown in the simulation of the purely parallel RLC circuit in Figure 2, and is not repeated in the the displays of Figure 4. Instead simulated responses with (a) R1=0.1, R1=0.5, R1=1, and R1=3 reveal the sensitivity of the mixed RLC circuit to the serial resistance in transiting from an underdamped to an overdamped response as R1 is increased.



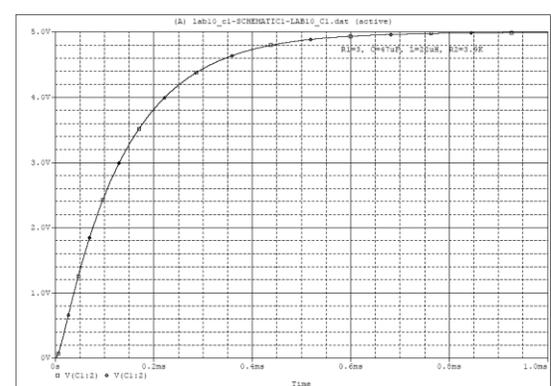
(a)



(b)



(b)



(d)

Figure 5: PSpice simulations for the circuit of Figure 4 with  $R_2=3.9\text{K}\Omega$ ,  $L_1=22\mu\text{H}$ ,  $C_1=47\mu\text{F}$ , and varying  $R_1$  to: (a)  $R_1=0.1\Omega$ ; (b)  $R_1=0.5\Omega$ ; (c)  $R_1=1\Omega$ ; (d)  $R_1=3\Omega$ .

At this stage usually the lab students are elated in having solved the “puzzle” of the contrast between theoretical or simulated calculations and the actual circuit response as measured in the lab. This is also usually the time when suggestions, comments and relevant interesting questions start to freely flow from different students in a spontaneous braistorming sessions.

“Wow! Can we actually find the serial resistance in the circuit –coming from wires and internal resistance of the voltage source- by matching the measured response to the best fitting simulated response when varying  $R_1$ ?” ( And later on they do that in their comprehensive lab report. The total series resistance is slightly different for each lab team due to small differences in wires and power supply resistances, but they usually fall between 1 to 3 Ohms.)

“Can we do anything in order to observe an underdamped response in our RLC circuit, since those hidden serial resistance are there and cannot be taken away?” (Now some of the students usually find the answer: “Maybe we can change the values of the capacitor or the inductor?”) We take advantage of that burst of creativity and good thinking by asking all groups to simulate and measure the response of the circuit with different values of the storage elements. For example, replacing the original  $47\mu\text{F}$  capacitor with a  $0.1\mu\text{F}$  element, and keeping  $R_1=3\Omega$  typical of the serial hidden resistances, yields an underdamped response, shown in Figure 6.

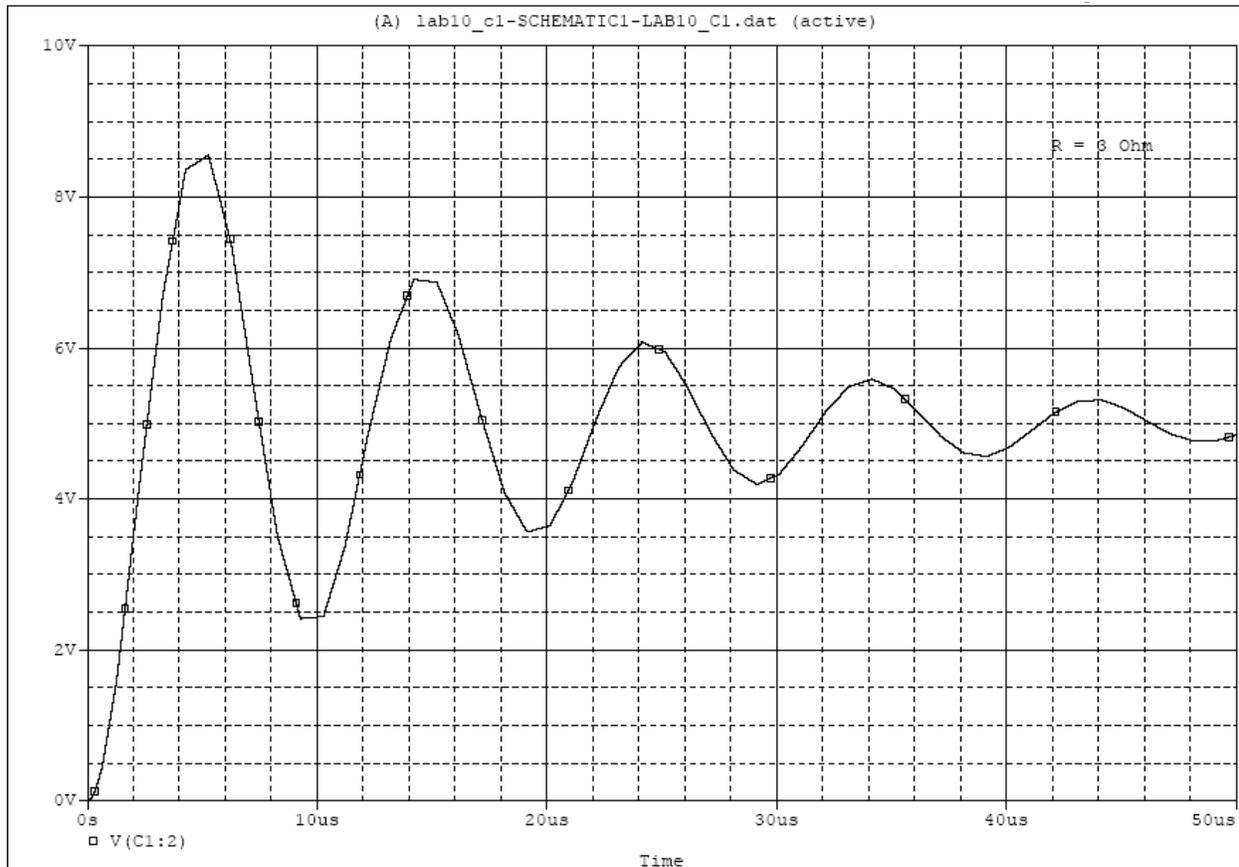


Figure 6: Simulation of the RLC circuit with  $R_1=3\Omega$ ,  $R_2=3.9K\Omega$ ,  $L_1=22\mu H$ ,  $C_1=0.1\mu F$ . A similar fitting response was also directly measured.

From here this lab usually moves to conclusion with a discussion of the expected magnitude and rate of damping, and the students try to refer the damping to the ratio of the energy dissipated at every cycle (mostly related to the value of  $R_1$ ) to the energy stored in the system (mostly dependent on the magnitudes of  $C$  and  $L$ .) This analysis and the connected calculations are then performed and expanded in the final report to be submitted one week after completion of the lab exercise.

Students usually significantly benefit from this interactive “trick and think” approach, and their understanding of different details and subtleties of RLC circuits is greatly enhanced.

While it is usually difficult to evaluate and assess the contribution of a single lab exercise to the development and/or enhancement of different skills for the students, in this particular case we can offer some considerations and even report some loosely acquired quantitative estimates. This is due to two principal reasons: first, we have been teaching this circuits lab for many years at our institution, and during many yearly offerings we have tried different approaches, including the one described above in this paper. An alternate approach was sometimes used, where the measured circuit was chosen with component values far away from the sensitive edge of critical damping. In this alternate circuit the hidden resistances would not significantly influence the response and the students would calculate and simulate the circuit in the prelab, and then

measure essentially the same response during lab time. Second, this lab is part of a Circuit Analysis course, and homework exercises are assigned once or twice per week on the relevant topics learned in the course. A couple of homework assignments on RLC circuits are usually assigned immediately after the material is covered in class and also after the lab is performed. As such the instructors could evaluate the students' solutions to those homework problems, and assess and compare their performances in different years characterized by different lab handouts.

To be more specific, the assessment was obtained by including in one of the weekly homework assignments two questions carefully crafted to test the students' skills in both solving a complex RLC problem and designing a circuit to comply with performance specification. This homework was always assigned after the students had completed the associated RLC lab, including a final written analysis and report. The class grade average on these two specific questions was used as an estimator of the students' improved skills in understanding and designing RLC circuits. Overall one could note an improvement between 15% and 25% percent in the overall grades in these RLC homework assignments for the years when the "trick and think" approach was used, as contrasted with years when the same lab was performed with component values far from the critical transition, and, as a consequence, the students were not exposed to the subtleties of the circuit and to the discovery process of the hidden parameters, as described above.

In summary, we do believe that this approach helps in catching the students' interest and motivation during this RLC lab exercise, resulting in a better understanding and mastering of the topics considered; and also in a better understanding of what should be the interplay between theoretical preparation and practical alert and focus for possible additional hidden considerations during experimental work and measurements.

### Acknowledgment

While we developed this "trick and think" approach independently over many years in teaching the Circuit Analysis course and lab, we would like to acknowledge a comment by one of the reviewers who pointed out that a similar exercise – featuring a hypothetical interactive exchange between a professor and a student<sup>[8]</sup> – can be found in the text book "Engineering Circuit Analysis" by Hayt and Kemmerly, McGraw-Hill publishers, since the third edition (1978.) We have used three or four textbooks for these courses over the years, but had never been exposed to Hayt and Kemmerly. (We found out that the original authors have deceased, and a third author, Steven Durbin has joined the care and renewal of this textbook . The 2012 eighth edition still includes the mentioned exercise as an exchange between a professor and a student.) We are grateful to the reviewer, as this lead us to discover this excellent textbook, especially admirable in treating the chapters analyzing RLC circuits. The exercise mentioned therein addresses a measurement of the frequency response of an RLC circuit and the inconsistent results for the resonant frequency and Q-factor measurements with respect to theoretical predictions, as some hidden circuit features are not considered at first. The "trick and think" approach described above in this contribution addresses measurements in the time domain, where perhaps the disparity between underdamped and overdamped response offers a more dramatic and immediate visual effect.

## References

- [1] "Confidence-Building in a Circuits Course," by Ilan Gravé, in Proceedings of the ASEE 2005 Conference in Portland, OR, June 2005.
- [2] "Study of the phase relationships in resonant R C L circuits using a dual-trace oscilloscope," by Zdenek Hurych in Am. J. Phys. 43, 1011 (1975.)
- [3] "Electronic device of didactic and electrometric interest for the study of RL C circuits," by Angel L. Pérez Rodríguez, Juan José Peña Bernal and Benito Mahedero Balsera in Am. J. Phys. 47, 178 (1979.)
- [4] "On the use of local fitting techniques for the analysis of physical dynamic systems," by A Page, P Candelas and F Belmar in Eur. J. Phys. 27 273 (2006.)
- [5] "An Introduction to Electric Circuits," by James A. Svoboda, Richard C. Dorf, Wiley, 9th edition (2014.)
- [6] "Fundamentals of Electric Circuits," by Charles K. Alexander and Matthew N. O. Sadiku, McGraw Hill, 5th edition (2013.)
- [7] See, for example, chapter 7 in "Basic Engineering Circuit Analysis" by Irwin and Nelms, John Wiley and Sons, 10th edition (2011,) for a detailed solution of this circuit.
- [8] "Engineering Circuit Analysis" by Hayt and Kemmerly, McGraw-Hill, 3rd edition (1978,) pages 468-470, exchange between a student named Pat and a professor, Dr. Noe; and/or "Engineering Circuit Analysis" by Hayt, Kemmerly and Durbin, McGraw-Hill, 8th edition (2012,) pages 642-643, exchange between a student named Sean and a professor, Dr. Abel.