

## **AC 2008-207: ACTIVE AND COLLABORATIVE LEARNING EXERCISES FOR A FIRST COURSE IN FLUID MECHANICS**

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# ACTIVE AND COLLABORATIVE LEARNING EXERCISES FOR A FIRST COURSE IN FLUID MECHANICS

## **Abstract**

The education literature clearly shows that classroom instruction that requires students to actively participate is superior to the teacher-centered lecture mode of instruction. Moreover, instructional activities that require student interaction and collaboration also promote learning. The superiority of active and collaborative learning to traditional methods applies to any number of measures.

To implement active and collaborative learning strategies in a junior-level fluid mechanics class, the authors have developed a number of in-class exercises. These exercises range from activities that consume a large portion of a class period to those that require just a few minutes, or less. Although many instructors may see the benefits of active and collaborative learning strategies, they may be reluctant to use them in their classes because they lack information on how to apply them to specific mechanical engineering subjects. Here we present twenty-three in-class exercises useful for instruction in a first course in fluid mechanics. The attributes of each exercise are delineated. These attributes include the approximate amount of class time required, the degree of collaboration involved (individual effort, pairs, or groups of three or four students), the educational objectives, and the specific subject area(s).

Survey results show that our students are highly receptive to these collaborative learning exercises, welcoming them over a traditional lecture format. We also show that these exercises may be adapted readily by others and present limited evidence illustrating their effectiveness in improving student learning.

## **Introduction**

We follow Prince<sup>1</sup> and define active learning as a classroom activity that requires students to do something other than listen and take notes. In such activities, students respond to a situation presented by the instructor by writing, sketching, discussing, formulating, solving, or responding in some other designated way. We further adopt Prince's<sup>1</sup> definition of collaborative learning as an instructional method that requires students to interact in some way to achieve a common goal.

A wealth of information exists showing the effectiveness of both active and collaborative learning in achieving a wide range of educational outcomes.<sup>1-5</sup> Prince provides an excellent summary.<sup>1</sup> For example, research shows that active learning is superior to the traditional lecture approach in the following measures:<sup>1, 6-9</sup>

- Short-term retention of subject matter
- Long-term retention of subject matter
- Conceptual understanding
- Positive student attitudes
- Motivation for further study

Improved educational outcomes associated with collaborative learning over individual or competitive learning include the following:<sup>1, 8-10</sup>

- Academic achievement
- Quality of interpersonal interactions
- Self esteem
- Student activities
- Retention in academic programs

These results provide a compelling case for introducing active and collaborative activities into engineering courses generally.

A primary purpose of this paper is to present a number of active and collaborative exercises that have been specifically developed over a period of years for a first course in fluid mechanics. To illustrate the ease with which others can adapt and use these exercises, we also discuss how these exercises have been adapted and used by the second author.

### **Overview of Exercises**

To provide the reader with an overview of the 23 exercises, we have categorized each with respect to several characteristics in Tables 1-4. These characteristics include class time required (Table 1); the specific nature of the exercise, e. g., the reinforcement of a definition, the development of a procedural skill, etc., (Table 2); the specific fluid-mechanics topic, e.g., mass conservation, momentum conservation, etc. (Table 3); and, finally, the degree of student collaboration required (Table 4).

**Table 1. Class Time Required for the Exercises**

<b><u>Time (minutes)</u></b>	<b><u>Exercise No.</u></b>
< 1	3, 7, 20
1 – 2	6
2 – 5	1, 2, 8, 13, 17-19
5 – 10	10, 15, 16
10 – 20	5, 9, 21, 23
> 20	4, 11, 12, 14

From Table 1, we see that ten of the exercises require 5 minutes or less, with three exercises requiring less than a minute. Four of the exercises require 20 minutes or more. Two of these lengthy exercises (Nos. 4 and 11) are relatively complex examples (Table 2) and deal with mass and momentum conservation, respectively (Table 3). The other two lengthy exercises (Nos. 12 and 14) involve complex derivations and the development of procedural skills, respectively (Table 2); topics dealt with are the application of momentum conservation to a differential control volume and a dimensional analysis of the Navier-Stokes equation, respectively (Table 3).

**Table 2. Specific Nature of the Exercises**

<u>Category</u>	<u>Exercise No.</u>		
Definitions	3, 10, 18, 22		
Exploration or Discovery	2, 4, 15, 17, 19, 20, 22		
Drill or Reinforcement	1, 7-9, 11, 13, 21, 23		
Procedural skills	4-6, 9, 12, 14, 16		
	<u>Simple</u>	<u>Intermediate</u>	<u>Complex</u>
Derivations	17, 18	14, 16, 21	5, 12
Examples	13	1, 2, 8, 9, 23	4, 11

**Table 3. Specific Topics Treated by the Exercises**

<u>Subject Area</u>	<u>Exercise No.</u>	<u>Subject Area</u>	<u>Exercise No.</u>
Control Volumes	1, 2	Navier-Stokes Equation	13, 14, 16
Mathematical Concepts	3, 6, 7, 10-13, 15, 16, 20, 21	Dimensional Analysis	14, 22
Mass Conservation	4-7	Internal Flows	15-18
Rigid Body Acceleration	8, 9	Flat-Plate Boundary Layers	19-21
Momentum Flow	10	Other External Flows	22
Momentum Conservation – Integral Control Volumes	11	Mechanical Energy Equation	23
Momentum Conservation – Differential Control Volumes	12, 13		

Table 3 provides a point of entry to the exercises by topic, which should be useful to readers desiring to adapt some of these exercises for their classes. In general, the exercises were developed to deal with specific points that students fail to remember or have a difficult time grasping. Although the exercises do not deal with every topic one might encounter in a first course in fluid mechanics, their coverage is reasonably comprehensive.

**Table 4. Student Collaboration Associated with the Exercises**

<u>Type of Effort</u>	<u>Exercise No.</u>
Individual	2, 3, 6, 7, 10, 13, 18, 19, 20
Individual followed by consultation with neighbors	1, 8, 15-17
Pairs	5, 9, 12, 21, 22
Teams of 3	4, 11, 14, 23

Table 4 illustrates the degree of student collaboration associated with each exercise. As implemented by the authors, somewhat less than one-half of the exercises are designed for individual effort; all of these exercises are short and fall into the 5 minutes or less category. An intermediate category of collaboration is for students to work individually at first and then compare and discuss their results with a neighbor. This is similar to the “think-pair-share” technique.<sup>5, 11</sup> Five of the exercises fall into this category (Table 4); however, many of the individual-effort exercises could be modified to add a share-with-neighbor component if desired.

In general, the more complex and lengthy the exercise, the greater the collaboration required. In implementing the exercises that require more than 5 minutes, the instructor typically walks around the classroom making sure that students are collaborating and on task, giving hints, and answering (or asking) questions. When it is recognized that many students are having the same problem, the instructor interrupts the class to get everyone on track. Most students work intently and interact vigorously with their partners. It is not unusual for the period to end without the students recognizing it is time to adjourn. Even after indicating that the period is over, many students continue to work.

To conclude some of the lengthier exercises, the instructor reconvenes the class to discuss the exercise before everyone has completed it. This prevents those who are finished from becoming bored and makes sure that everyone benefits from the exercise. This closure of the exercise frequently generates a lot of questions and discussion. This lively give-and-take clearly demonstrates the students’ engagement with the subject matter. Furthermore, seeing the students so engaged is highly motivating to the instructor.

### **Representative Exercises**

The first 5 of the 23 exercises are presented below. These exercises illustrate a range of characteristics: the shortest to the longest; all three of the levels of participation; and various

types: a derivation, several examples, a definition, one drill and reinforcement, and one procedural skills development. The topics considered include control volumes, mathematical concepts, and mass conservation. The remainder of the exercises appears in Appendix 1.

### **In-Class Exercise No. 1**

**Title:** Choosing Control Volumes to Study the Space Shuttle Main Engine

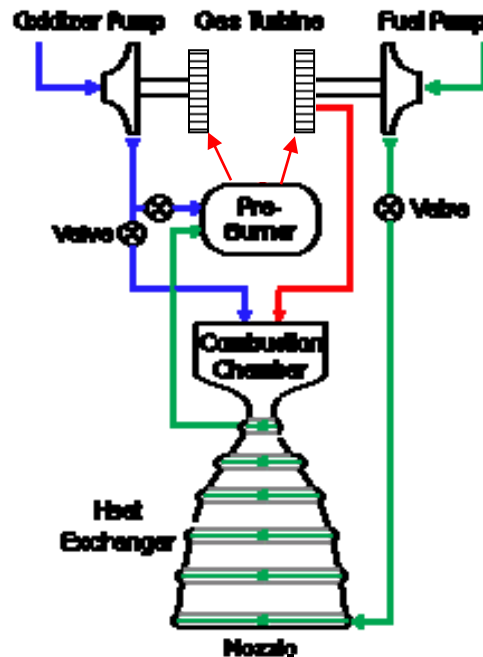
**Subject Area:** Basic Concepts – Control Volumes

**Duration:** ~5 minutes

**Participation:** Individual effort followed by consultation with neighbors

**Educational Objective:** To have students to discover the importance of control volume selection and the problem-dependent nature of such selection.

**Description of Activity:** Students are given a handout with a drawing of the basic components of the Space Shuttle main engine. See Fig. 1.



**Figure 1. Schematic of Space Shuttle main engine adapted from Reference 12 with permission.**

Using the drawing, the instructor briefly describes the operation of the engine. Students are asked to sketch on the drawing appropriate control volumes in response to the questions:

1. What if you wanted to find the oxidizer pump power requirements?
2. What if you wanted to find the power produced by the fuel pump turbine?

3. What if you wanted to find the temperature in the preburner?
4. What if you wanted to find the thrust produced by the engine during a static test on a test stand?

The activity is concluded by bringing the class together as a whole. The instructor asks for volunteers to describe their choices and the reasoning behind their selection. This can go quickly.

**Comments:** This activity builds upon students' previous work with control volumes in their study of thermodynamics. This activity provides a reinforcing link to previous knowledge.

### **In-Class Exercise No. 2**

**Title:** Choosing Control Volumes for Flow through a Pipe

**Subject Area:** Basic Concepts – Control Volumes

**Duration:** ~2 minutes

**Participation:** Individual effort

#### **Educational Objectives:**

1. To have students understand the importance of control volume selection and the problem-dependent nature of such selection.
2. To have students begin to think about the types of forces that are important in fluid mechanics, i.e., pressure forces and viscous (frictional) forces.

**Description of Activity:** This activity can be used in conjunction with Exercise 1. Students are shown a sketch of flow through a pipe (Fig. 2) and are asked to reproduce the sketch in their notes.



**Figure 2. Flow through a pipe.**

Students are asked the following questions:

1. What control-volume choices do you have for this situation?
2. What are the effects of your choice in each case?

The activity is concluded by bringing the class together as a whole. The instructor asks for volunteers to describe their choices and the reasoning behind their selection.

**Comments:** The instructor can also use this exercise to foreshadow topics covered later in the course, i.e., pipe flow and friction.

### **In-Class Exercise No. 3**

**Title:** Differential Area  $dA$  for Cylindrical Systems

**Subject Area:** Mathematics; Pipe Flow; Flow Rates

**Duration:** ~30 seconds

**Participation:** Individual effort

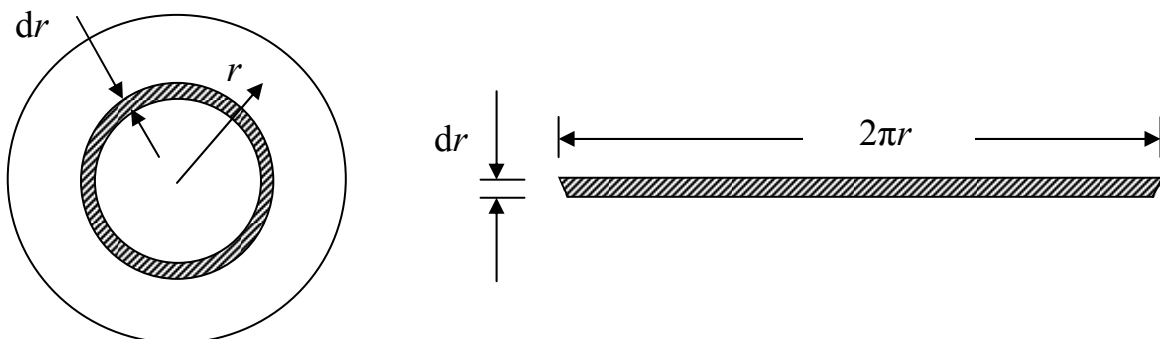
**Educational Objectives:**

1. To facilitate long-term retention of the fact that the appropriate differential area  $dA$  for integrating over the cross-sectional area of a pipe flow is  $2\pi r dr$ .
2. To help students become confident in working in cylindrical coordinate systems.

**Description of Activity:** The instructor defines the mass flow rate for a flow through a pipe having a circular cross-section as:

$$\dot{m} = \int_{A_{x\text{-sec}}} \rho u(r) dA$$

Students are asked, first, to sketch what might be an appropriate  $dA$  for this situation and, second, to provide an algebraic expression this differential area involving the radial coordinate  $r$ . After the students have had a chance to think and write, the instructor asks for volunteers to give their answers and to explain their reasoning. The instructor concludes the exercise by drawing a sketch showing  $dA$  and how this annular strip can be laid out linearly such that  $dA \approx \text{length } (2\pi r)$  times width ( $dr$ ). See Fig. 3.



**Figure 3. Differential control volume for pipe flow.**

**Comments:** This concept is used (and revisited) many times throughout the course.



### In-Class Exercise No. 4

**Title:** Unsteady Flow from One Tank to Another

**Subject Area:** Mass Conservation; Unsteady Flow

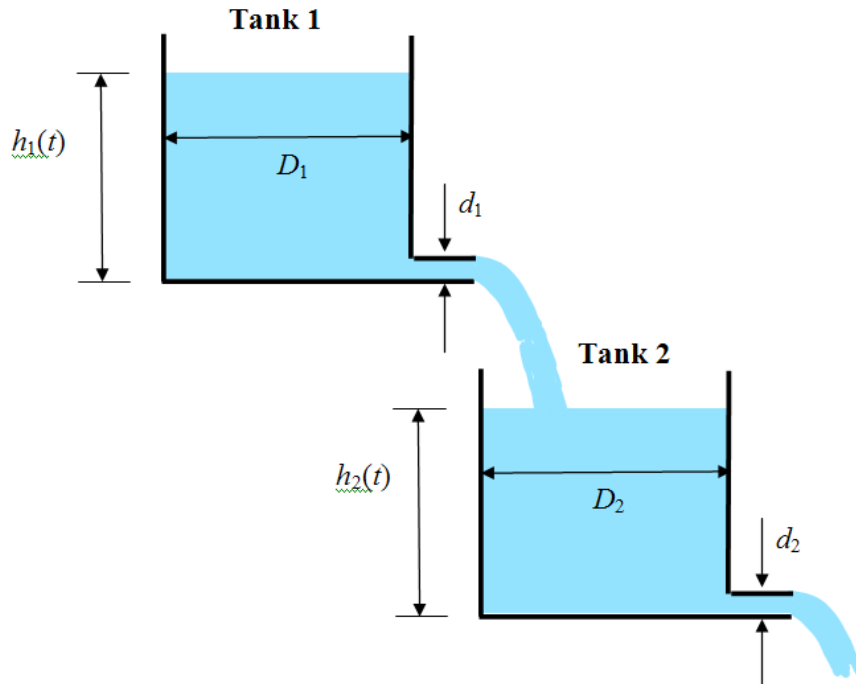
**Duration:** 20-30 minutes

**Participation:** Students work in groups of three. The instructor walks around the classroom offering help and clarification to make sure everyone is engaged and making progress.

#### **Educational Objectives:**

1. To help students understand and internalize the conservation of mass principle.
2. To have students develop confidence in their analytic capabilities.
3. To have students discover how ordinary differential equations arise in the context of unsteady mass-conservation problems.

**Description of Activity:** Students are given a handout that illustrates the filling of one tank from another (Fig. 4).



**Figure 4. Sketch illustrating unsteady flow from one tank to another.**

The sketch provides all of the important geometric parameters, shows important time-dependent variables, and the initial conditions. Expressions relating the exit velocities from each tank to the height of the liquid in the respective tank are also provided on the handout. The students are asked to formulate an analysis that allows them to determine the following:

1. An expression for  $h_1(t)$ .
2. An expression for  $h_2(t)$ .
3. An expression for the time to empty both tanks.

Students are given ample time to formulate answers, with students achieving various degrees of completion. The class is brought together with the instructor asking students about their approach as the instructor develops the solution.

**Comments:** The students appear to be fully engaged and ask many good questions both when they are working in their small groups and during the class-as-a-whole portion of the activity. Perceptive students also point out the need to deal in some way with the fluid in transit from one tank to the other.

### **In-Class Exercise No. 5**

**Title:** Mass Conservation – Differential Control Volumes

**Duration:** 15-20 minutes

**Participation:** Students work in pairs. The instructor walks around the classroom offering help and clarification to make sure everyone is engaged and making progress.

#### **Educational Objectives:**

1. To have students see the underlying physics (conservation of mass) in the continuity equation.
2. To have the students develop some comfort level with partial differential equations.
3. To have students connect their previous study of mathematics to its use in fluid mechanics.

**Description of Activity:** Students are given a handout that contains a sketch of a cubic control volume having dimensions  $\Delta x$ ,  $\Delta y$ , and  $\Delta z$ . The handout also contains the following items along with space for the students' work:

**Given:**  $V = u\hat{i} + v\hat{j} + w\hat{k}$

#### **Assume:**

- i. The mass flow through each face can be characterized by a local density and velocity on that face, e.g., at the  $x + \Delta x$  face, the mass flow rate can be expressed as  $[\rho u]_{x+\Delta x} \Delta y \Delta z$ .
- ii. The density within the control volume can be characterized by an appropriate average value, but the flow is not necessarily incompressible or steady.

**Objective:** Derive a partial differential equation (PDE) for mass conservation at a “point.”

**Task A:** Draw arrows to indicate a mass flow through each face. Write out a mass conservation statement for the control volume having dimensions  $\Delta x$ ,  $\Delta y$ , and  $\Delta z$ .

**Task B:** Group the *in* and *out* flows from the same coordinate directions as pairs. Do some dividing by delta quantities and take limits as these go to zero (or to the continuum limit). Discover your PDE.

After the students have been working an appropriate amount of time, the instructor convenes the class as a whole. The instructor then works through the exercise by interacting with the class, the final result being the continuity equation in Cartesian coordinates.

**Comments:** This exercise engages students and allows them to take ownership of the continuity equation. This exercise has been found to be much more effective than an instructor-centered derivation. After completing this exercise, it is a simple matter to present and discuss the general vector and cylindrical-coordinate forms of the continuity equation. Note that this exercise is easily modified to use a Taylor series expansion approach to the derivation.

### Ability of Other Instructors to Use Exercises

In the fall of 2007, we (the first two authors) taught separate sections of an undergraduate fluid mechanics course. I (LLP) was talking with some students from SRT’s section one day and they mentioned that they liked the way their section was being taught. I asked them for details, and they told me about student problem solving in class. I then talked with SRT and told him I was interested in learning more. He soon gave me a packet of samples (included in this paper) and invited me to sit in on his class to see how an exercise was implemented. At this point I was concerned about how I would add active-learning components, as I had not tried anything like this before. Would I lose control of the class? Would students stay on task?

I found my visit to SRT’s class to be very helpful. It showed me how to present the activities and how to guide the students. Watching a class also assured me that the class would not get out of control when students were given some time to work on the questions. Since you are not able to attend one of SRT’s classes, I will try to describe here what I observed.

At the beginning of class, SRT listed an outline of the lecture: Momentum conservation, momentum flow or flux, definitions, examples, integral CV. SRT then defined momentum flow in words, verbally and written:

Momentum flow  $\equiv$  flow of momentum across a control surface, i.e. , time rate at which momentum crosses boundary.

SRT asked, “So if we were to have a little quiz today, how would you define momentum?” He then gave the students a few seconds to think about their answer before he called on someone. This gave everyone a chance to come up with their answer before hearing an answer. SRT wrote

the momentum flow term using an averaged velocity and using an integration across the control surface. SRT then asked, “Is this a vector or scalar?” Again he waited a few seconds before looking for a hand to call on. He then presented Exercise 10 from Appendix 1. “Let’s take an example of flow through a pipe. At the inflow, let’s have a uniform velocity. (SRT starts drawing the sketch.) At the outlet we will have our laminar parabolic flow. (Draws outflow profile and writes the equation for the velocity profile.) The question I would like to ask is: What are the momentum flows at 1 and 2? Let’s see what you can do by yourself for a minute. If you get stuck, you can then talk with someone else. Let’s all get on the same page and use  $\vec{i}$  as the unit vector in the z-direction.” SRT then wrote:

Momentum flow at station 1 =

Momentum flow at station 2 =

SRT walked through the class to see how students were doing. He said, “If you think you have it, just wave at me.” Some students raised their hand. After one minute, some students started talking to their neighbors. After three minutes, SRT announced, “OK. Let’s reconvene. Let’s first look at the inlet.” He started with the momentum definition, asked for student responses and worked through the term. SRT then said “And now the next one is just a little tougher algebraically” and worked through the outflow momentum term, asking for student input. Since the students had time to think about the problem, there were many students who raised their hands with answers. SRT then derived the conservation of momentum equation and started Exercise 11 from Appendix 1. He said, “What I would like you to do is to work out this example.” He then drew the figure (Fig. 6) and announced, “Find the friction force, and I want you to follow the procedure on the handout. I think that it would be best to work in pairs.” Students pulled out the handout “Detailed Procedure to Apply Conservation of Linear Momentum to an Integral Control Volume” included in Table 8 of Appendix 1. The last ten minutes of class were used by the students to work on this problem. There was not time to discuss the problem as a class, but SRT told me that he started the next class with a discussion.

Observing SRT’s class showed me (LLP) how easily active-learning components can be added to a course. Almost any derivation or example problem can be used as an active-learning exercise. Giving the students a few minutes to work on the problem themselves before a class discussion allowed more students to be prepared to participate in the group discussion. It also allowed the students to check their solution and understanding of the problem. SRT’s walking through the classroom was important to keep the students on task. When he saw some students inactive, he walked up and asked how they were doing. This started the student working or caused the student to ask a question. Other students raised their hands as SRT walked past.

I now felt ready to try active-learning components in my classroom. After the first exam, I announced to the class that I would be adding active-learning components to the lectures. I presented a quote from Felder<sup>13</sup> to describe the benefits:

*“Teacher-centered instructional methods have repeatedly been found inferior to instruction that involves active learning, in which students solve problems, answer questions, formulate questions of their own, discuss, explain, debate, or brainstorm during class.”*

In that lecture, I introduced momentum analysis. I used SRT's Exercise 1 to discuss selection of a control volume. I presented the cross-sectional averaged momentum equation and then asked the class to find the reaction force for a curved nozzle given inflow and outflow velocities and pressures. I first drew a figure for the curved nozzle and then divided the analysis into the following three steps:

- Draw a CV used to find the force required to hold the nozzle on the pipe. Define a coordinate system. Identify the forces acting on the nozzle.
- Find  $Mom_x =$  and  $Mom_y =$
- Put terms together in  $x$ -direction and  $y$ -direction.

For each bullet I wrote the wording and discussed the step. After presenting each bullet, I stopped the class lecture for a minute or two and allowed the students to work on that step of the problem. I timed the minutes that I gave students since the time often seemed much longer than what actually transpired. After a minute or two, I asked students in the class to share their solution for that step of the problem. I then moved to the next bulleted step and continued the process.

For most lectures, I usually spent less than five minutes of class time on the active-learning activities, so I did not find a reduction in the material that I covered in a lecture. I found that it was important to walk around the class, especially to the back of the class since students who sat towards the back tended to prefer observing rather than participating in class. It was very easy for me to add the active-learning components. After using this method in a few classes, I started seeing every derivation or example as being an opportunity for active learning. In the lectures between the first and second exams, I included one to three active-learning activities in each class. I found that the shorter exercises worked better for my class. In longer exercises, it was more difficult to keep the class focused on the activity. Some students walked out of the class to use the restroom or get a drink during longer active-learning activities. I also found that example problems were more successful as active-learning activities. Derivations, such as the differential continuity equation (Example 5), were rather difficult for many students. I walked around the class helping individual students and also gave some general hints to the class along the way; however, most students were not able to work through the derivation, even when working with a neighbor.

Studies have shown that women students are more hesitant to answer questions without having time to thoroughly consider the problem. I noticed that the women students were particularly responsive to the active-learning activities and were more likely to raise their hand with input after they were given a minute or two to work on the problem.

I found my experiences in using active-learning components in my course to be enjoyable and successful. Walking through the classroom while the students worked on a problem allowed me better to gauge the students' understanding. It also allowed me to have an individual discussion with students who did not come to my office hours. Also, I hoped this would "break the ice" in some cases and encourage these students to come to office hours when a question arose. After the students worked on an active-learning exercise, I found that many more students raised their hands when I asked a question. In previous semesters, student participation in class was often

poor. To increase student participation in the past, I prepared simple questions to ask during the lecture, but asking these questions did not significantly improve student participation. Adding active-learning components in Fall 2007 required some planning before class, but not any more than I had done previously in preparing questions to ask the class. The resulting student participation, however, was greatly improved when I added the active-learning exercises. The authors encourage others to also try these simple techniques to improve student participation and student learning during class.

### **Student Perception and Performance**

Although this paper is not research-focused methodological study, we have collected some basic assessment data to support the use of the active learning activities. During the 2007 fall semester Some direct and indirect assessment data were collected in the two sections of fluid mechanics taught by the authors in the fall 2007 semester. The active learning techniques were used for the entire semester in section 1 by Instructor 1, who has been using active learning techniques for quite some time. In section 2, the active learning techniques were used for only the second half of the semester by Instructor 2, who had not previously used active learning. For assessment, we addressed the following questions:

1. How do students' perceptions of the course differ between the two sections?
2. What is the impact of active learning on student performance?

The data collection and results are discussed below.

#### **Assessment of Student Perception**

To assess students' perceptions of the active learning techniques, students in both sections were surveyed twice during the semester. Students completed an 11-item Likert-type scale in which they were asked to rate their perceptions of the course and their ability to learn the material. The items used a 5-point scale that ranged from *strongly disagree* to *strongly agree*, which was coded from 1 through 5 for analysis. Students were also asked to provide suggestions for changes to the course and to provide any other comments on the course. A copy of the survey is included in Appendix 2.

Here we discuss only the end-of-semester survey results. For this survey, 66 students completed the survey in section 1, and 45 students completed the survey in section 2. Table 5 displays the descriptive statistics for the end-of-semester survey. Independent *t*-tests are used to test for differences in the average item scores between the two sections.

Section 1 received more positive ratings for several of the questions, including Q1, Q3, Q6, Q8, and Q9. However, the averages for the items were quite positive in both courses. There were no statistical differences in the students' perceived ability to think through a problem in fluid flow (Q2), the perception of the course pace (Q4), the perceived ability to discuss problems presented in the course (Q7), the perceived opportunity to practice course material (Q10), and perceived helpfulness of the approach of material (Q11). It is important to note that the students in both sections rated the desire to be more actively involved with the course (Q8) as the lowest item.

**Table 5. Comparison of Survey Results by Section**

Item	<u>Section 1 – Active Learning Throughout</u>		<u>Section 2 – Active Learning Second Half</u>		<u>Comparison</u>
	Mean	Standard Deviation	Mean	Standard Deviation	<i>t</i> -Statistic ( <i>p</i> -value, degrees of freedom)
Q1	4.24	0.703	3.73	0.618	-3.93 (0.000, df=109)*
Q2	4.11	0.636	3.98	0.583	-1.08 (0.283, df=109)
Q3	4.14	0.875	3.49	0.815	-3.93 (0.000, df=109)*
Q4	4.12	0.734	3.86	0.632	-1.90 (0.060, df=108)
Q5	2.88	0.937	3.44	1.119	2.79 (0.007, df=109)*
Q6	4.15	0.614	3.76	0.712	-3.13 (0.002, df=109)*
Q7	4.29	0.650	3.68	0.829	-4.28 (0.000, df=108)*
Q8	2.82	0.910	2.89	0.832	0.42 (0.678, df=109)
Q9	3.92	0.975	3.47	0.894	-2.47 (0.015, df=109)*
Q10	3.02	1.030	3.02	0.892	0.04 (0.970, df=109)
Q11	3.94	0.839	3.69	0.763	-1.60 (0.112, df=109)

\* Indicates statistical significance at  $p < 0.05$ .

In addition to the student surveys, measures of teaching effectiveness were examined. Near the end of each semester, students at our university are asked to evaluate the teaching effectiveness in each of their classes. A standard Rating of Teaching Effectiveness (SRTE) instrument is used in these evaluations and consists of three parts: University Items, Department Items, and Instructor Items. A 7-point scale is the same for all items, with 1 being the lowest rating and 7 being the highest. In recent years, the first author has asked students to rate the use of the in-class exercises with the following instructor item:

*Rate the effectiveness of the in-class, active-learning exercises to the overall learning experience in this class.*

Table 6 shows the students' responses to this question for the first author's two most recent fluid mechanics classes. The fall 2007 class was a regular section with an enrollment of 79 students. Seventy-three students responded to the survey. The fall 2006 class was an honors section with an enrollment of 17. Fifteen students responded to the survey.

From the data in Table 6, we see that students thought that the exercises were quite effective in their learning of fluid mechanics. In both classes, over 80 percent of the students responding gave ratings of 5 or above. For the larger class, 66 percent of the responses were in the two highest categories. An even higher percentage (74%) rated the exercises in the two highest categories for the honors class. No students gave ratings in the two lowest categories. To

calibrate this rating scale for the reader, teaching effectiveness scores of 6 and higher are coveted ratings. Written student comments expressing enthusiasm for the exercises support the ratings shown in Table 6.

**Table 6. Student Rating of Effectiveness of In-Class Exercises**

Rating	<u>Number of Responses (%)</u>						
	1	2	3	4	5	6	7
Fall 2007 (N = 73) <sup>1</sup>	-	-	2 (3%)	9 (12%)	14 (19%)	32 (44%)	16 (22%)
Fall 2006 (N = 15)	-	-	2 (13%)	1 (7%)	1 (7%)	4 (27%)	7 (47 %)

<sup>1</sup> Some students misinterpreted the directions on the SRTE regarding the added question and wrote comments rather than provided the rating. Two raters examined each comment and assigned a conservative rating to it based on the adjectives used. These ratings are included in this data set.

The open-ended questions on the student surveys also shed some light on the impact of the activities. In section 2, before the active learning techniques were introduced, students made the following comments when asked what changes should be made to the course:

- Active learning via in-class activities really helps to keep my attention.
- More asking audience questions.
- I like the idea of a more interactive learning style. While I think there is a good balance between information covered and example problems, I don't feel like, given a fluid flow problem not discuss in class, I would necessarily know where to begin.
- Let the students think in class rather than just having a lecture.

Some students seemed to enjoy the active learning activities in both sections. The following are some student quotations from the surveys in both sections of the course:

- The class activities are beneficial to the learning process.
- I like the demonstration that [the professor] did in class. That really helped me to think about something related to real life.
- [The exercises] were very effective. It gave me a practical understanding of the material and equations presented.
- They made you think and kind of put you on the spot to see if you knew what was going on.
- I think the in-class, active learning exercises are extremely beneficial in aiding the learning process.

### Assessment of Student Performance

To understand the impact of the active learning techniques in section 2, in which active learning techniques were implemented for the first time, final exam scores were compared to those from a previous semester. Ninety percent of the items on the final exam were identical to those on the final exam used in the previous semester. Both exams were graded by Instructor 2, and the same partial credit point assignment was used. Table 7 displays the average and standard deviation for



the final exam scores during the two semesters when active learning was used and when it was not.

**Table 7. Comparison of Exam Scores during Semesters with and without Active Learning**

	<u>Exam Average</u>	<u>Exam Standard Deviation</u>	<u>t-Test (p-value)</u>
No active learning	78.94	11.92	-2.228 (0.024, df =109)
Active learning	83.87	10.75	

The independent *t*-test supports the conclusion that final exam scores were higher for the semester that the active learning techniques were utilized. Although this result supports the idea that active-learning techniques improved student performance, other uncontrolled factors may have influenced this finding, e.g., differences in the background levels of the students.

#### Summary of Assessment

Overall, the students had a generally positive perception of the techniques in both course sections. The results of the student survey show that students positively perceive the benefit of the class activities, the opportunity to be actively involved in the course, and the approach of the material. In addition, limited evidence suggests that active learning techniques had a positive impact on students' understanding of the course material.

#### Summary and Concluding Remarks

Active and collaborative learning techniques have been shown by many researchers to have a positive effect on student learning. Such techniques are making inroads in engineering education. To assist mechanical engineering educators in implementing such strategies, we presented twenty-three in-class exercises developed for instruction in a first course in fluid mechanics. These exercises were presented in detail and categorized to allow the reader to understand their various attributes and to ascertain how they might be implemented in the reader's classes. Although not discussed explicitly in the paper, an underlying objective in presenting these exercises is to show how easily active-learning exercises can be created to suit the needs on an individual instructor and his or her class.

We shared the experiences of the second author in successfully implementing active and collaborative learning for the first time, with the hope that this sharing of a highly positive experience might inspire others to adopt these strategies.

Survey results were also presented showing that students are highly receptive to these collaborative learning exercises, welcoming them over a traditional lecture format. We also presented limited evidence illustrating the effectiveness of these exercises in improving student learning in fluid mechanics.

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## Appendix 1 – Additional Exercises

### In-Class Exercise No. 6

**Title:** Simplifying the Continuity Equation for Special cases

**Duration:** 1or 2 minutes

**Participation:** Individual effort

**Educational Objective:**

1. To have students develop skill in simplifying mathematical representations of fluid flow.
2. To introduce the use of cylindrical coordinates.

**Description of Activity:** Students are asked to do the following:

1. Simplify the Cartesian-coordinate form of the continuity equation, given below, for an incompressible fluid.

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

2. Simplify the cylindrical-coordinate form of the continuity equation, given below, for a steady, two-dimensional  $(r, z)$  flow.

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial (r \rho u_r)}{\partial r} + \frac{1}{r} \frac{\partial (\rho u_\theta)}{\partial \theta} + \frac{\partial (\rho u_z)}{\partial z} = 0$$

**Comments:** This exercise introduces students to the idea that relatively complex, and possibly intimidating, equations can be easily simplified for many flows.

### **In-Class Exercise No. 7**

**Title:** Flows that Satisfy Continuity

**Duration:** ~30 seconds

**Participation:** Individual effort

**Educational Objectives:**

1. To illustrate one way that the continuity equation can be used.
2. To give students some practice in dealing with partial derivatives.

**Description of Activity:** Students are given a velocity field associated with the flow of an incompressible fluid and asked to determine if this flow satisfies the continuity equation. For example, a flow that does satisfy continuity is given by

$$\vec{V}(x, y, t) = (0. + 0.8x)\hat{i} + (91.5 + 2.5 \sin \omega t - 0.8y)\hat{j}.$$

The instructor can poll the class by show of hands, or other means, to see if the students were successful in solving the problem.

**Comments:** This simple exercise also reinforces the idea that the velocity is a vector and can be decomposed into components.

### **In-Class Exercise No. 8**

**Title:** Rigid-Body Acceleration – Fish Tank in an Elevator

**Duration:** 2-5 minutes

**Participation:** Individual effort followed by consultation with neighbors

**Educational Objective:** To reinforce the relation between pressure and acceleration for a simple situation, i.e., for the case in which the acceleration and the gravity vectors are collinear.

**Description of Activity:** Students are asked to consider a fish tank filled with water to a depth  $h$  in an elevator. Students are asked to determine the pressure at the bottom of the fish tank for two cases:

1. When the elevator and the tank are accelerating downward at  $0.5g$ .
2. When the elevator and the tank are free-falling without drag.

**Comments:** This exercise provides a simple case to explore students understanding of  $\bar{\nabla}P = \rho(\bar{g} - \bar{a})$ . Also, the concept that a free fall produces “weightlessness” is intrinsically interesting.

### **In-Class Exercise No. 9**

**Title:** Rigid Body Motion with Uniform Linear Acceleration

**Duration:** 15-20 minutes

**Participation:** Students work in pairs or groups of 3. The instructor is available for consultation with student groups.

**Educational Objectives:** For students to learn how to graphically represent the mathematical expression of rigid body motion for a fluid, i.e.,  $\bar{\nabla}P = \rho(\bar{g} - \bar{a})$ .

**Description of Activity:** Students are given a handout with the following information and questions. Space is provided for their work on the handout.

Consider the rigid-body acceleration of a fluid in standard earth gravity. For the acceleration vectors  $\bar{a}$  given below, perform the following operations:

- i. Draw a vector diagram showing  $(\bar{g} - \bar{a})$  and  $\theta$ , the angle that the free surface makes with the horizontal.
  - ii. Sketch the location of a constant-pressure line.
  - iii. Find the magnitude of  $(\bar{g} - \bar{a})$ . Your answer should be expressed in  $gs$ .
  - iv. Find the value of  $\theta$  in degrees.
- A. Magnitude and direction of  $\bar{a}$ : 1  $g$  horizontally to the left
  - B. Magnitude and direction of  $\bar{a}$ : 1  $g$  to the right, upward at  $30^\circ$  from the horizontal
  - C. Magnitude and direction of  $\bar{a}$ : 2  $g$  to the right, downward at  $30^\circ$  from the horizontal
  - D. Magnitude and direction of  $\bar{a}$ : 2  $g$  vertically downward

**Comments:** The instructor can follow up with the class to make sure that all students have the correct answers. Item B can also be used to discuss what happens if the fluid is contained in a tank with a lid. The effect of various vent locations can also be explored for the case with the lid.

### In-Class Exercise No. 10

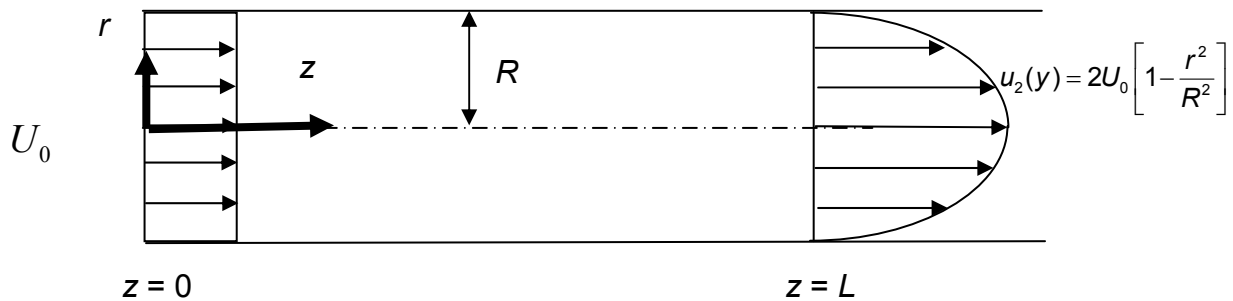
**Title:** Calculating Momentum Flows from Velocity Distributions

**Duration:** ~5 minutes

**Participation:** Individual effort

**Educational Objectives:** For students to obtain an improved understanding of the formal definition of momentum flow and to become comfortable with the integral in this definition.

**Description of Activity:** The instructor draws a sketch of a pipe in which the flow enters with a uniform velocity distribution and exits with a parabolic velocity distribution, as shown in Fig. 5.



**Figure 5. Flow enters a pipe with a uniform velocity distribution and exits with a parabolic distribution.**

The coordinate system is defined with  $\hat{i}$  as the unit vector in the axial direction. Students are also reminded of the formal definition of a momentum flow as given by

$$\int_{A_{x\text{-sec}}} \rho \vec{V} (\vec{V} \cdot \hat{n}) dA$$

Students are then asked to substitute the appropriate quantities into this expression, including all unit vectors, necessary to evaluate the momentum flow at the inlet and at the outlet of the pipe. Students are asked not to solve the integral for the outlet, but just to do the set up.

The instructor asks for volunteers to share their results and leads a discussion. The discussion includes keeping track of the unit vector products that result in the inlet momentum flow to be negative and the outlet momentum flow to be positive. The instructor provides the results from the integrations:  $-\dot{m} V_{avg} \hat{i}$  and  $+(4/3)\dot{m} V_{avg} \hat{i}$ .

**Comments:** The exercise causes students to recall that  $dA = 2\pi r dr$ , which was the subject of Exercise No. 3.

### In-Class Exercise No. 11

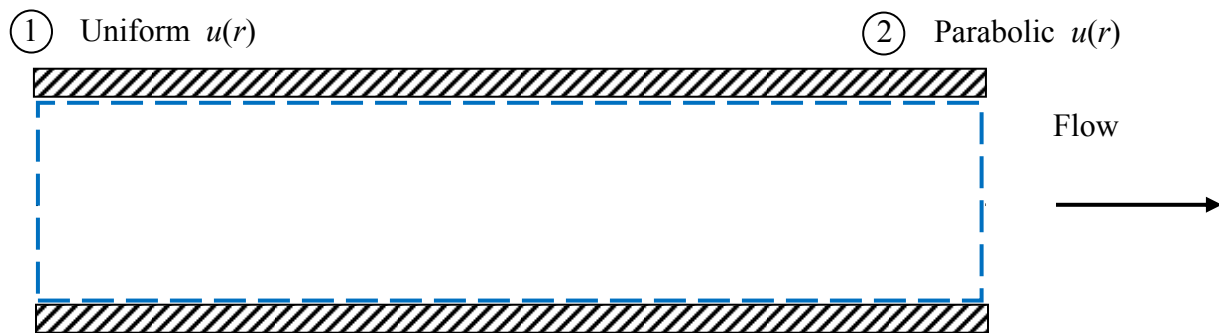
**Title:** Applying the Conservation-of-Linear-Momentum Principle to Integral Control Volumes

**Duration:** 30-40 minutes

**Participation:** Students work in groups of three. The instructor walks around the classroom offering help and clarification to make sure everyone is engaged and making progress.

**Educational Objectives:** For students to learn how to set up and solve problems that require application of the conservation-of-linear-momentum principle to integral control volumes.

**Description of Activity:** Students are given the handout “Detailed Procedure to Apply Conservation of Linear Momentum to an Integral Control Volume” shown in Table 8. The instructor presents the class with the problem shown in Fig. 6. Known quantities here are the inlet and outlet pressures, the mass flow rate, the fluid density, and the pipe diameter. The inlet velocity is uniform, whereas the outlet velocity distribution is parabolic. Students are asked to follow the procedure outlined in Table 8 to find the total frictional force retarding the flow  $F_{\text{fric}}$ .



**Figure 6. Control volume selected to expose viscous frictional forces at the walls and pressure forces at the ends.**

After students work for 15-20 minutes, the class is reconvened to discuss the problem solution. The problem can then be reframed to solve for the mechanical forces in the pipe walls at the inlet, assuming the pipe empties to the atmosphere. This reframed problem connects to Exercise 2, which dealt with control-volume choices for pipe flow.

**Comments:** This exercise also builds on Exercise 10. If students are not aware that the outgoing momentum flow for the parabolic distribution is given by  $(4/3)mV_{\text{avg}}$ , then this information should be provided as a known quantity. This exercise is particularly engaging; students are very eager to find the correct solution and will work for an extended time.

**Table 8. Detailed Procedure to Apply Conservation of Linear Momentum to an Integral Control Volume**

1. Carefully select & draw a control volume.	Your choice of CV is very important. It can make the problem easy, difficult, or impossible to solve. Try to choose a control volume that relates to the information given in the problem, while excluding irrelevant, or difficult to find, information. For example, a CV <b>inside</b> a pipe exposes the fluid shear stresses, which may be unknown, while a CV <b>outside</b> of the pipe is exposed to atmospheric pressure, a well-known quantity.
2. Draw a separate force diagram.	Identify all of the forces acting at the surface of the control volume (pressure, viscous shear, or mechanical forces where the CV cuts through a solid wall, bolt, etc.). Imagine walking over the surface. Be sure to include the body force (weight) unless told to neglect it. Indicate all forces with arrows. Surface force arrows should be drawn <b>external</b> to the CV pointing in the appropriate directions. Label each arrow.
3. Draw a separate momentum flow diagram.	Whenever fluid crosses the control surface, indicate the momentum flow with an arrow pointing in the correct direction. Show the $x$ - and $y$ -components for 2-D problems. Check to see if the momentum within the CV, $(m\mathbf{V})_{CV}$ , is changing with time. If so, draw a squiggly arrow inside the CV to indicate this. Label each arrow.
4. Using your 2 diagrams as guides, write out the $x$ , $y$ , $z$ component equations expressing momentum conservation.	If your diagrams are correct, this will be easy. Recall that the sum of the forces acting in a given direction plus the net momentum flow <b>into</b> the CV equals the time rate of change of the CV momentum in the same direction. For steady-flow problems, the CV momentum change rate is zero.
5. Invoke mass conservation as needed.	Some velocities may be unknown, but can be easily determined using mass conservation.
6. Using the equations from steps 4 & 5, solve for the unknown quantities.	Solve for the unknowns algebraically before substituting numbers.
7. Check to see if your results make sense.	Is some number unusually large or small? Is this appropriate? Are the solved-for forces or velocities of appropriate sign (direction)?

**In-Class Exercise No. 12**

**Title:** Applying the Conservation-of-Linear-Momentum Principle to Differential Control Volumes

**Duration:** 20-30 minutes

**Participation:** Students work in groups of two or three. The instructor walks around the classroom offering help and clarification to make sure everyone is engaged and making progress.

**Educational Objectives:**

1. To engage students in an important and somewhat difficult derivation.
2. To create a background development that allows students to understand the physical concepts embedded in the Navier-Stokes equation.

**Description of Activity:** Students are given the handout “Conservation of Linear Momentum: Differential Form” shown in Table 9. Students are reminded that the velocity vector is expressed as  $\mathbf{V} = u\hat{i} + v\hat{j} + w\hat{k}$ . They are then asked to apply the general concept of linear momentum,

which is given in integral form on the handout, to a small control volume having dimensions  $\Delta x$ ,  $\Delta y$ , and  $\Delta z$ . A table in the handout facilitates writing out the momentum flows in Part A. In Part B, students are asked to modify the integral form of the time-rate-of-change of the control volume momentum to apply to a differential form. In Part C, students merely have to identify the surface forces (pressure and viscous) acting on the control volume, whereas in Part D, they are asked to write out the body force, i.e.,  $\vec{W} = \rho \vec{g} \Delta x \Delta y \Delta z$ . In Part E, the instructor leads the class through the mathematic development that results in the net momentum flow per unit volume term:

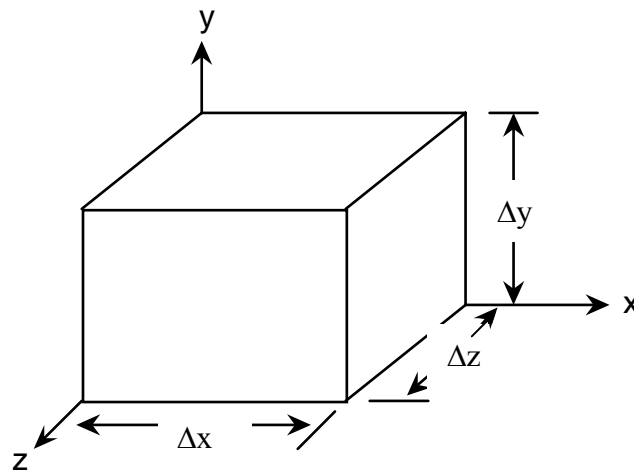
$$-\frac{\partial(\rho u \vec{V})}{\partial x} - \frac{\partial(\rho v \vec{V})}{\partial y} - \frac{\partial(\rho w \vec{V})}{\partial z}.$$

**Comments:** Students have a difficult time recognizing on their own that the momentum flow through a face involves both the velocity component perpendicular to the face for the mass flow rate, e.g., the  $u$  in  $\rho u \Delta y \Delta z$ , and the velocity vector  $\vec{V}$  associated with that same face. For example, the momentum flow exiting the  $x$ -face at  $x + \Delta x$  is  $[\rho u \vec{V}]_{x+\Delta x} \Delta y \Delta z$ . After allowing students to struggle a bit with this and seeing how they are doing, the instructor can provide some help to get the class on the right track. After students have completed Parts A-D on their own, they are prepared to follow the mathematical treatment in Part E presented by the instructor.

**Table 9. Handout - Conservation of Linear Momentum: Differential Form**

In general, momentum conservation is expressed as  $\sum_{Inlets} \dot{m}_i V_i - \sum_{Outlets} \dot{m}_e V_e + \sum F_{CV} = \frac{d}{dt} (mV)_{CV}$ .

In the following, you will apply this expression, step-by-step, to the differential Cartesian control volume below:





A. Write out the momentum flows through each of the six faces:

<u>Face</u>	<u>Area</u>	<u>Inlet Momentum Flow</u>	<u>Outlet Momentum Flow</u>
$x$	$\Delta y \Delta z$		
$y$	$\Delta x \Delta z$		
$z$	$\Delta x \Delta y$		

B. Simplify the unsteady term for the differential control volume, i.e.,

$$\frac{d}{dt}(mV)_{CV} =$$

C. Combine all of the momentum flows from part A and the unsteady term from part B and simplify.

D. List the two types of surface forces acting on the control volume.

E. Write out an expression for the body (gravitational) force acting on the control volume.

### **In-Class Exercise No. 13**

**Title:** Expanding the Vector Form of the Navier-Stokes Equation

**Duration:** 3-5 minutes

**Participation:** Individual effort

#### **Educational Objectives:**

1. To help students become proficient at expanding the common operators used in fluid mechanics.
2. To help students develop some comfort level with the Navier-Stokes equation expressed in vector form.

**Description of Activity:** Students are asked to write out one of the Cartesian components of the following expression of the Navier-Stokes equation:

$$-\bar{\nabla}P + \mu \nabla^2 \bar{V} + \rho \bar{g} = \rho \frac{D\bar{V}}{Dt}$$

**Comments:** This exercise helps students to see the usefulness of the compact vector notation as well as building their confidence in working with vector notation.

### **In-Class Exercise No. 14**

**Title:** Dimensionless Boundary-Layer Equations

**Duration:** 30-45 minutes

**Participation:** Students work in groups of 3. The instructor is available for consultation with student groups. The instructor also interacts with the class as a whole, offering guidance and hints, as needed.

**Educational Objectives:** For students to see how making the governing equations dimensionless generates useful dimensionless parameters. Students see first-hand how the Reynolds number appears in the non-dimensional axial momentum equation.

**Description of Activity:** Students are provided a handout that includes the  $x$ -component of the Navier-Stokes equation together with a series of tasks:

$$-\frac{\partial P}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + \rho g_x = \rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right)$$

- A. Simplify  $x$ -direction component of N-S equation above for:
  - i. Steady, incompressible flow
  - ii. 2-D ( $x, y$ ) flow
  - iii. Flat plate (no pressure gradient, i.e.,  $U_\infty = \text{constant}$ )
  - iv. Horizontal orientation
  - v. Negligible normal viscous stresses
- B. Define dimensionless variables using a single characteristic velocity  $U_\infty$  and a single characteristic length  $L$ .
- C. Express dimensional variables in terms of their dimensionless counterparts and substitute into your governing equation (Part A).
- D. Clean up your result from Part C, creating as many unity (numerical one) coefficients as possible.

**Comments:** This exercise introduces students to the concept of dimensionless equations in a very effective manner. This exercise removes some of the mystique in working with dimensionless variables, e.g.,  $u^* = u/U_\infty$ ,  $y^* = y/L$ , etc.

### **In-Class Exercise No. 15**

**Title:** Wall Shear Stress in the Developing Region of Flow through a Pipe.

**Duration:** 5 minutes

**Participation:** Individual effort followed by consultation with neighbors

**Educational Objective:** For students to develop skill at interpreting velocity distribution information

**Description of Activity:** The instructor shows the development of an initially uniform velocity profile to a parabolic profile in a pipe and defines the entrance length. Following this presentation and discussion of the developing region in a pipe flow, students are asked to sketch the wall shear stress as a function of distance from the pipe entrance to a location beyond the entrance length. Students are reminded of the relationship between wall shear stress and the velocity gradient at the wall, i.e.,  $\tau_w = \mu[\partial u / \partial y]_{y=0(\text{wall})}$ .

**Comments:** During closure of this exercise, the instructor asks for students to volunteer their results and draws them all on a single  $\tau_w$  versus  $x$  plot. The instructor then plots the velocity profiles in a standard mathematical way with  $u(y \text{ or } r)$  on the vertical axis and  $y$  (or  $r$ ) on the horizontal axis. Students can then easily visualize the slope at the wall and conclude which student-generated plot is correct. This exercise helps students to transform velocity profiles superimposed on a sketch of the flow (developing region of a horizontal pipe) to a standard mathematical representation with which they are familiar, i.e., a  $90^\circ$  rotation is required.

### **In-Class Exercise No. 16**

**Title:** Simplifying the Navier-Stokes Equation for Fully-Developed Flow in a Pipe

**Duration:** ~5 minutes

**Participation:** Individual effort followed by consultation with neighbors

**Educational Objectives:**

1. For students to understand how various assumptions and restrictions can simplify the Navier-Stokes equation for fully-developed pipe flow.
2. For students to develop a familiarity and comfort with cylindrical coordinates.

**Description of Activity:** Students are provided a handout containing the three components of the Navier-Stokes equation in cylindrical coordinates ( $r$ ,  $\theta$ , and  $z$ ). The instructor discusses the underlying basis for simplifying these component equations: steady flow, fully-developed, no swirl, and  $z$ -axis horizontal. Students are then asked to cross out terms based on the given conditions and assumptions. They then retrieve the following simple result for the  $z$ -component:

$$\mu \frac{1}{r} \frac{d}{dr} \left( r \frac{du}{dr} \right) = \frac{\partial P}{\partial z}$$

**Comments:** This exercise reinforces previous efforts at simplifying the Navier-Stokes equation in Cartesian coordinates. Students usually have no trouble with this exercise.

### **In-Class Exercise No. 17**

**Title:** Pipe Flow: The Connection between the Velocity Distribution and Head Loss

**Duration:** ~5 minutes

**Participation:** Individual effort followed by consultation with neighbors

**Educational Objectives:** For students to develop a hands-on feel for how the details of a flow, i.e., the velocity distribution, determine the head loss, an integral control-volume quantity.

**Description of Activity:** The exercise begins with a lecture-based discussion of how mechanical energy and conservation of momentum can be applied to fully-developed pipe flow for an integral control volume, i.e., for some pipe of arbitrary length  $L$ . Students are then asked to use the fully-developed velocity profile for laminar flow  $u(r) = u_{\max}(1-r^2/R^2)$  to derive algebraic expressions for the wall shear stress  $\tau_w$  and the head loss  $h_L$ . The key relationships from the integral control-volume analyses that the students use are  $\tau_w = \mu[\partial u / \partial y]_{y=0(\text{wall})}$  and  $h_L = 4\tau_w(L/D)/\rho g$ . The results are  $\tau_w = 4\mu u_{\text{avg}}/R$  and  $h_L = 8\mu u_{\text{avg}}L/(\rho g R^2)$ .

**Comments:** This exercise reinforces the connections between flow details derived from differential control-volume analysis (the velocity distribution) and integral quantities appearing in integral control-volume analyses of mechanical energy and axial momentum (wall shear stress and head loss).

### **In-Class Exercise No. 18**

**Title:** Friction Factor for Laminar Flow in a Tube

**Duration:** ~2-4 minutes

**Participation:** Individual effort

**Educational Objectives:** To reinforce the importance of dimensionless parameters and for students to see first-hand how the Reynolds number relates to the friction factor for laminar flow.

**Description of Activity:** The instructor defines the Darcy (Moody) friction factor as

$$f = \frac{h_L}{(L/D)(u_{\text{avg}}^2/2g)}.$$

Students are then asked to show that  $f = 64/Re_D$  using the results from Exercise 17.

**Comments:** This exercise helps to integrate a number of concepts.

## In-Class Exercise No. 19

**Title:** Introduction to Boundary-Layer Flows

**Duration:** ~3-5 minutes

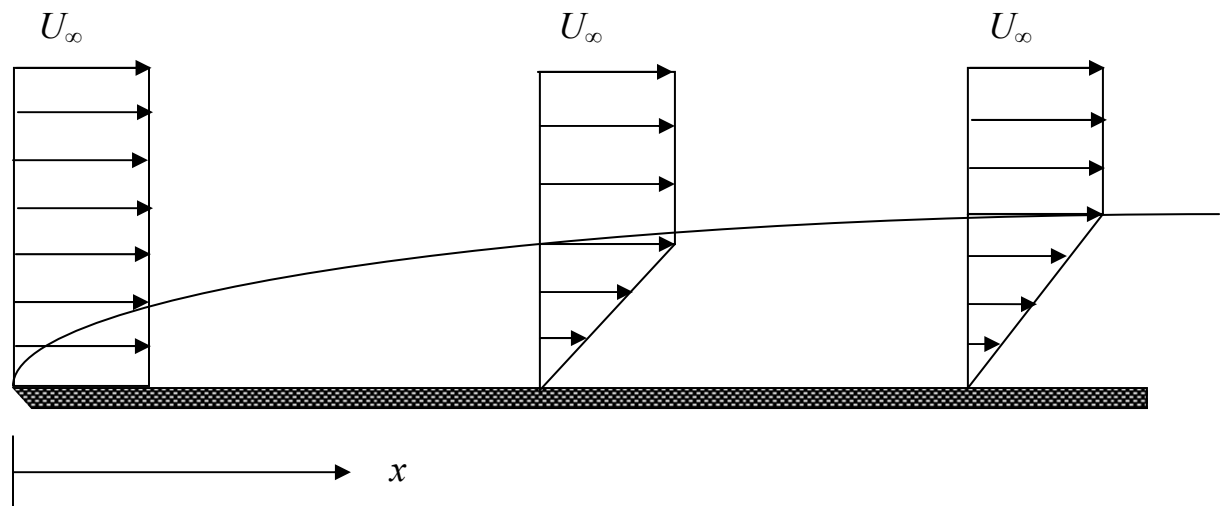
**Participation:** Individual effort

**Educational Objectives:** For students to discover first-hand how a growing boundary layer results in an ever decreasing wall shear stress for flow over a flat plate, provided the flow is in a single regime.

**Description of Activity:** Students are presented with a sketch of a boundary layer growing on a flat plate in a uniform flow as shown in Fig. 7. For simplicity, the velocity distribution through the boundary layer is approximated as a linear function going from zero at the wall to  $U_\infty$  at the outer edge of the boundary layer  $\delta$ . Students are given two tasks:

First, students are asked to draw this simplified profile at two or three  $x$ -locations downstream.

Second, students are then asked what their sketch implies about the axial distribution of the shear stress at the wall and to draw a plot of  $\tau_w$  versus  $x$ .



**Figure 7. Boundary layer growing on a flat plate in a uniform flow in which the velocity profile is approximated as a linear function.**

**Comments:** With the crude approximation given for  $u(y)$ , students should conclude that  $\tau_w \approx \mu U_\infty/\delta$ . This result can then be compared with what students know about the axial shear stress distribution in developing flow in a pipe.

## **In-Class Exercise No. 20**

**Title:** Frictional Drag on a Flat Plate

**Duration:** ~1 minute

**Participation:** Individual effort

**Educational Objectives:**

1. For students to appreciate that the local wall shear stress determines the frictional force.
2. To provide an opportunity for students to recognize that a mathematical integration is a useful tool.

**Description of Activity:** Following the discussion of Exercise 19, the following question is asked: *If you had an expression for  $\tau_w(x)$ , how would you determine the drag force  $F_D$  exerted on the top surface of a plate having a width  $W$  and length  $L$ ?*

**Comments:** This exercise together with Exercise 19 sets the stage for the analysis of the development of a boundary layer over a flat plate and motivates the need to find  $u(x, y)$ ,  $\delta(x)$ ,  $\tau_w(x)$ , and  $F_D$ .

## **In-Class Exercise No. 21**

**Title:** Mixed Laminar and Turbulent Boundary Layers on a Flat Plate

**Duration:** 15 minutes

**Participation:** Students work in pairs. The instructor walks around the classroom offering help and clarification to make sure everyone is engaged and making progress.

**Educational Objectives:** For students to understand the assumptions built into the correlations commonly used for mixed flow over a flat plate.

**Description of Activity:** The instructor provides a sketch illustrating the model used to estimate the drag force for a flat plate for which both laminar and turbulent contributions are important (see Fig. 8). In this model, the turbulent boundary layer at the transition point is assumed to be the same thickness as a turbulent boundary layer growing from the leading edge, as shown. Students are told to treat the following as known quantities: the free stream velocity  $U_\infty$ , the fluid and its properties, the plate length  $L$ , and the critical Reynolds number  $Re_{\text{crit}} = 0.5 \cdot 10^6$ .

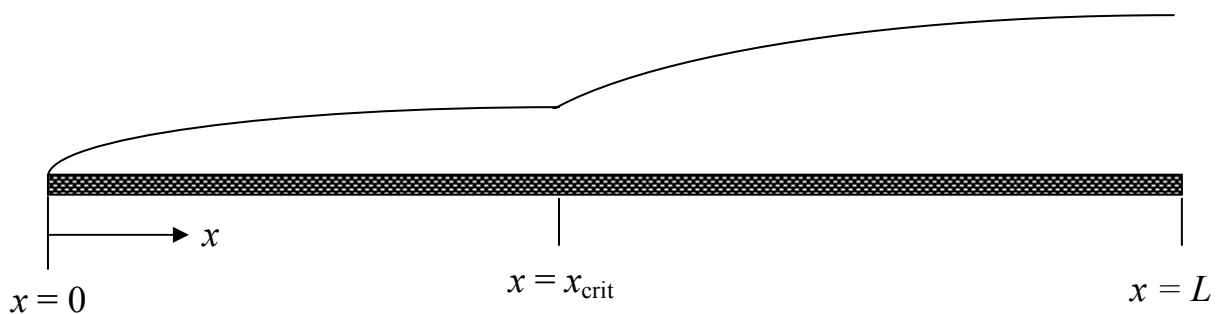
Students are also provided the necessary local friction coefficient correlations for laminar and turbulent flow:

$$\tau_{\text{wall, lam}} = \frac{1}{2}\rho U_{\infty}^2 C_{f,x} = \frac{1}{2}\rho U_{\infty}^2 (0.664 \text{Re}_x^{-1/2})$$

$$\tau_{\text{wall, turb}} = \frac{1}{2}\rho U_{\infty}^2 C_{f,x} = \frac{1}{2}\rho U_{\infty}^2 (0.0592 \text{Re}_x^{-1/5})$$

With this information, the students can proceed to determine  $x_{\text{crit}} (= 0.5 \cdot 10^6 \nu / U_{\infty})$  and set up the following integrals:

$$F_D = \int_0^{x_{\text{crit}}} \tau_{\text{wall, lam}} W dx + \int_{x_{\text{crit}}}^L \tau_{\text{wall, turb}} W dx$$



**Figure 8. Mixed laminar-turbulent boundary layer growing over a flat plate in a uniform flow.**

**Comments:** This exercise provides both a conceptual and mathematical framework for the mixed boundary-layer problem. Once students get their solutions set up and understand how to proceed, the exercise can be terminated and the final result presented without the students having to get bogged down by performing the integrations and dealing with the algebra.

### **In-Class Exercise No. 22**

**Title:** External Flows – Drag Force on a Flagpole

**Duration:** 15 minutes

**Participation:** Students work in groups of 2 or 3. The instructor walks around the classroom offering help and clarification to make sure everyone is engaged and making progress.

### Educational Objectives:

1. For students to see how fluid mechanics intersects their previous study of statics and strength of materials.
2. To reinforce how experimental and theoretical data are represented using dimensionless parameters and how this provides a wonderful generalization to a wealth of problems.

**Description of Activity:** Students are presented with the context of designing a 6-in diameter, 120-foot long flagpole to withstand a 100-mph hurricane wind. To begin their design, students must estimate the force exerted on the flagpole and the moment at the base. The students are provided with a drag coefficient versus Reynolds number plot for circular cylinders in a cross-flow. Additional information is provided so that a Reynolds number can be calculated:  $P = 100$  kPa,  $T = 298$  K, and  $\mu = 184.6 \cdot 10^{-7}$  N-s/m<sup>2</sup>. Students proceed to use the  $C_D$ -versus- $Re$  plot to determine a value for  $F_D$ .

**Comments:** For the conditions given, the Reynolds number ( $\sim 4.3 \cdot 10^5$ ) falls in the region of the drag crisis. Here the drag coefficient falls rapidly with  $Re$  because of the transition from a laminar to a turbulent boundary layer. This complication can be used to discuss factors-of-safety in the design and to set the stage for a more detailed discussion of the drag crisis.

### In-Class Exercise No. 23

**Title:** Mechanical Energy Equation, Pumps, Pump Efficiency, and Head Loss

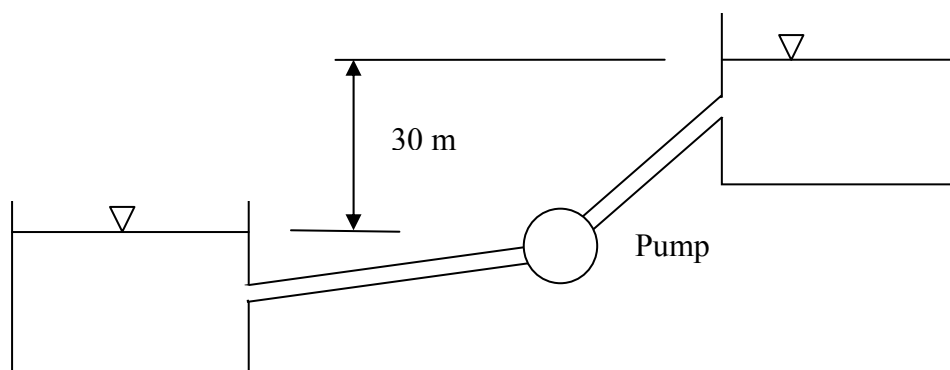
**Duration:** ~20 minutes

**Participation:** Students work in groups of 2 or 3. The instructor walks around the classroom offering help and clarification to make sure everyone is engaged and making progress.

### Educational Objectives:

1. To provide an opportunity for students to practice simplifying the mechanical energy equation.
2. To provide a concrete example of the relationship between head loss and pump efficiency.

**Description of Activity:** Students are provided with the sketch shown in Fig. 9.



**Figure 9. Sketch of water being pumped from one large tank to another.**



The students are also given the following information:

- i. The atmospheric pressure  $P_{\text{atm}}$  is 100 kPa.
- ii. The mass flow rate  $\dot{m}$  is 0.921 kg/s.
- iii. Both tanks are very large such that there is negligible change in the elevation difference between them during the process.
- iv. Pump head loss is 3 m.
- v. Total head loss (pump plus piping, etc.) is 9.7 m.

The students are asked to determine the power supplied to the pump  $\dot{W}_{\text{pump}}$  and the pump efficiency.

**Comments:** This exercise should follow a discussion of some of the common ways to simplify the mechanical energy equation, i.e., negligible velocities at the free surface of a large reservoir, atmospheric pressure at both surfaces, etc. Students should also have previously seen the definition of pump efficiency. The given information also allows students to see clearly how the head loss of the pump proper relates to the pump efficiency.

### Appendix 2– Student Survey

The purpose of this questionnaire is to collect your perceptions of how the course is going so far. Please respond to the following questions honestly. The responses are anonymous and confidential. The information will be used to improve the course for this and future semesters.

Check the box that best reflects your agreement with each statement.

	Strongly disagree	Disagree	Neutral	Agree	Strongly Agree
1. The class activities and assignments fit together in a way helpful to my learning.					
2. I am able to think through a problem or argument in fluid flow.					
3. My interest in fluid flow has increased as a result of this course.					
4. The pace of the course is appropriate for my learning.					
5. My mind often wanders during this class.					
6. I am able to relate the course material to other things I know.					
7. Students are able to discuss the problems presented in the course.					

8. I wish I had more opportunities to become actively involved with the course material in this course.					
9. I feel encouraged to ask questions and/or contribute comments in class.					
10. I wish I had more opportunities to practice the material in this course.					
11. The way that the material is approached is helpful to my learning.					

10. What changes should be made in this course to enhance student learning?

11. Any other comments on the course (continue on back if necessary)?