Active and Cooperative Learning of Markov Processes in a Half-Term Course

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Abstract

This paper discusses the design of a new half-term undergraduate course on Markov processes that has weekly lab exercises. The motivation for designing this course was two-fold. First, the Curriculum 2000 initiative in the University of Michigan College of Engineering has resulted in major curriculum redesign and in the introduction of half-semester courses. Second, we wanted to create a more engaging classroom environment for teaching Markov processes. The lab exercises are designed to introduce topics to students before lecture. The exercises are intended to help students develop intuition about certain properties of Markov processes as well as to encourage students to participate actively and cooperatively in the learning process. In this paper we present the background for the course development, discuss lab designs, and give one lab exercise example.

1. Introduction

This paper discusses the design of a new undergraduate course, *Introduction to Markov Processes*, in the Department of Industrial and Operations Engineering (IOE) at the University of Michigan. This course, henceforth referred to as "the new course", teaches the basics of Markov Processes to sophomores and juniors. Students taking this course have previously completed a semester of introductory probability and statistics. The new course differs from usual Markov processes courses in two key ways. First, it is a two-credit half-semester course consisting of four contact hours per week for seven weeks. Second, one hour per week is devoted to a lab that is designed to promote active and cooperative student learning.

"Cooperative learning is the instructional use of small groups so that students work together to maximize their own and each other's learning."[1] Research has shown that students who participate in cooperative learning groups develop higher reasoning strategies and critical thinking skills than students who engage in the traditional competitive or individualistic learning strategies.[2] Additionally, lectures have been found to focus on lower levels of cognition and learning.[3] Students' education can be enhanced if professors not only lecture but also include active learning activities in their classrooms. In active learning, "students solve problems, answer questions, formulate questions of their own, discuss, explain, or brainstorm during class."[4] Active and cooperative learning encourages students to draw upon prior knowledge and focus on higher levels of understanding.[5]

Because many of the concepts regarding Markov processes require higher-level thinking skills, an active and cooperative learning experience seems suitable for teaching these skills and developing students' intuition about Markov processes. In particular, we have developed laboratory classes for the new course in order to achieve an active and cooperative learning environment. Furthermore, the labs have been designed to encourage students to question results and eventually derive for themselves, concepts and theorems of Markov processes. The labs facilitate higher-level thinking skills, as students must synthesize lecture concepts and lab observations. Students may not have had previous exposure to a particular concept but will have seen related topics. They must use their knowledge as they explore new concepts and extrapolate when necessary. Students also need to draw on concepts from their probability and statistics courses as well as from their mathematical modeling courses to complete lab projects.

Students will first explore a topic in the lab through the lab exercise. Core concepts will be formally discussed in the lectures following the lab. For example, one day in lab students will do an exercise introducing them to continuous-time Markov chains. They explore the concept drawing on their experiences with discrete-time Markov chains. In the following lecture, the professor formally defines a continuous-time Markov chain and discusses properties and theorems associated with continuous-time Markov chains. Because the students have already been exposed to the material in their lab exercise, they will be able to apply what they are hearing in lectures to what they experienced in lab. Questions arising from the lab can be answered in lecture. Such correlation between lab and lecture will lead to more interactive lecture discussions and help students gain better intuition about the concepts.

This paper is organized as follows. In Section 2 we give the background for the course and describe the Curriculum 2000 initiative that motivated the design of this course. Section 3 gives an overview of the lab exercises. Section 4 discusses a specific lab exercise. Lastly, Section 5 concludes the paper.

2. Course Background

In an effort to comply with Curriculum 2000 at the University of Michigan College of Engineering, the Department of Industrial and Operations Engineering created several new undergraduate courses designed to address the new curriculum requirements. Curriculum 2000 was undertaken to try to bring the average number of years required to graduate down from its current value of 4.7 years per student. It was felt that if students were only taking four courses concurrently, rather than the usual load of five, fewer students would need to drop courses due to excessive loading. Therefore, core to Curriculum 2000's design is the requirement that all courses should be two or four credits (in the past, most courses have been three credits). Two-credit courses will be taken over half a semester. Therefore all courses will meet for four hours a week and a regular load will be to have four such courses at any time.

Under the old curriculum, Markov processes were taught as one unit in a general 300-level probability and stochastic processes course. This course was the first introduction to randomness that IOE students received at college; it was traditionally considered to be a difficult course. Students would later take one or more statistics courses. Under the new curriculum, IOE students' introduction to randomness occurs at the 200-level in a four-credit probability and statistics course. Thus randomness is now introduced alongside one of its key application areas, namely statistics. This course is a pre-requisite for our new course.

Given the opportunity to design a new course, we wanted to increase the level of active and cooperative learning in the classroom. Consequently, in planning the course in the summer of 1998, we created lab activities intended to actively engage students in the learning process. As mentioned above, research on student learning has shown that students maintain information better when engaging in cooperative learning groups. We hope that these activities will create a more enriching educational experience for our students. The new course will be taught for the first time in winter 1999.

3. Class Outline and Lab Design

In the new course students study both discrete-time and continuous-time Markov chains. In particular, the course includes discrete-time Markov chains, Markov decision processes, continuous-time Markov chains, Poisson processes, and queueing theory. The textbook for the course will be V.G. Kulkarni's *Modeling, Analysis, Design, and Control of Stochastic Systems*, which is soon to be available in print. The expected enrollment for the course is 75 students per term with an offering of once per semester. This section outlines the general format for the lab exercises.

There are several components to each lab exercise. If appropriate, background material is first discussed. Next, students are presented with a scenario that involves randomness of a Markovian nature. Students are asked to consider a viable mathematical model for the scenario presented. At the end of the lab exercise, students revisit the mathematical model and examine the modeling assumptions. Revisiting the mathematical model reinforces concepts as well as helps develop good modeling skills. Students also simulate the scenario and make appropriate measurements, calculations, and observations. After completing the simulation, students examine the outcomes, discuss relevant theoretical concepts, and possibly derive their own theoretical results. In the lectures following the lab, students will have the chance to return to the lab concepts, thus getting additional reinforcement and an opportunity to clarify any concepts.

Each lab has specific objectives. These objectives include: introducing discrete-time and continuous-time Markov chains, modeling problems as Markov processes, exploring steady-state, learning to generate exponentially distributed random variables, deriving the distribution of the minimum of several exponential random variables, and studying the relationship between the Poisson process, the exponential distribution, and the uniform distribution. As mentioned above, these objectives are reinforced in the lectures following the lab exercises. Students draw their own conclusions from the lab exercises and apply lecture concepts back to their lab experiences. The exercises guide students into addressing lab objectives.

Calculators are used to facilitate the lab exercises. Because the labs require simulating Markov processes, students need to be able to generate random numbers and make appropriate calculations. Therefore, students need access to a random number generator and need mathematical calculating capabilities. However, given the large class size, it is infeasible to hold the labs in a computer lab and IOE does not currently require students to own laptop computers. An alternative solution is needed. However, students are required to have at least a TI-83 (Texas Instruments-83) or equivalent graphing calculator for their pre-requisite calculus courses. Therefore, all students in the course have access to a graphing calculator in their lab groups. Graphing calculators have the random number generation and conditional programming capabilities necessary for these labs. Additionally, this calculator has the ability to share variables and programs through a link, thus making program dissemination easier.

The labs will contain a number of discussion periods. Here the students will be asked to discuss the questions in their group before the lab facilitator brings the group together for a full class discussion. The facilitator will have some concepts in mind that need to be presented and will manage the discussion to highlight these concepts.

Key to the lab designs is the participation of students in small groups. These groups should consist of two to three students. Working in groups facilitates active learning because students can help each other if stuck and will have more confidence in providing answers that they know to come from the group rather than just themselves.

4. Continuous-time Markov Chain Lax Exercise: An Infectious Disease Model

This section describes a continuous-time Markov chain lab exercise which require the use of programmable calculators. In this exercise, the objective is to introduce continuous-time Markov chains and to explore the concept of the minimum of exponential random variables. The lab is divided into three parts. Part 1 introduces continuous-time Markov chains. Part 2 explores the minimum of exponential random variables. Part 3 involves a discussion of the model and applications of the concepts discussed in Parts 1 and 2 to the original example.

Suppose we have a closed population of N individuals, some of whom are infected with a particular virus (for example, the flu) and others who are susceptible to the virus. Each susceptible individual will become infected from an infected individual after an exponentially distributed length of time. Each infected person has an exponentially distributed length of time (healing period) before becoming healthy, and hence susceptible, again.

This problem can be modeled as a continuous-time Markov chain. Let N = number of individuals in the population, $X(t)$ = number of susceptible people at time t, and $Y(t) = N - X(t)$ = number of infected people at time t. Also, let λ = rate at which a person gets cured and μ = rate at which a person becomes infected by each infected individual. Hence $\mu*(N-X(t))$ = rate at which a person becomes infected by someone in the general infected population.

In this lab we will simulate this model and examine some of the model properties. The first part of the lab will consist of the problem simulation. The second part of the lab will be a discussion on the concept of a minimum of N exponentially distributed random variables.

4.1 PART I: Simulation

4.1.1 Simulation Materials:

- Green and red flags each marked with a group number (one of each color for each group)
- Graphing calculators (one per group of two - three students)
- Records sheets (two per group, one each for the healthy and the infected records)
- Blackboard (to keep track of the state of the system)

4.1.2 How to run the class simulation:

Divide students into groups. Each group will serve as a single entity. Each group needs to have at least one graphing calculator. Groups will also want to keep track of the amount of time they are well and the amount of time they are infected.

The facilitator will set the rates for getting infected and getting well, μ and λ, respectively. The facilitator will also establish how many infected and how many susceptible groups will be in the initial population. Note that the simulation can be executed a number of times with different initial populations. The facilitator should keep track on the blackboard of the state of the system and of the time between state transitions. The state of the system is the number of susceptible groups at time *t*.

At the beginning of the simulation, let i = number of susceptible (well) groups at time 0. Then N-i is the number of infected at time 0. Infected groups will display a red flag indicating their infected status. Susceptible groups will display a green flag, indicating their healthy status. Each of the infected groups will calculate its time until recovery as $-(1/\lambda)*\ln(u)$, where u is a Uniform(0,1) random variable (i.e., the time until recovery will be distributed exponentially with mean $1/\lambda$). Each susceptible group will calculate its time until becoming infected as the minimum of N-i samples of $-(1/\mu)*\ln(u)$ (i.e., the minimum of N-i exponential random variables). In Part II of the lab we explore the fact that the minimum of exponential random variables is exponential, and hence each susceptible group could have generated the time to become infected as $-(1/(\mu\,(N-i)))*\ln(u)$.

After generating the times until becoming infected or susceptible, all groups compare times and the group with the minimum time until changed status will become infected or susceptible, as appropriate. If an infected group changes status and becomes susceptible, then the state of the system becomes i+1. This newly formed susceptible group will generate N-i-1 samples of $-(1/\mu)*\ln(u)$ and find its time until re-infection. All other susceptible groups will no longer be able to be infected by this formerly infected group. If a susceptible group changes status and becomes infected then the state of the system becomes i-1. This formerly susceptible group will calculate its time until recovery as

- $(1/\lambda)*\ln(u)$. All other susceptible groups will be able to be infected by this formerly susceptible group and must calculate the time until this occurs as - $(1/\mu)*\ln(u)$. Groups record their status as well as their status with respect to other groups on a record sheet. In particular, groups record time to being healthy and time to being infected on the record sheet.

Once the state reaches N, we stop the simulation and proceed to discussion.

4.1.3 Discussion

Students first discuss the following questions as they examine the outcomes of their simulation.

1. Why did we reach an absorbing state (namely state N)?
2. Was this outcome surprising? Why or why not?
3. What other possible outcomes are there?

The facilitator then discusses the answers to these questions with respect to the following system intuition.

Steady-state distribution for this problem has all members of the population as susceptible. Hence there are no infected people and the population is stable. Note that there is a nonzero probability of everyone becoming well. That is, as long as there is a positive probability of getting cured, then we will never be able to keep the population completely and permanently infected. We will expect to be able to get to state N and once at state $X(t) = N$, we have reached steady-state because no one can become infected. The steady-state of this system is not very interesting because when all members of the population are well, nothing changes in the system. In this example, we would be more interested in transient state distributions.

4.2 PART II: Minimum of exponential random variables

In this part we explore the concept of the minimum of exponential random variables. Students begin by generating the minimum of two exponentially distributed random variables. Students will download lab-specific calculator programs from the facilitator's calculator. Using these calculator programs, students repeat this calculation K times and examine the graph of the K minimums. Next they explore the minimum of three exponentially distributed random variables and draw conclusion based on these two examples. After establishing some intuition for the concept of the minimum of exponential random variables being exponential, we show the proof.

4.3 PART III: Bringing it all together

In this part, we get the students to apply the knowledge gained in Part 2 to Part 1. In other words, we discuss how to apply the concept of the minimum of exponential random variables being exponential to the first part of the lab. In particular, lab calculations in Part I can be simplified by noting that, given we are in state $X(t) = i$, we need only to generate a random variable with mean $(N-i)(\lambda+\mu i)$ to find the transition time to the next state. We can construct a rate diagram and derive the transition probability matrix from the rate diagram. As much of this information should be derived by the students as possible. Students may need to be reminded of the memoryless property for exponential random variables, which makes this replacement possible. If time permits we perform a simulation using this simplified process.

Next we tell the students that this type of model is a continuous-time Markov chain and get them to think about differences and similarities between discrete-time and continuous-time Markov chains. The lab ends with the following discussion on the realism of the model.

4.3.1 Discussion on the model

According to our model, the steady-state distribution for this model has no infected individuals. However, in real life, infectious diseases still exist. Populations may not reach the steady-state case of no infected individuals. (*Smallpox has reached steady-state.*) Students should be asked:

1. Why don't all infected populations achieve steady-state, that is why don't all populations become completely cured?
2. Why doesn't a disease get eradicated through the natural evolution of the system?

One possible explanation is that the time horizon is too short. It may take a very long time to reach steady-state and not enough time has elapsed for the population to reach steady-state. Consequently there are still infected individuals in the population. Another possible explanation is that our model may be incomplete. Students should be asked "what sorts of elements have not been included in the model?" Some possible answers include:

- Too short time horizon
- Inaccurate distribution assumptions (exponential)
- Exclusion of outside/environmental factors
- Inability to get cured
- Incubation period for the disease

The objectives of the infectious disease lab are to introduce students to continuous-time Markov chains and to study the minimum of exponential random variables. Students participate in this lab before the professor introduces continuous-time Markov chains. From the lab, students witness a working continuous-time Markov chain. They gain intuition about the properties of continuous-time Markov chains and how exponential random variables are key in their definition.

5. Conclusions

We have developed supplemental class material to enhance students' classroom experience. Students traditionally have trouble understanding or developing the intuition for certain topics on Markov chains. Markov chains have traditionally been taught almost exclusively using lecture format. Such a teaching style has not been very effective in developing students' intuition and hence retention of material about Markov chains. We have devised a set of lab exercises to address some of these more difficult concepts and to help students develop the intuition for these concepts through experiential learning. These concepts get additional reinforcement in the lectures, where students can draw upon their lab experiences when listening to the lectures.

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