AC 2009-1251: ADAPTABLE HOMEWORK SOLUTIONS IN FLUID MECHANICS USING MATHCAD

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Adaptable Homework Solutions in Fluid Mechanics
Using Mathcad

Introduction

One challenge in teaching undergraduate courses is to develop new homework and exam problems each semester. Problems from the textbook are often not used more than one or two semesters because students find the solutions posted online or get solutions from a fellow student.

This paper describes the development of homework problems and solutions for an undergraduate fluid mechanics course using Mathcad. Mathcad is well suited for this purpose since publication quality output is generated with equations in a publication-style format, embedded figures, and graphs. Mathcad also includes parameter dimensions in the calculations. Using the Mathcad-generated homework problems allows an instructor to easily create modified homework problems each semester. Assigning modified homework problems each semester encourages students to work on the assignment instead of seeking a previously posted solution. The Mathcad-generated problems also allow the instructor to easily modify problems for use on exams.

Mathcad-solved problems have been incorporated into many engineering textbooks and homework probems[1,2]. The Mathcad web site provides a list of Mathcad textbooks and engineering textbooks using Mathcad for problem solving[3]. In many texts, Mathcad is used as a problem solving tool by students for more advanced problems. Student assignments and projects using Mathcad have also been reported[4]. Example problems include solutions of ODE's, solutions of set of linear equations, and iterative solutions. The present paper, however, demonstrates the use of Mathcad by the instructor to generate homework problems and solutions that can be easily revised.

Instructor Problem Solutions

Four adaptable homework problems for an undergraduate fluid mechanics course have been developed and are included in the Appendix. The Mathcad files can be sent to instructors by email request to the author.

The first page of many of the Mathcad files is the "instructor work space". The original problem statement is given at the top of the page. Problem parameters that can be varied are then listed in blue highlighted areas. On the "instructor work space", the instructor can add reminders and notes about the problem. To assist the instructor, the input section of the worksheet includes a range of typical values that would be assigned for the problem. Some critical variables might also be calculated in the "instructor work space" to assure that the problem is defined as intended. For example, pipe flow and boundary layer flow problems

might include the calculation of Reynolds number in the "instructor work space" to check if the flow is laminar or turbulent under the stated flow conditions. At the bottom of the "instructor work space" the problem statement is repeated with the instructor specified values of the parameters. This problem statement can be pasted into a homework assignment.

The problem solution to be posted for the students starts on the second page of the Mathcad file. The problem statement is listed at the top of the problem solution page. The problem statement is the same as at the top of the "instructor work space" except that the instructor specified values of the parameters are included. The problem solution then follows with all problem solving steps listed including given, find, properties, assumptions, solve, solution, and comments. The final answer is boxed and highlighted.

The four example adaptable homework solutions in the Appendix were selected to demonstrate various capabilities of Mathcad. Example 1 shows a laminar pipe flow problem. In the "instructor work space" the Reynolds number is calculated so that the instructor can verify that the selected values describe laminar flow having a Reynolds number below 2300. Example 2 is a manometer problem with the fluid and column height specified by the instructor. The third example demonstrates the analytical solution and plotting capabilities of Mathcad. The fourth example solves a turbulent pipe flow problem. The Reynolds number is again calculated in the "instructor work space" to verify that the selected values describe turbulent flow conditions. The solution given to the students indicates that the Darcy friction factor is found using the Moody Diagram. In the Mathcad solution, however, the Darcy friction factor is solved in the "instructor work space" using the explicit relation by S.E. Haaland.[5] It is also possible to solve for the Darcy friction factor iteratively in the "instructor work space" using the Colebrook Equation. The value calculated for the Darcy friction factor is then displayed in the student solution where the students would use the Moody diagram. This demonstrates how the "instructor work space" can be used for "behind the scenes" calculations not expected of the student and therefore not shown in the solution given to the students.

Using Mathcad for Problem Solutions

Preparing problem solutions in Mathcad required several commands that might not be commonly used in Mathcad calculations. These particular commands and methods are listed here as a reference for others instructors wanting to develop Mathcad solutions.

Variable Subscripts. Many times Mathcad variables contain an unformatted character string, similar to that used in other programming languages. To make the Mathcad solution more similar to a printed text, a variable name can be written with a subscript by entering a period before the subscript. When the variable is entered as "P.1" Mathcad will display the variable as "P_1". (The input and variable name in Mathcad do not include the quote marks.)

Highlight Solution. To box and highlight an equation, mark the box and click on Format→ Properties→ Display. You can then "show border" and "highlight region" in a desired color.

In the Mathcad examples, I have highlighted the problem solution and the instructor specified variables in the "instructor work space".

Display Values of Variables in Equation. Published solutions usually include equations in variable form, equations with values substituted for the variables, and then the final answer. Mathcad can show the substitution of numbers into an equation explicitly using "explicit, ALL"[6]. To show the variable substitution in an equation, enter "[ctrl sft .] explicit,ALL". (The three keystrokes in the brackets are pressed down at the same time. No spaces are included in the command line.) This can also be done using the Symbolic window by typing the name of the variable being solved, clicking on "explicit" in the symbolic window, and then typing ",ALL". Example 2 in the Appendix uses "explicit,ALL" in the solution as shown below:

$$P_A := SG_{oil} \rho_{water} \cdot g \cdot h$$

$$P_{.A} \text{ explicit, ALL} \rightarrow 0.86 \cdot \left[998 \cdot \left(\frac{kg}{m^3} \right) \right] \cdot \left[9.81 \cdot \left(\frac{m}{s^2} \right) \right] \cdot (2 \cdot cm)$$

The gravitational acceleration, g, is a built-in constant having a value of 9.807 m/s^2 unless redefined by the user. I found that the built-in value of g is not substituted into the equation when using "explicit,ALL". I needed to redefine the variable in the "instructor work space" to have the value appear in the variable substitution.

Using "explicit,ALL" shows the desired substitution of parameter values into the equation. The Mathcad equation, however, includes the phrase "explicit,ALL" and makes the solution appear more as a program step than a printed solution. To remedy this, I created a textbox mask with "P_A=" and placed it over the beginning of the equation. To make the textbox large enough to cover the "P_A explicit,ALL $\rightarrow$" I included several tab spaces before the text to lengthen the mask and a carriage return before and after to increase the height of the mask.

Example 1 also used "explicit,ALL" for several calculations. In this case, the defining equation was used as a mask for the "explicit,ALL" words. Several problems can be seen in the hf calculation. The numerical value for V was not substituted into the equation. Instead, the previous equation defining V in terms of Q and d was inserted and the numerical values for Q and d displayed. In addition, the Darcy friction value previously calculated is not rounded in the "explicit,ALL" equation. Attempts to resolve these problems were not successful.

Solving Equations Analytically. Equations can be solved symbolically (analytically) using [cntl .]. (The two keystrokes in brackets are pressed down at the same time.) An example of this can be seen in Example 3 in the appendix. When the values of some variables are specified earlier in the solution, the symbolic solver substitutes known variable values and continues to solve the other variables symbolically. An example of this is above the plot in Example 3. Note that the units of velocity have become separated into the numerator and

denominator. Using "explicit,ALL" described in the previous paragraph results in the same splitting of dimensions. I do not yet know how to remedy this problem.

Solution Headings. Common Solution headings such as "Given" and "Find" are also Mathcad functions. Adding a space after the title name will change the heading from function to text mode. The heading can then be formatted.

Steam Tables. In a thermodynamics course, steam properties are used in many problems. Mathcad does not have built-in steam properties but others have developed these functions to be used in Mathcad[7].

Changing Display of Units. Mathcad will combine units into commonly used single units when possible. If you do not want the units to be simplified, go to Format→Results and turn off the "Simplify Units when Possible" option.[8]

Figures. Figures can be created in Mathcad using the SmartSketch LE application included with Mathcad. (The version of SmartSketch included with Mathcad is a limited version.) Dimensions in the SmartSketch figure can be linked to values in the Mathcad program. This allows the figure dimensions to be modified when values in Mathcad are changed. This capability seems ideal for modifying problem solutions. However, I found SmartSketch LE to be non-intuitive and I was not able to successfully create an active sketch after reading the help materials provided. Static figures can also be created in another application, such as Word, and then cut and pasted into Mathcad. The figures in appendix of this paper were created in MS Word using the shapes and text box. The separate components were grouped together and pasted into Mathcad. The quality of the figures in MS Word was good but the transfer to Mathcad degraded the quality. Several "special paste" options were tested without improvement of the figures.

Observations and Conclusions

Developing these few Mathcad examples took much time. Some of the formatting and solution commands described in the previous section were new to me. When I was unfamiliar with the commands, I found it difficult to locate the appropriate help section. The tutorial instructions for some commands, such as the explicit solution, lacked some important details. In some cases, I found the needed information in articles written by users instead of the Mathcad tutorial and help information. I needed to piece together information from several locations to create the output format I desired. After developing these four examples, I think that developing other solutions will be much easier by modifying one of these existing examples.

There are several aspects of the solution that I would like to improve. I would like to input figures into Mathcad without degrading the quality. I would like to develop a better control of the solution formatting such as the placement of the dimensions at the end of Example 3. Currently the problem statement is not automatically updated with the user values. I have not

found a way for values in the text to be updated without also inserting the variable names in an equation. I would also like to develop a way to allow the user to specify the fluid and have the Mathcad solution insert the appropriate property values. More text description will be added to the problem solutions as they are refined.

Overall, I find the current Mathcad solutions to be readable and easily modified with new values of the flow variables. I plan to develop more problem solutions using Mathcad so that homework and exam problems can be easily modified each year.

References

[1] F.P. Beer & E.R. Johnston "Vector Mechanics for Engineers"
http://www.mhhe.com/engcs/engmech/beerjohnston/vm/solutions.mhtml

[2] Bedford, Keith W., Streeter, Victor L., Wylie, E. Benjamin, *Fluid Mechanics 9th Ed.*, WBC/McGraw-Hill, New York, NY.ISBN 0-07-062537-9(Bundled with *The Student Edition of Mathcad 7.*)

[3] http://www.mpassociates.gr/software/distrib/science/mathsoft/allbooks.html

[4] B.K. Hodge, R.P. Taylor, (2002), "Piping-System Solutions Using Mathcad" *Computer Applications in Engineering Education* **10(2)**:59-78.

[5] S.E. Haaland, (1983), "Simple and Explicit Formulas for the Friction Factor in Turbulent Pipe Flow," *Journal of Fluids Engineering*, March 1983, pp. 89-90.

[6] L. Bondaryk, "Identifying the Use and Source of Numbers" at PTC Express
http://www.imakenews.com/ptcexpress/e_article001032571.cfm?x=b11,b3jsqcsB,w

[7] A.A. Alexandrov, V.F. Ochkov, K.A. Orlov, and A.V. Ochkov "Finding the Values of Steam Properties with Mathcad" at http://www.imakenews.com/ptcexpress/e_article001062509.cfm?x=b11,b3jsqcsB,w and http://www.icpws15.de/papers/01_Ind-07_Orlov.pdf .

[8] L. Bondayrk, "Using Units in Mathcad" at
http://www.imakenews.com/ptcexpress/e_article001008459.cfm?x=bc7qVh0,b7qh6mCp

Appendix
Example 1. Laminar Pipe Flow
Example 2. Manometer
Example 3. Flow Acceleration
Example 4. Turbulent Pipe Flow

INSTRUCTOR WORK SPACE

Example Problem 1
Fluid Mechanics

What is the pressure drop produced by a <u>0.001 m³/s</u> flow of oil through a horizontal <u>5 meter</u> length of 2 <u>cm</u> diameter <u>iron</u> pipe?

Several parameters highlighted in blue can be changed in this problem:

Enter flow rate: $Q := 0.001 \dfrac{m^3}{s}$ Try values from 0.0005 to 0.01 m³/s. Then check that the Reynolds number shown in yellow is less than 2300.

Enter pipe diameter $d := 2cm$ Try values from 1 to 10 cm.

Enter pipe length $L := 5m$

For water, $\rho := 888 \cdot \dfrac{kg}{m^3}$ $\mu := 0.29 \cdot \dfrac{kg}{m \cdot s}$

Check the Reynolds Number. Needs to be less than 2300 for this problem. $Red := 4 \cdot \rho \cdot \dfrac{Q}{\pi \cdot \mu \cdot d} = 194.937$

The revised problem description is:

What is the pressure drop produced by a 0.001 m³/s flow of oil through a 5 meter length of 2 cm diameter iron pipe?

SOLUTION for Example Problem 1

What is the pressure drop produced by a 0.001 m³/s flow of oil through a 5 meter length of 2 cm diameter iron pipe?

GIVEN $\qquad Q = 1 \times 10^{-3} \dfrac{m^3}{s} \qquad\qquad d = 0.02\,m \qquad\qquad L = 5\,m$

FIND Pressure drop in the pipe.

PROPERTIES for oil $\qquad \rho = 888 \dfrac{kg}{m^3} \qquad\qquad \mu = 0.29 \dfrac{kg}{m\cdot s}$

SOLVE

Find the average flow velocity in the pipe.

$$V := \frac{Q}{\frac{\pi}{4}\cdot d^2} \;\rightarrow\; \frac{0.001\cdot\left(\dfrac{m^3}{s}\right)}{\left[\dfrac{\pi}{4}\cdot(2\cdot cm)^2\right]} = 3.183\,\frac{m}{s}$$

Calculate the Reynolds Number.

$$Re := \rho\cdot V\cdot\frac{d}{\mu}\,.L \;\rightarrow\; 888\cdot\left(\frac{kg}{m^3}\right)\cdot\left[\frac{0.001\cdot\left(\dfrac{m^3}{s}\right)}{\left[\dfrac{\pi}{4}\cdot(2\cdot cm)^2\right]}\right]\cdot\left[\frac{2\cdot cm}{\left[0.29\cdot\left[\dfrac{kg}{(m\cdot s)}\right]\right]}\right] = 194.937$$

If Re<2300, the flow is laminar. For laminar pipe flow, the Darcy friction factor is:

$$f := \frac{64}{Re} = 0.328$$

The frictional head loss in the pipe is

$$hf := \left(\frac{V^2}{2\cdot g}\right)\cdot\left(\frac{L}{d}\right)\cdot f \;\rightarrow\; \frac{\left[\dfrac{0.001\cdot\left(\dfrac{m^3}{s}\right)}{\left[\dfrac{\pi}{4}\cdot(2\cdot cm)^2\right]}\right]^2}{(2\cdot g)}\cdot\left[\frac{5\cdot m}{(2\cdot cm)}\right]\cdot 0.32831058361839272 = 42.401\,m$$

The mechanical energy equation

$$\frac{P_1}{\rho \cdot g} + \frac{\left(V_1\right)^2}{2} + z_1 = \cdot\frac{P_2}{\rho \cdot g} + \frac{\left(V_2\right)^2}{2} + z_2 + h_s + h_f$$

Simplifies for this problem since the velocity and height do not change from point 1 to point 2. The pressure drop is

$$P_1 - P_2 := \rho \cdot g \cdot hf = 3.692 \times 10^5 \, Pa$$

INSTRUCTOR WORK SPACE

Example Problem 2
Fluid Mechanics

What is the pressure in pipe A if the oil manometer has a reading of <u>20 cm</u>?

Several parameters highlighted in blue can be changed in this problem:

Enter fluid height in the manometer $h := 20cm$

Typical manometer readings range from 2 to 30 cm.

Enter the oil specific weight $SG_{oil} := 0.86$

$g := 9.81 \dfrac{m}{s^2}$

Other manometry fluids can also be used:
Gasoline: SG=0.68
Ethyl Alcohol: SG=0.79
Mercury: SG=13.61
SAE 30 Oil: SG=0.89
Glycerin: SG=1.27

For water, $\rho_{water} := 998 \cdot \dfrac{kg}{m^3}$

The revised problem description is:

What is the pressure in pipe A if the oil manometer has a reading of 20 cm?

SOLUTION for Example Problem 2

What is the pressure in pipe A if the oil manometer has a reading of 20 cm?

GIVEN

    height of manometry fluid is $h = 0.2\,m$

    Specific Gravity of oil is

                         $SG_{oil} = 0.86$
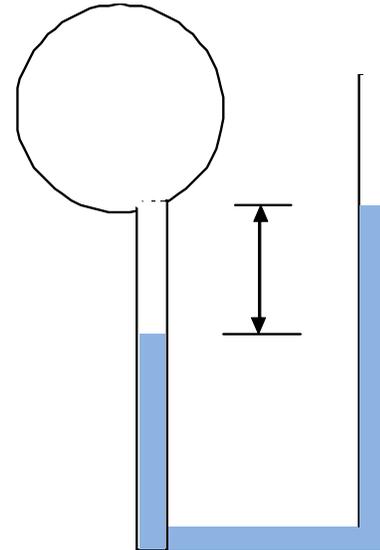
FIND

    The gage pressure in the pipe.

PROPERTIES

    density of water is $\rho_{water} = 998\,\dfrac{kg}{m^3}$

SOLVE

$$P_A := SG_{oil} \cdot \rho_{water} \cdot g \cdot h$$

$$P_A = 0.86 \cdot \left[998 \cdot \left(\frac{kg}{m^3} \right) \right] \cdot \left[9.81 \cdot \left(\frac{m}{s^2} \right) \right] \cdot (20 \cdot cm)$$

$$\boxed{P_A = 1.684 \times 10^3\,Pa}$$

Example Problem 3
Fluid Mechanics

From inviscid flow analysis, the velocity along the line AB is found to have only one component of velocity with
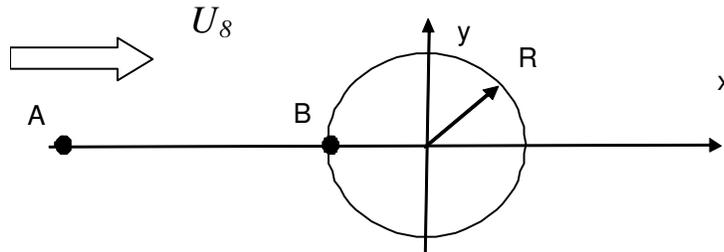
$$u(x) := U_0 \cdot \left[1 - \left(\frac{R^2}{x^2}\right)\right]$$

   a. Find the location of maximum deceleration when U0=3 and R=0.5

     Find the deceleration

$$u(x) \cdot \left(\frac{d}{dx} u(x)\right) \rightarrow -\frac{2 \cdot R^2 \cdot U_0{}^2 \cdot \left(\frac{R^2}{x^2} - 1\right)}{x^3}$$

     Find the location where ax is minimum

$$a(x) := u(x) \cdot \left(\frac{d}{dx} u(x)\right) \rightarrow -\frac{2 \cdot R^2 \cdot U_0{}^2 \cdot \left(\frac{R^2}{x^2} - 1\right)}{x^3}$$

$$\frac{d}{dx} a(x) \rightarrow \frac{4 \cdot R^4 \cdot U_0{}^2}{x^6} + \frac{6 \cdot R^2 \cdot U_0{}^2 \cdot \left(\frac{R^2}{x^2} - 1\right)}{x^4}$$

b. Plot the acceleration when

$$U_0 := 3\frac{m}{s}$$
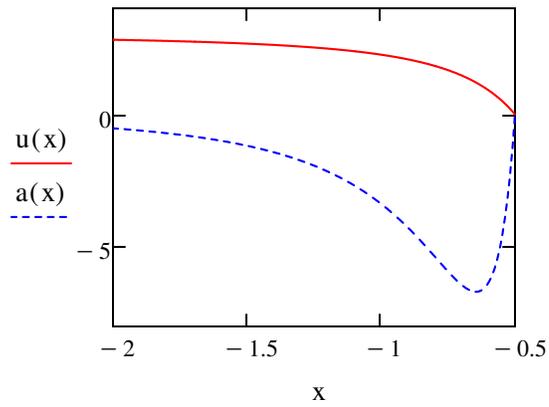
$$R := 0.5m$$

$$u(x) := U_0 \cdot \left[1 - \left(\frac{R^2}{x^2}\right)\right]$$

$$u(x) \rightarrow \frac{3 \cdot m \cdot \left(1 - \dfrac{0.25 \cdot m^2}{x^2}\right)}{s}$$

$$a(x) := u(x) \cdot \left(\frac{d}{dx} u(x)\right)$$

$$a(x) \rightarrow -\frac{4.5 \cdot m^4 \cdot \left(\dfrac{0.25 \cdot m^2}{x^2} - 1\right)}{s^2 \cdot x^3}$$

u(x)

a(x)

x

INSTRUCTOR WORK SPACE

Example Problem 4
Fluid Mechanics

What is the pressure drop produced by a 0.2 m³/s flow of water through a horizontal 50 meter length of 10 cm diameter cast iron pipe?

Several parameters can be changed in this problem:

Enter flow rate:
$$Q := 0.2 \frac{m^3}{s}$$

Enter pipe diameter
$$d := 10 cm$$

Enter pipe length
$$L := 50 m$$

Enter wall roughness for cast iron
$$\varepsilon := 0.26 mm$$

For water,
$$\rho := 998 \cdot \frac{kg}{m^3} \qquad \mu := 1 \cdot 10^{-3} \cdot \frac{kg}{m \cdot s}$$

Check the Reynolds Number. Needs to be greater than 2300 for this problem.
$$Red := 4 \cdot \rho \cdot \frac{Q}{\pi \cdot \mu \cdot d} = 2.541 \times 10^6$$

Find friction factor
$$f := \left[\frac{1}{-1.8 \log\left[\left(\frac{\varepsilon}{3.7 \cdot d}\right)^{1.11} + \frac{6.9}{Red}\right]} \right]^2 = 0.025$$

Or an iteration scheme can be set to solve the Colebrook Equation

The revised problem description is:

What is the pressure drop produced by a 0.2 m³/s flow of water through a 50 meter length of 10 cm diameter cast iron pipe?

SOLUTION for Example Problem 4

What is the pressure drop produced by a 0.2 m³/s flow of water through a 50 meter length of 10 cm diameter cast iron pipe?

GIVEN $Q = 0.2 \dfrac{m^3}{s}$ $d = 0.1\,m$ $L = 50\,m$

FIND Pressure drop in the pipe.

PROPERTIES $\rho = 998 \dfrac{kg}{m^3}$ $\mu = 1 \times 10^{-3} \dfrac{kg}{m \cdot s}$

SOLVE

Find the average flow velocity in the pipe.

$$V := \frac{Q}{\frac{\pi}{4} \cdot d^2} = 25.465 \frac{m}{s}$$

Calculate the Reynolds Number.

$$Re := \rho \cdot V \cdot \frac{d}{\mu} = 2.541 \times 10^6$$

If Re>2300, the flow is turbulent. Find the wall roughness: $\dfrac{\varepsilon}{d} = 2.6 \times 10^{-3}$

Use the Moody Diagram to find: $f = 0.025$

The frictional head loss in the pipe is

$$hf := \left(\frac{V^2}{2 \cdot g}\right)\left(\frac{L}{d}\right) \cdot f = 417.369\,m$$

The mechanical energy equation

$$\frac{P_1}{\rho \cdot g} + \frac{(V_1)^2}{2} + z_1 = \cdot \frac{P_2}{\rho \cdot g} + \frac{(V_2)^2}{2} + z_2 + h_s + h_f$$

Simplifies for this problem since the velocity and height do not change from point 1 to point 2. The pressure drop is

$$P_1 - P_2 := \rho \cdot g \cdot hf = 4.085 \times 10^6\,Pa$$