



# Adaptive Frequency Estimation in Smart Grids

Jinnan Hussain and Wolf Peter Jean Philippe

Date: 4/6/2015

## Introduction

The system frequency of a power grid is the frequency derived from the phase angle of the three phase voltage generated by an alternating current power. Frequency estimation techniques, such as Kalman filtering and least mean square, have been utilized to ensure the power system is operating at a nominal frequency in real time while maintaining a balance between the generation and load. Since the demands of electric power at the generation and load sites are changing as a result of technology advancing, the implementation of smart grids in power systems provide new challenges in tracking the system's frequency. These challenges are: imbalance between the generation and load at the main and micro grid sites, faults arisen from voltage sags, and harmonics. Thus the research introduces an alternative adaptive frequency estimation framework utilizing the Complex Least Mean Square and Augmented Complex Least Mean Square Model. These models address the challenges and provide an accurate and fast frequency tracking method in response to unexpected frequency variations from the systems nominal value.

## Background

Due to the voltages of a three phase power system represented as a complex value signal, it is necessary to simplify the signal utilizing Clarke transform.

In a noise free environment, the three phase voltage can be represented in discrete form as follows:

$$\begin{aligned} v_a(k) &= V_a(k) \cos(\omega k \Delta T + \phi) \\ v_b(k) &= V_b(k) \cos\left(\omega k \Delta T + \phi - \frac{2\pi}{3}\right) \\ v_c(k) &= V_c(k) \cos\left(\omega k \Delta T + \phi + \frac{2\pi}{3}\right) \end{aligned}$$

Since six phase voltages exist within a three-phase system, Clarke's transform is employed to map the time dependent three phase voltage to the orthogonal  $\alpha\beta\theta$  matrix to produce:

$$\begin{bmatrix} v_a(k) \\ v_b(k) \\ v_c(k) \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} v_\alpha(k) \\ v_\beta(k) \\ v_\theta(k) \end{bmatrix}$$

Note: for a balance system the  $\theta$  component is zero and the two other components are orthogonal.

For the adaptive frequency estimation, the desired voltage requires only the use of the alpha and beta voltage components and is given by:

$$v(k) = v_\alpha(k) + jv_\beta(k)$$

Where the signal can be estimated iteratively as:

$$\begin{aligned} v(k+1) &= A(k+1)e^{j(\omega(k+1)\Delta T + \phi)} \\ &= Ae^{j\omega\Delta T}e^{j(\omega k \Delta T + \phi)} = v(k)e^{j\omega\Delta T} \end{aligned}$$

And the instantaneous system frequency is represented by the phasor:

$$e^{j\omega\Delta T} (f = (\omega/2\pi))$$

## Framework of Frequency Estimation

### Complex Least Mean Square Model

Given two signals  $x$  and  $y$ , let's estimate  $y$  knowing  $x$  by the real valued estimator  $E[y|x]$ .

When  $x$  and  $y$  are zero mean and jointly distributed, the solution for above system is:

$$\hat{y} = x^T h$$

Where  $h$  is a vector of fixed filter coefficient and  $x$  is the regressor vector, and  $(.)^T$  is the transpose operator.

In Complex domain the same form of estimator leads to the standard complex least means square error estimator:

$$\hat{y} = \hat{y}_r + j\hat{y}_i = x^T h$$

Thus the voltage signal can be summarized as:

$$\begin{aligned} \hat{v}(k+1) &= v(k)w(k) \\ e(k) &= v(k+1) - \hat{v}(k+1) \\ w(k+1) &= w(k) + \mu e(k)v^*(k) \end{aligned}$$

And the system frequency is estimated as:

$$\hat{f}(k) = \frac{1}{2\pi\Delta T} \sin^{-1}(\Im(w(k)))$$

The linear estimator is written as:

$$\hat{g} = h^T x + g^T x^* = x^T h + x^H g$$

Where  $h$  and  $g$  are complex-value coefficient vectors. The widely linear model collapses into strictly linear model for proper data, with  $g=0$ .

Thus the voltage signal can be summarized as :

$$\begin{aligned} \hat{v}(k+1) &= (A(k)h(k) + B^*(k)g(k))e^{j(\omega k \Delta T + \phi)} \\ &\quad + (A^*(k)g(k) + B(k)h(k))e^{-j(\omega k \Delta T + \phi)} \end{aligned}$$

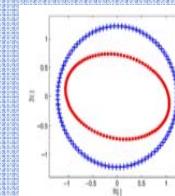
And the system frequency estimated as:

$$\hat{f}(k) = \frac{1}{2\pi\Delta T} \sin^{-1}(\Im(h(k) + a_1(k)g(k)))$$

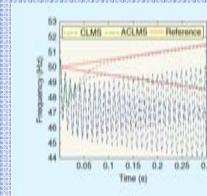
## Simulations

Estimation frequency is frequently done in presence of one of these situations: voltage sag, mismatch between generation and load, and harmonics in the system caused by certain loads and imbalance of active and reactive power.

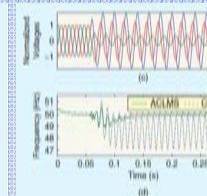
A voltage sag is defined by the IEEE standard 1159-1995 as: "decrease in the root mean square (RMS) voltage at the power frequency for durations from 0.5 cycles to 1 minute." From figure 1, observe that, in normal operating conditions, samples of  $v(k)$  are located on a circle in the complex plane, while when two of the phase voltages go onto sag, samples of  $v(k)$  are located on an ellipse. It is possible in principle to identify the type and parameters of a voltage sag within a quarter of frequency cycle, providing a very fast indication of a system fault.



(Fig 1:  $v(k)$  in complex plane)



(Fig 2: Estimation for a mismatch)



(Fig 3: Estimation for a 3 phase sag)

The simulations were performed for signals sampled at 5 KHz, and a step-size  $\mu=0.01$ . Originally, the system was balanced and was operating at a frequency  $f=50$  Hz, and all estimation algorithm were initialized at a frequency  $f_0=50.5$  Hz.

In Table 1, it shows that CLMS and ACLMS have similar performances in term of estimation of error over a certain range of SNR in balanced systems.

[TABLE 1] ABSOLUTE ERROR % FOR THE ESTIMATION IN BALANCED SYSTEMS.

SNR [DB]	50	40	30	20	10
CLMS	0.047%	0.17%	0.53%	2.73%	14.73%
LMP	0.032%	0.11%	0.33%	1.05%	13.72%
ACLMs	0.048%	0.17%	0.53%	1.83%	13.21%

In figure 2, we can observe when there a type D voltage unbalanced three-phase voltage sag, that the ACLMS algorithm has a superior performance over the CLMS. Another observation is, ACLMS technique converges to the true system frequency after an initialization period of around of 0.05 s, whereas the CLMS produced a biased estimate with large error variance.

A real-world three phase sags were recorded at a 110/20/10 kV transformer station by an ABB REL 531 numerical relay line distance protection. Device was set to record whenever a phase voltage value dropped below 90% of its nominal value for more than 20 ms. We considered the case which at time  $t=0.07$  s, phase vb experienced a shortcut with earth, resulting in a 65.32% voltage sag and 79.25% and 21.92% voltage swells in phases va and vc respectively. Figure 3 shows that CLMs was not adequate for this unbalanced situation, while ACLMS was able to accurately estimate the real system frequency at 50 Hz.

## Conclusion

By utilizing widely linear modeling, the system frequency in three phase power systems can be accurately estimated, particularly for unbalanced systems since the system becomes noncircular when unbalanced.

## Future Opportunities

In future smart grid applications, where severe frequency variations are expected due to the on-off switching of subgrids, dual roles of generator and loads (e.g., EVs), and false alarms due to voltage sags. Widely linear estimation in this context will enhance solution for:

- Rapid frequency trackers
- Identification and Classification of system fault
- Loss of main detection in real time
- Optimal operation of microgrids
- Rate of change frequency trackers
- Low voltage ride and transient stability

## References

Xia, Yili, Scott C. Douglas, and Danilo P. Mandic. "Adaptive Frequency Estimation in Smart Grid Applications." IEEE Signal Processing Magazine 29.5 (2012): 44-54. Web.

A.C. Monteith , C.F. Wagner (ed), Electrical Transmission and Distribution Reference Book 4th Edition, Westinghouse Electric Corporation 1950, page 6

## Acknowledgments

Thank you to Dr. Tomislav Bujanovic , the College of Engineering and Science, and John Michael-Vilarde for making this research opportunity possible.