Algebra-based physics for all disciplines

Abstract
A physics education strategy has been developed by which all STEM (Science, Technology, Engineering and Mathematics) students, regardless of major, take an algebra-based year-long physics sequence (General Physics). The model is based on programs of study found in other countries, and follows the approach used by international examining boards such as International Baccalaureate (IB) and those administering the General Certificate of Education advanced level (GCE A-level) curricula. With these programs of study, students typically complete a rigorous algebra-based sequence before their university studies. We are emulating that process by having all STEM students take General Physics, and then building on that for those majors that need further development. The curriculum outline and findings from the first two years are presented.

The General Physics sequence automatically meets the requirements for the “traditional” algebra-based (College Physics) population; for students needing the “traditional” calculus-based (University Physics) course, the algebra-based sequence forms the first tier of studies. The second tier of studies comprises a pair of courses titled “Calculus Applications in Physics”. This pair of courses focuses on those areas from General Physics in which calculus plays a significant role, and extends the traditional realm of topics from University Physics into advanced calculus and basic differential equations. In conjunction with the mathematics group, the calculus sequence and physics sequences have been arranged into one, almost seamless, 24 semester-hour block of courses.

The new sequence offers pedagogical benefits by separating the conceptual framework from the rigorous mathematical applications, and scheduling benefits for students already invested in a high quality algebra-based physics sequence. For these students, the entry-level physics courses are now more challenging and interesting, and less tedious and repetitive.

We discuss some of the issues that have arisen during this transition, and present the full 24 semester-hour framework. Samples of assessments and results are included for comparison to local standards.

Introduction
In the small college setting, enrollment in introductory physics courses is often low and resources limited. A common approach to meeting enrollment criteria is to combine the algebra-based and calculus-based physics sequences into one course, with the calculus-based students meeting for an extra hour per week or so. This approach was tried at first, but we found that “extra time” to be an ineffective tool. The problem lies in the degree to which different topics need continuing development. Some, such as geometric optics, have almost no use for calculus; others, such as the relationship between potentials and fields, are covered minimally in algebra-based physics, but require advanced calculus and significant class time for the calculus-based students.

With the prospect of offering different classes looking bleak, and the current system failing to meet outcomes, we designed an alternative system. As it developed, we started to recognize the
possibilities that it offered for overcoming some of the major problems typically associated with a calculus-based physics sequence, i.e. meeting the needs of the students who have already completed a rigorous physics sequence and the pedagogical issues associated with separating out the conceptual physics development from the problem-solving, mathematical, development. As the enrollment crisis eased, we chose to offer two sections of this new system rather than return to the old system of separate tracks based on major.

**Development of the new sequence**

The traditional University Physics sequence has at least two primary demands on its time – to develop problem solving skills, and to introduce the fundamental concepts that govern the physical world. The exact degree to which these (and other) outcomes are emphasized vary from program to program, and from instructor to instructor. Through this restructuring, we have assigned primary responsibility for each of these focus-areas to a different course.

The new physics sequence first puts all of our STEM (science, technology, engineering and mathematics) students through a rigorous concept-driven algebra-based physics sequence. This General Physics sequence concentrates on the concept development, while introducing basic skills in mathematical modeling and problem solving through basic application of those concepts. Since the vast majority of a University Physics sequence involves no calculus, it is appropriate to spend most of the time (8 semester-hours) in this area.

Rather than try to offer an extra hour or so each week, the calculus part of the physics sequence starts as a regular course, meeting 4 hours per week, at midterm. In a modification of the spiral pedagogical approach,¹,² (see also Collura et al.³ for application of spiral approach to the engineering curriculum), students are shown how physics concepts from earlier in the semester, and mathematical tools from the parallel calculus sequence, can be brought together to solve more complex problems. This second course, Calculus Applications in Physics (CAP), emphasizes problem-solving skills. The three sides of basic engineering – mathematical tools, scientific concepts, and problem-solving skills – are developed coincidentally. Certainly, problem-solving skills cannot be mastered without some understanding of the mathematics and scientific theories involved, but the ability to apply those tools and concepts drives the interest in learning more, and advancing further.

The developmental layout of the physics-calculus sequence is shown in figure 1.

![Diagram of course sequence]

**Figure 1**
The schematic layout of a student’s schedule passing though the 24-hour calculus-physics sequence. Solid arrows show the pre-requisite chain, and dotted arrows show co-requisites/pre-requisites.
In addition to the pedagogical merit of separating these difficult outcomes, the sequence is also schedule-friendly. Students who have performed at a high level in Advanced Placement B (AP-B) or International Baccalaureate Higher Level (IB-HL) physics at High School, or in College Physics, will usually find much of a University Physics sequence repetitive and boring; most of the sequence is not significantly different to a high quality algebra-based preparation. For these students, the opportunity to “top-up” their conceptual framework with Calculus Applications makes the entry into engineering or physics much more appealing, and allows them to be meeting another graduation requirement instead of a “busy work” requirement.

One of the problems associated with breaking down a course into two components comes about when those two components are instructed by different faculty. This is nothing new, and also causes problems even when the two semesters of a year-long sequence are divided between faculty. Expectations can vary from person to person, and if the first instructor does not cover the material expected by the second instructor, students are often left bewildered by missing information. The problem is further compounded for students transferring between schools, or skipping a period of time between courses in a sequence.

While the standard algebra-based texts (e.g. Cutnell & Johnson, Giancoli, Tipler) all cover similar material in a similar way, there is no recognized college-physics curriculum. In order to bring some continuity to the courses, we wanted a curriculum outline showing what topics should be covered in each semester, and at what depth. In the absence of a college curriculum, we looked to programs for advanced High School students, the AP-B\textsuperscript{4} and IB-HL\textsuperscript{5} programs. These offer a college-level algebra-based physics sequence in a very structured curriculum, using assessments common to multiple schools. Looking farther afield, the GCE A-level curriculum\textsuperscript{6} is also well structured, and performs a similar task. Using information from these curricula models, as well as a combination of commonly used algebra-based and calculus-based physics texts, we have started the long process of putting together a curriculum outline for this sequence.

Although a detailed curriculum is still far from completion, we are reasonably confident in the progress we have made. When our local school district approached us earlier this year looking for ideas for advanced course offerings, we were able to offer them a cross-listing with our General Physics sequence. Over the two years since this program has been in place, enrollment in the physics year-long sequence has risen from an average of 15 per year to a high this year of 38 students in college, and over 60 students in high school.

**Curriculum Outline**

The 24 semester-hour sequence is laid out as shown in figure 1. The Calculus sequence and General Physics sequence are independent of each other. It is possible to take one without the other; physics without calculus is fairly common, but the full calculus sequence without at least one physics course is much more rare.

First semester
The first semester of the General Physics sequence covers topics in kinematics, forces, circular motion and rotational kinematics, harmonic motion, energy, momentum, fluid statics and dynamics, and electric forces, fields, and circuits.

By the time the first Calculus Application course starts, students have covered 1.5 semesters of calculus, and are finishing their dynamics sections from General Physics. The Calculus Applications course follows topics in:

- Derivatives and Integrals in Physics
  This area starts by showing how the kinematics equations for constant acceleration may be derived, before looking more generally at cases of non-constant acceleration, including rotational motion. Simple applications of derivatives and integrals are also applied to topics from energy/, and momentum/impulse.

- Vector Calculus and Vector Products
  By this point in the course, students have already been exposed to vectors and the dot product in physics, and to dot and cross products in calculus. This section works with vector notation and introduces integration of vectors. The section wraps up by revisiting rotational motion, torques, and angular momentum as examples of applications of the cross product.

- Simple Differential Equations
  Differential equations are handled throughout the calculus sequence; they are introduced at the very beginning, and solution of simple separation of variables problems is an element of the first semester. In this section, the differential equation is introduced as a mathematical tool for solving physics problems such as the simple harmonic oscillator, exponential growth and decay, and variable-mass kinematics, including rocketry. Oscillations about equilibrium are also introduced as a prelude to perturbation theory, and an application of Taylor Series, which was introduced earlier in the semester in calculus.

  Complex numbers have been removed entirely from the calculus sequence and placed into the calculus applications sequence. As students are covering electrical circuits towards the end of the semester in General Physics, they are also seeing solutions to the second order differential equations associated with LRC circuits, and the equivalent description of damped harmonic oscillators.

Second Semester

The second semester is offered at the same time as General Physics II, and the combined Calculus III / Differential Equations course. The second semester of physics covers topics such as wave motion, magnetic fields, electromagnetism, AC circuits, light, and thermal physics. Options include the conventional geometric optics, nuclear physics, and atomic physics, as well as biomedical physics, relativity, and astrophysics. The optional materials are offered at the instructor’s discretion, and may be based on student preference. The Calculus Applications sections include:

- Integrals over shapes
By this time, three-dimensional integration has been introduced as a tool in the calculus sequence, in different coordinate systems. This section works with locating appropriate centers (e.g. mass, gravity) of irregular objects, calculating moments of inertia, and calculating electric potentials from non-uniform charge distributions.

- Computer-based numerical solutions to differential equations
  As the calculus sequence covers the mathematical basis of solving differential equations, the calculus applications sequence looks at using software, particularly Derive to obtain numerical solutions.

- Old math, new physics
  The final part of the course wraps up topics from the second semester of physics using basic mathematical skills with topics such as the Biot-Savart Law, and energy considerations of expanding systems.

Assessment

A detailed comparison of student achievement old-to-new is unavailable because of the low-enrollment problem that first generated this project. A meaningful assessment of how well students have truly mastered the coursework is best assessed in the upper-level courses that build on the material. However, going though upper level courses now, we have only 1 student from the old system and 3 from the new system, insufficient for a meaningful study. However, other factors suggest that the program is, at a minimum, not hurting students. We have also seen an increase in interest from mathematics majors in taking upper level coursework in sciences and applied sciences for which calculus-based physics is a prerequisite, suggesting that they are more comfortable with physics than in the past. The enrollment in the physics sequence has also increased dramatically, from an average of 15 or less per year over several years, to 27 and 38 in the two years since the new system was implemented, as shown in Figure 2. Again, these observations do not conclusively demonstrate that the program has helped, and these are comparisons to the old system of algebra-based physics + one extra hour per week, not to a program of separate tracks.

Successful retention/graduation of prospective engineering students has also increased, as shown in the table below:

<table>
<thead>
<tr>
<th>Class of</th>
<th>Entering class</th>
<th>Current /final enrollment</th>
<th>Retention</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2006</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2007</td>
<td>4</td>
<td>3</td>
<td>0.75</td>
</tr>
<tr>
<td>2008</td>
<td>6</td>
<td>4*</td>
<td>0.67</td>
</tr>
</tbody>
</table>

* enrolled in the sequence this academic year
Figure 2
The enrollment trends in the physics sequence. The new system was started in Fall, 2004. The old system included all students in Algebra-based Physics, with 1 hour per week extra for students needing Calculus-based Physics. Spring semesters tend to attract fewer students because many majors require only one semester of Physics.

Further Work
We are continuing to develop a detailed syllabus, but do not have any immediate plans for formal assessment of the methods described in this paper, primarily for lack of a comparative audience. With a detailed curriculum in place, we will approach some larger universities, with multiple sections of calculus-based physics, to invite collaboration with the assessment.

Bibliography