Alternative Approaches to Teaching Extended Surface Heat Transfer

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Introduction

Extended surface heat transfer is a topic that is always covered in a standard undergraduate class in heat transfer. In fact, we tell our students that fins are one of the two heat transfer systems that they will most likely encounter as practicing mechanical engineers (the other being heat exchangers). Therefore, it becomes clear that the mechanical engineering student needs to have a firm foundation in the physical aspects of extended surface heat transfer, and an understanding of the mathematical models available for the required calculations.

Two tools have been developed to facilitate the students' learning in these directions. The first tool consists of a set of physical models of the different types of fins. This allows the students to get a "hands-on" feel for fins. It is particularly useful for students whose primary learning style is kinesthetic. The second tool is an excel spreadsheet that calculates the standard fin parameters and presents the temperature distribution for a variety of fins using the three standard boundary conditions at the fin tip: adiabatic, convective, and tip at fluid temperature. A calculation option that is available to the user will determine the number of fins in an array that is required to provide a specified heat transfer rate requirement (may be heating or cooling). This option allows the students to perform some simple design studies and compare the performance of different fin types (for example rectangular versus triangular) and arrangements, including the tradeoffs in length, thickness, and orientation. For visual learners, the spreadsheet also includes a photograph of the physical model and a diagram relating the geometric parameters for each type of fin geometry.

These tools have been used in heat transfer classes at both Olivet Nazarene University and Michigan State University. At Olivet Nazarene the spreadsheet was used as a virtual design lab activity, while at Michigan State the spreadsheet was used as an extended homework assignment.

This paper continues with the development of the physical models. The corresponding mathematical models and the Excel spreadsheet are then presented. Next, the assignments for both classes are discussed along with results of student surveys on the tools and assignments. Final remarks conclude the paper.

Physical Background and Models

Fins are used to enhance the heat transfer to or from a surface by increasing the surface area. To introduce this idea to students, the following scenario is proposed to them. Consider a computer circuit board that contains electronic components producing heat through I^2R heating. To ensure proper operation and eliminate short lifetimes, the board must be maintained at a specified

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temperature, T_{max}. A cooling system has been designed for this application which involves blowing air past the board to achieve convective cooling or

$$q = h_c A(T_{max} - T_{air})$$

where q will be equal to the electrical power dissipation. Of course, one of the things that is true about computers is that the customer always wants the machine to go faster and be more powerful. To achieve this a company might put even more electronic components on the board, which would lead to more electrical power dissipation, and if not addressed this would likely overheat the circuit board. So to maintain the new circuit board at the maximum operating temperature the cooling system must be redesigned. To gather a sense of what we might want to do let us look at our Newton's law of cooling.

$$q = h_c A(T_{board} - T_{air})$$

So if q is fixed, the students are asked what might be done to lower T_{board}. Three possible design alternatives are:

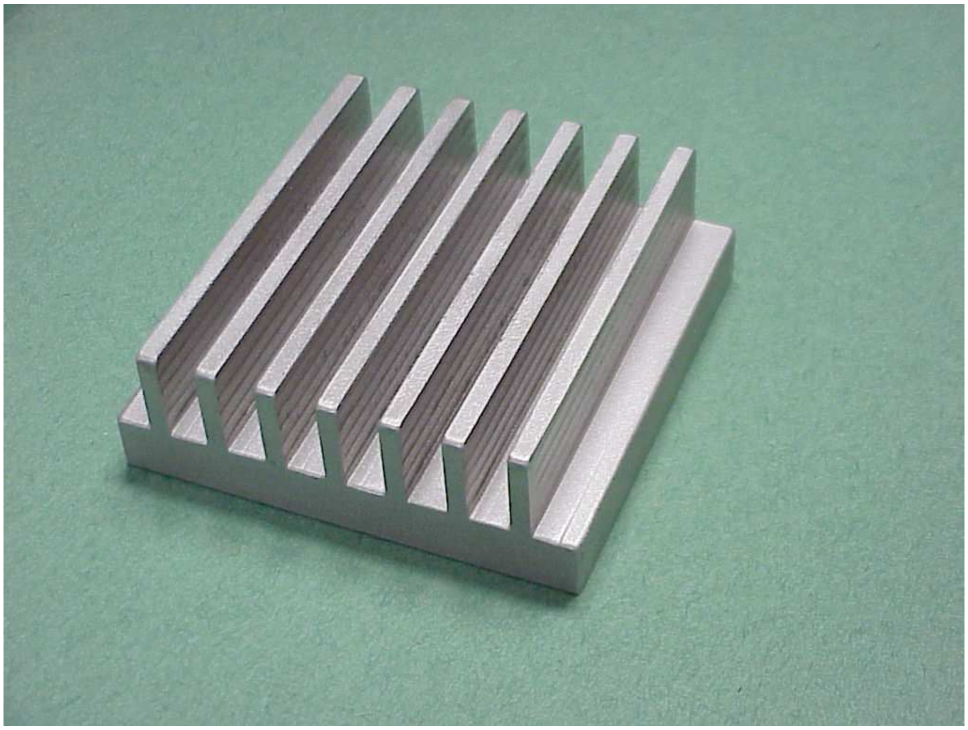
- increase h_c - increase air velocity or change to another fluid
- decrease T_{air} - refrigerate the air
- increase A - add extended surfaces or fins to the circuit board

The addition of fins is argued to the students as being the simplest and probably most effective design alternative. It needs to be pointed out to the students that there are two major downsides to adding fins:

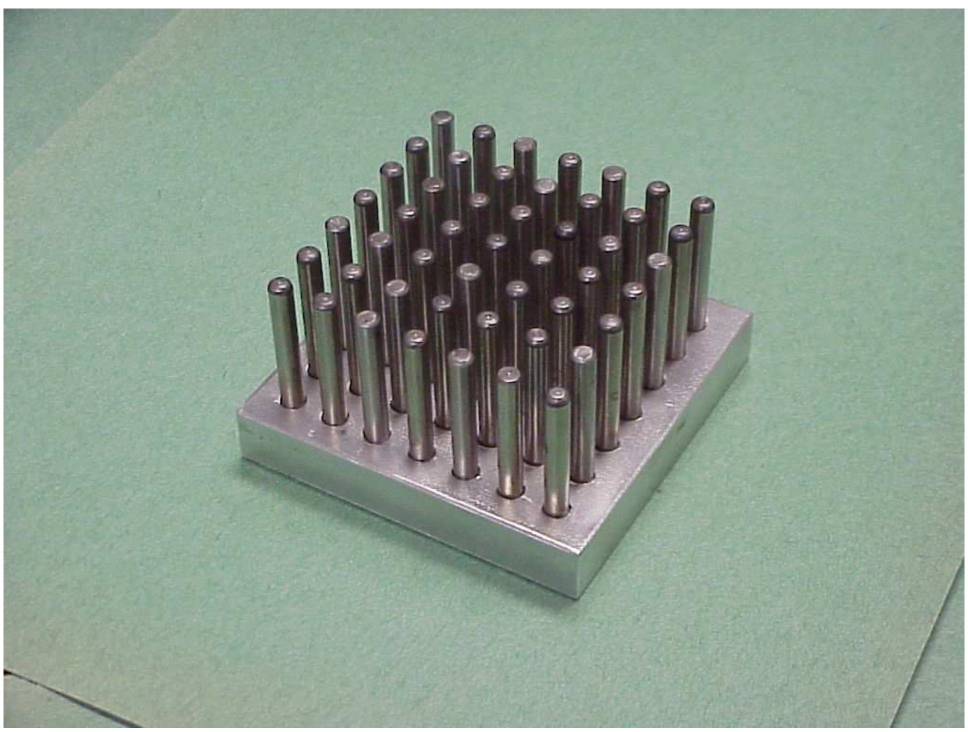
(i) may negatively affect the flow of air around the circuit board and hence h_c
(ii) because of conduction the fin cannot be entirely at T_{board}

At this point it proves useful to discuss the various types of fins. Physical models of four different fin types are available to pass around the classroom, so that the students may get a "feel" for what a fin array consists of. These physical models are shown in Figure 1. They were constructed by one of the authors (Ryan Harrier) as an undergraduate at Michigan State University. Detailed drawings and fabrication details may be obtained from the lead author at somerton@egr.msu.edu. Our experience has been that use of these props creates a much stronger understanding about the physical nature of fins than the pictures in a typical heat transfer text book. One of the authors has used some junked heat sinks from a computer shop that might be available from computer services on most campuses. Though not as simple and direct as the models made to match the textbook formulas, they still provide a hands-on experience for the students.

Figure 1 Pictures of Physical Models

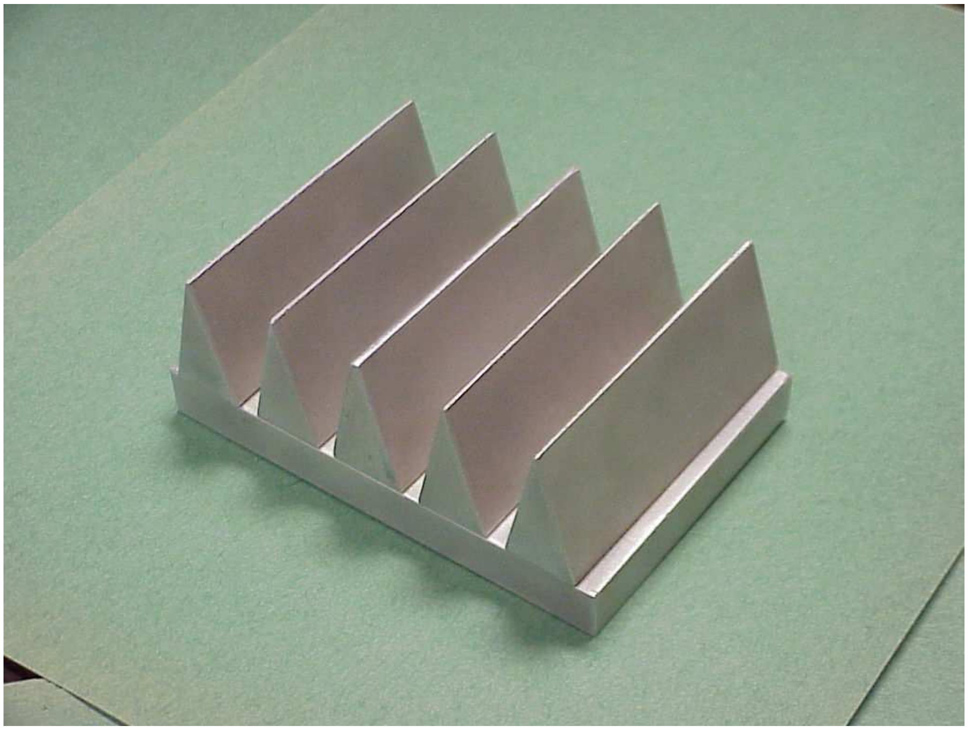


Rectangular Fin Array



Cylindrical Fin Array

Figure 1 Pictures of Physical Models (continued)

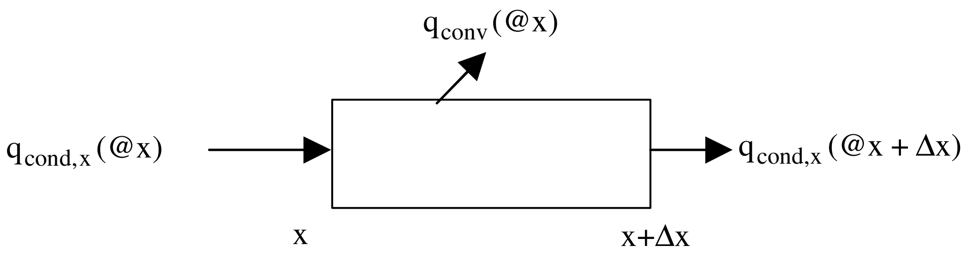


Triangular Fin Array

Annular Fin Array

Mathematical Background and Excel Workbook

The primary mathematical model for fins consists of the differential energy equation for the fin. The following derivation is used in our classes, and for completeness is included here. We consider the fin shown in Fig. 2. Then we consider a differential element Δx of this fin of arbitrary geometry and varying cross-sectional area, as shown below.



Performing an energy balance on this element yields

$$q_{cond,x}(@x) = q_{cond,x}(@x + \Delta x) + q_{conv}(@x)$$

To make this useful we now need to use our constitutive equations to relate the heat flows to temperature. Then we have

$$-kA_c(x)\left.\frac{\partial T}{\partial x}\right|_{at\ x} = -kA_c(x + \Delta x)\left.\frac{\partial T}{\partial x}\right|_{at\ x+\Delta x} + h_c \Delta A_s \left(T(x) - T_f\right)$$

The differential surface area in general can be given by

$$\Delta A_s = P(x)\Delta x$$

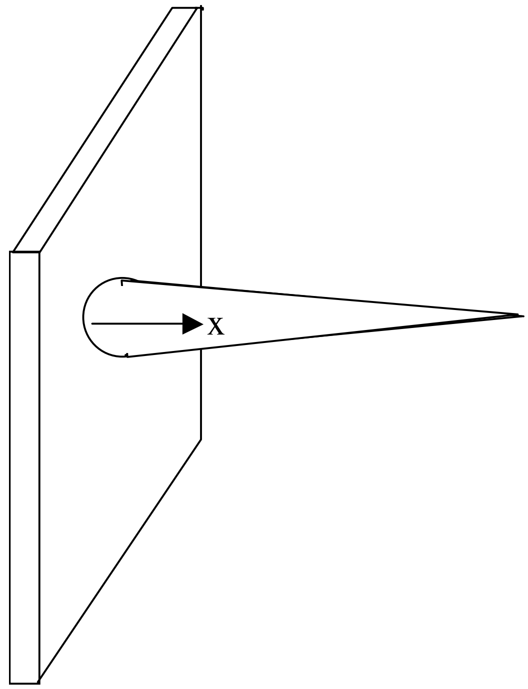
where P(x) is the perimeter of the fin. Substituting

$$-kA_c(x)\left.\frac{\partial T}{\partial x}\right|_{at\ x} = -kA_c(x + \Delta x)\left.\frac{\partial T}{\partial x}\right|_{at\ x+\Delta x} + h_c P(x)\Delta x\left(T(x) - T_f\right)$$

Now dividing through by $k\Delta x$ and rearranging

$$\frac{A_c(x + \Delta x)\left.\frac{\partial T}{\partial x}\right|_{at\ x+\Delta x} - A_c(x)\left.\frac{\partial T}{\partial x}\right|_{at\ x}}{\Delta x} = \frac{h_c P(x)}{k}\left(T(x) - T_f\right)$$

Figure 2 Physical Model for Fin Equation Derivation



and taking the limit as $\Delta x \Rightarrow 0$ and using the definition of the derivative

$$\frac{d}{dx}\left(A_c(x)\frac{dT}{dx}\right) - \frac{h_c P(x)}{k}\left(T(x) - T_f\right) = 0$$

This second order, variable coefficient, non-homogeneous differential equation is the most general form of the energy equation for heat transfer through a fin. Though various geometries could now be considered and the differential equation solved, a typical heat transfer class only has time to develop the mathematical solution for the case of a fin of constant cross-sectional area. For this case the differential equation simplifies to

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

where

$$m = \sqrt{\frac{h_c P}{k A_c}}$$

and we have defined an excess temperature, θ,

$$\theta = T(x) - T_f$$

The general solution will take the form

$$\theta = D_1 \cosh(mx) + D_2 \sinh(mx)$$

where the constants D_1 and D_2 will be evaluated from the boundary conditions. At the base of the fin, x = 0, we typically assume a known temperature, or

at $x = 0$, $T(0) = T_{base}$ or $\theta(0) = T_b - T_f$

At the tip of the fin we may have any of the three standard thermal boundary conditions. If the fin is long enough, we might suppose that it would reach the surrounding fluid temperature at the tip

at $x = L$, $T(L) = T_f$ or $\theta(L) = T_f - T_f = 0$

Again if the fin is long enough, we might argue that all of the heat will leave by the sides before it gets to the tip, so that the tip would be adiabatic (have zero heat flow). Then our boundary condition would be

$$\text{at } x = L,\ -k\left.\frac{dT}{dx}\right|_{x=L} = 0 \ \text{ or } \ \left.\frac{d\theta}{dx}\right|_{x=L} = 0$$

Finally, the most general boundary condition (not concerned with having a "long" fin) is the convective condition or

$$\text{at } x = L,\ -k\left.\frac{dT}{dx}\right|_{x=L} = h_{tip}\left[T(x=0) - T_f\right] \ \text{ or}$$

$$\left.\frac{d\theta}{dx}\right|_{x=L} + \frac{h_{tip}}{k}\theta(x=L) = 0$$

Note that the convective heat transfer coefficient at the tip as been denoted as a different variable (h_{tip}) than the heat transfer coefficient acting on the side (h_c). This will allow us to take the convective tip solution and obtain the other two cases by letting h_{tip} go to zero (the adiabatic tip case) or letting h_{tip} go to infinity (the tip at fluid temperature case).

Applying the general convective tip boundary condition gives the following expression for the excess temperature

$$\theta = \theta_b \cosh(mx) - \frac{\theta_b\left[\frac{h_{tip}}{k} + m \cdot \tanh(mL)\right]}{m + \frac{h_{tip}}{k}\tanh(mL)}\sinh(mx)$$

From this expression and the standard definitions the fin heat transfer rate, fin thermal resistance, and fin efficiency can all be obtained as shown below.

$$q_{fin} = \frac{\sqrt{hPkA_c}\,\theta_b\left[\frac{h_{tip}}{m \cdot k} + \tanh(mL)\right]}{1 + \frac{h_{tip}}{m \cdot k}\tanh(mL)}$$

$$R_{fin} = \frac{1 + \frac{h_{tip}}{m \cdot k}\tanh(mL)}{k \cdot m \cdot A_c\left[\frac{h_{tip}}{m \cdot k} + \tanh(mL)\right]}$$

$$\eta_{fin} = \frac{\sqrt{hPkA_c}\left[\frac{h_{tip}}{m \cdot k} + \tanh(mL)\right]}{hA_{s,fin}\left[1 + \frac{h_{tip}}{m \cdot k}\tanh(mL)\right]}$$

The Excel spreadsheet FinSolver.xls has been developed using these expressions. The workbook is composed of five spreadsheets, each for a different fin geometry. As indicated by the worksheet tabs, calculations may be made for rectangular fins, cylindrical fins, triangular fins, annular fins, or parabolic fins. Each worksheet has a similar layout, so for demonstration purposes, we consider the rectangular fin spreadsheet. The spreadsheet is divided into three parts. Fig. 3 shows part 1 of the rectangular fin spreadsheet. At the top is a drawing that labels the pertinent dimensions of the fin and a picture of the physical model of a rectangular fin array. Beneath these pictures, cells are provided for the required inputs for the fin analysis. As shown in Fig. 4, part 2 of the spreadsheet provides the results of the fin analysis for the inputs specified. This includes the temperature, heat transfer rate, efficiency, and thermal resistance for the three different boundary condition cases discussed above. The temperature distribution is provided graphically for the three cases, so students can clearly see the differences or similarities among the solutions as input parameters are varied. The third and final part of the spreadsheet (see Fig. 5) has been set up to allow students to explore some design issues associated with fin arrays. The student will specify the heat transfer required and the base surface area available, and the spreadsheet will calculate the number of fins and their mass per the inputs provided in part 1 to fulfill the requirements. The spreadsheet may be downloaded from the web at

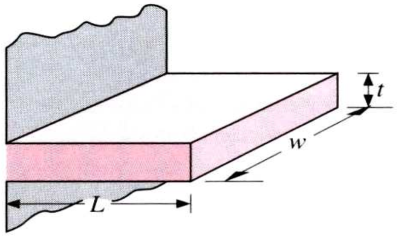
http://www.msu.edu/~somerton/Fins

Assignments and Student Feedback

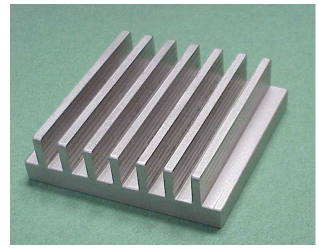
At Olivet Nazarene University FinSolver.xls was used in a virtual design lab activity for the junior-senior level heat transfer course, ENGN 385. The assignment and a suggested procedure are shown in Figs. 6 and 7. At Michigan State University FinSolver.xls was used in an extended homework assignment for the senior level heat transfer course, ME 410, and is shown in Fig. 8. For the ME 410 assignment, one lecture period was taken to introduce the students to FinSolver.xls using a computer projection system. For ENGN 385, the instructor spent about thirty minutes in the lab with students logged in on computers, so as to have the spreadsheet available in front of them, while the instructor, using a computer projection system, demonstrated features of the program and worked out a first trial case of the assignment. The clear difference in the nature of these two assignments shows the utility of the spreadsheet.

Figure 3 Part 1 of Rectangular Fin Spreadsheet

Rectangular Fins



Drawing of Rectangular Fin



Picture of Rectangular Fin Array

Inputs									
Width (m)	Length (m)	Thickness (m)	Heat Transfer Coefficient (W/m^2·K)	Base Temperature (°C)	Fluid Temperature (°C)	Thermal Conductivity (W/m·K)	Density (kg/m^3)	Tip Heat Transfer Coefficient (W/m^2·K)	Distance at which temperature is desired (m)
0.36	**0.05**	**1.00E-03**	**23**	**250**	**33**	**21.9**	**4500**	**100**	**0.05**

Figure 4 Part 2 of Rectangular Fin Spreadsheet

Outputs				
Perimeter (m)	Cross Sectional Area (m^2)	Surface Area (m^2)	m (m^{-1})	θ(x=0)
0.7220	**0.0004**	**0.0365**	**45.8943**	**217.0000**
Cases	Temperature at Specified Distance (°C)	Heat Transfer Rate (W)	Efficiency	Thermal Resistance (K/W)
Convective Tip	**72.5**	**77.2**	**0.4**	**2.8**
Adiabatic Tip	**76.3**	**76.9**	**0.4**	**2.8**
Tip at Fluid Temperature	**33.0**	**80.1**	**0.4**	**2.7**

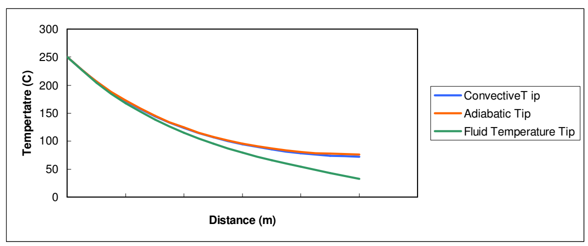


Figure 5 Part 3 of Rectangular Fin Spreadsheet

Design Analysis			
Input Required Heat Transfer (W)	**Input** Bare Surface Area (m^2)	**Output** Number of Fins	**Output** Mass of Fin Array (kg)
300	**0.13**	**4**	**0.324**

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Figure 6 Virtual Design Lab Assignment

Tiger Engineering

TO: Thermal Design Team
FROM: Product Development
Date: March 14, 2002

Due: Thursday, April 4, 2002.

The Product Development Team has produced a novel conceptual design that will display real-time stock quotes, basketball scores, and Krispy-Kreme delivery times in high-intensity purple and gold, using data downloaded from a wireless network. However, the sensitive electronics inside require that the housing temperature be maintained at or below 30 °C. The device has a peak power consumption rate of 25 W.

We require you to investigate possible finned heat-sink designs using aluminum, copper, and stainless steel. We are seeking the most cost-effective feasible design, since it is hoped that demand for the device will drive mass-production quantities.

Please document your design recommendations, and also your analyses of those designs considered, in a technical report so that designers for future products can build upon your work. Document assumptions made and any additional considerations which may be needed for design refinement or future product designs.

In addition, please determine the sensitivity of your design to the convective coefficient, h. The given value is a design estimate based on the flow enhancement available from the small fan.

Also be prepared to request improvements in the Fin Solver tool from the Technical Computing Department that would better facilitate these analyses.

You should base your analysis on the standardized design data that accompanies this memo.

Please submit your technical report by Thursday, April 4, 2002.

Figure 6 Virtual Design Lab Assignment (continued)

Design Data

Product	
Power dissipation	q = 25 W
max. allowable surface temperature	$T_{s,max}$ = 30 °C
surface available for heat sink	8.5 cm × 15 cm
aluminum	
unit cost	D_{Al} = $13.77 /kg
density	ρ_{Al} = 2770 kg/m^3
conductivity	k_{Al} = 177 W/m·K
copper	
unit cost	D_{Cu} = $9.15 /kg
density	ρ_{Cu} = 8850 kg/m^3
conductivity	k_{Cu} = 390 W/m·K
stainless steel	
unit cost	D_{SS} = $8.66 /kg
density	ρ_{SS} = 8055 kg/m^3
conductivity	k_{SS} = 15.1 W/m·K
Environment	
ambient temperature	T_∞ = 22 °C
avg design convection coeff	h = 25 W/m^2·K

Figure 7 Procedure for Virtual Design Lab Assignment

ENGN 385
Spring 2002
Suggested Procedure for Fin Design Lab

1. Copy the **Fin Solver.xls** spreadsheet from the 385 directory of the O: drive.
2. To start, choose one material (say, Aluminum) and one geometry (say, rectangular fins).
3. Enter the design parameters for input.
4. Begin with a "reasonable" design choice for one parameter (say, the length).
5. Determine the other design criteria (thickness of each fin) that satisfy the design problem requirements. Verify that the number of fins required is reasonable (that they will fit onto the surface with a reasonable spacing!)
6. You now have one **feasible design**, meaning one that is physically achievable (realistic) and that fits the bill on the design requirements.
7. Determine the weight and cost for this design. (You may want to add this calculation to your spreadsheet.)
8. Vary the length parameter chosen in step 3, and determine additional **feasible designs**. Determine the weight and cost for these designs.
9. As you begin to gain an understanding of the relationship between the parameters, you may want to use the *Goal Seek* and/or the *Solver* tools in Excel to quickly identify each feasible design, and even to optimize the design to minimize weight and cost.
10. Perform similar analyses with the other possible materials and fin geometries (rectangular, cylindrical pins, triangular, and parabolic).

Figure 8 Extended Homework Assignment

ME 410
Fin Analysis Using FinSolver.xls

For problems 1-3 consider a rectangular fin of width 15 cm and thickness 0.05 cm composed of plain carbon steel. The base temperature is 100°C and the fluid temperature is 20°C.

1. For a side convective heat transfer coefficient of 1 W/(m^2 K) and a length of 10 cm, determine the tip convective heat transfer coefficient that will
   (a) have the convective tip solution behave like the adiabatic tip solution
   (b) have the convective tip solution behave like the tip at fluid temperature solution

2. For the side and tip heat transfer coefficient equal to each other, determine the conditions (length and convective heat transfer coefficient) that will
   (a) have the convective tip solution behave like the adiabatic tip solution
   (b) have the convective tip solution behave like the tip at fluid temperature solution

3. Starting with a fin of length 10 cm and a convective heat transfer coefficient of 40 W/(m^2 K), determine how the fin efficiency varies with
   (a) thermal conductivity
   (b) fin length
   (c) fin thickness
   (d) fluid temperature
   (e) convective heat transfer coefficient
   (f) fin width

4. From the analyses performed above, evaluate how good of an approximation is the adiabatic tip model for a real fin (convective tip model).

5. A surface of area 0.13 m^2 has a heat flow of 300 W and a temperature of 250°C. For rectangular fin of 0.36 m wide, 0.05 m long, made of titanium, determine the fin thickness that will minimize the mass (and hence cost) of the fin array. The convective heat transfer coefficient may be taken to be 23 W/(m^2 K) and the fluid temperature is at 33°C.

6. Repeat problem 5 for a triangular fin array.

7. Comparing the results of problems 5 and 6, which fin geometry will cost less for this application.

Student feedback on the use of FinSolver.xls and the assignments were obtained via the survey form shown in Fig. 9. Results of these surveys are shown separately for MSU and Olivet-Nazarene in Fig. 10. It is striking to note that the student's assessment of how the assignments improved their understanding of fins was significant, as the average response from question 1a to question 1b went from 2.9 and 2.6 to 4.2 and 4.1. This increase in understanding is further supported by the students written comments:

"The project helped me to visualize the concepts of extended surfaces. I understood how to do fin problems, but now I have more than just a theoretical understanding."

"The ability to see the results graphically helped make the concepts sink in."

"This software helps to try different situations so it's easier to understand the behavior of the fins."

There is also a significant difference between the MSU response (3.7) and Olivet-Nazarene response (2.9) to question 2a, but this is due to the difference in the nature of the two assignments. The extended homework assignment made to the MSU students was clearly more focused on the heat transfer principles than the design assignment for the Olivet-Nazarene students. Another significant difference exists between the two cohort groups for question 5. This is probably due to the fact that the Olivet-Nazarene students were asked to use the Solver function Excel, and it is our experience that most students have little experience with this function. Finally, all students felt that they needed more direction on the assignments, but out experience is that students, in general, are very uncomfortable with these types of open ended assignments. The written comments were very supportive of the spreadsheet program, though several urged that a tutorial and a drop-down material database should be developed.

Final Remarks

A spreadsheet program, FinSolver.xls, has been used in heat transfer classes at both Michigan State University and Olivet-Nazarene University. Student feedback indicates that the assignments using the spreadsheet significantly improve the students understanding of fins. Based on the responses of students, a tutorial is being developed that will provide the students with additional direction in the use of the program.

Figure 9 Student Survey

Fin Project
Student Assessment Survey

Did not understand = (0)
Understood well = (5)

1. Rate your understanding of fins
   a) before starting this project and ⓪ ① ② ③ ④ ⑤
   b) after completing this project ⓪ ① ② ③ ④ ⑤

COMMENTS:

Did not understand = (0)
Understood well = (5)

2. How well did this project improve your understanding of
   a) the heat transfer principles of fins ⓪ ① ② ③ ④ ⑤
   b) the analysis of fins ⓪ ① ② ③ ④ ⑤
   c) the design of fins ⓪ ① ② ③ ④ ⑤
   d) the use of a spreadsheet as a design tool ⓪ ① ② ③ ④ ⑤

COMMENTS:

Did Not help= (0)
Enhanced very much = (5)

3. Did the ability to use the program to explore the effect of several parameters on fin performance enhance your learning about fins? ⓪ ① ② ③ ④ ⑤

COMMENTS:

Figure 9 Student Survey (continued)

Strongly disagree = (0)
Strongly Agree = (5)

4. I knew what I was expected to do for this project. ⓪ ① ② ③ ④ ⑤

COMMENTS:

5. I had the knowledge and tools to effectively complete this project. ⓪ ① ② ③ ④ ⑤

COMMENTS:

6. What would you suggest to improve this project?

7. What would you suggest to improve the fin solver program?

8. Give this assignment a grade A – F.

____

Figure 10 Survey Results

Questions	Averages	
	MSU	Olivet-Nazarene
1a) Rate your understanding of fins before starting this project	2.9	2.6
1b) Rate your understanding of fins after completing this project	4.2	4.1
2a) How well did this project improve your understanding of the heat transfer principles of fins	3.7	2.9
2b) How well did this project improve your understanding of the analysis of fins	3.8	3.4
2c) How well did this project improve your understanding of the design of fins	4.1	4
2d) How well did this project improve your understanding of the use of a spreadsheet as a design tool	4.1	4
3) Did the ability to use the program to explore the effect of several parameters on fin performance enhance your learning about fins?	4.0	4
4) I knew what I was expected to do for this project	3.1	3.1
5) I had the knowledge and tools to effectively complete this project	4.2	3.1
Give this assignment a grade A – F	3.22	3.13

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