ASEE's 123rd Annual • Conference & Exposition • New Orleans, LA • June 26-29, 2016

New Orleans JAZZED about Engineering Education

Paper ID #16022

# An Algebra- and Trigonometry-based Emitter Location Technique for Radar Warning Receivers

**Prof. Gene L. Harding , PE, Purdue University (Statewide Technology)**

GENE L. HARDING is an associate professor of Electrical and Computer Engineering Technology at Purdue University, where he has taught for 13 years. He has three years of industrial experience with Agilent Technologies, 28 years of combined active and reserve service in the United States Air Force, holds an MSEE from Rose-Hulman Institute of Technology, and is a licensed professional engineer.

# An Algebra- and Trigonometry-based Emitter Location Technique for Radar Warning Receivers


Abstract

This paper explores a radio frequency (RF) emitter location technique for a notional fighter aircraft. The goal is to have the students apply concepts, such as antenna directionality and signal propagation characteristics, to find an emitter's location using only algebra, trigonometry, and a straightforward graph of antenna gain—no higher-level mathematics. In the scenario the fighter aircraft carries a radar warning receiver (RWR) that uses a four-element antenna array. The antennas are arranged in 90° azimuthal increments around the perimeter of the aircraft, and each antenna has a gain response that falls off logarithmically as the receive angle shifts away from boresight.

The author, a retired officer from the U.S. Air Force, teaches this technique in an Electrical Engineering Technology course titled Military RF Electronic Applications. Such techniques are part of what the military calls electronic warfare (EW): using the electromagnetic spectrum for our advantage, preserving its use for friendly forces, and preventing the enemy from doing the same.

This paper begins with a brief description of electronic warfare and the course in which this technique is taught, then the physical configuration of the antenna array on the notional fighter aircraft. Finally, there is a detailed explanation of how an emitter's location can be calculated from the relative signal powers received by the RWR antennas.


## Introduction

Electronic warfare (EW) is the "art and science of preserving the use of the electromagnetic spectrum for friendly use while denying its use to the enemy."[1] EW is a critical part of modern warfare, and is used to protect friendly and attack adversary communications, radar, and navigation systems. Moreover, the principles and fundamental techniques used in military applications are essentially the same as those used in civilian applications. These dual-use technologies include such diverse topics as wave propagation, antenna design and deployment, information coding, decreasing susceptibility to jamming, signal amplification, emitter geolocation, and many others.

The author developed a course, Military RF Electronic Applications, that uses military applications to explore a number of dual-use technologies. The course is divided into three blocks: 1) Antennas and RF Propagation; 2) Military Systems and Electronic Warfare Processing; and 3) Search, Emitter Location, and Jamming. The dual-use technologies explored include antenna design, RF propagation, link analysis, radar, signal search, emitter location, and emitter identification.[2] In block three, multiple emitter location techniques are studied, including a technique implemented on many aircraft that uses several matched antennas, one on each "corner" of the aircraft.[1] This exercise uses only algebra, trigonometry, and antenna gain plots to reinforce the key concepts of antenna directionality, antenna gain variability versus angle off boresight, use of one type of antenna array, and signal attenuation as a function of propagation distance.

## Emitter Location Technique

A common type of antenna used in modern radar warning receiver (RWR) systems has a gain pattern that is linear in dB (which means it is really a logarithmic response) with respect to angle of arrival (AoA).[1] This linear response is key to determining both AoA of the received signal and distance to the emitter. Note the gain characteristic depicted in Figure 1. At boresight, or 0° azimuth with respect to the antenna, the antenna's gain is at a maximum, $A_{dB\_BS}$ (dB gain at boresight). At ±90° the gain diminishes to a minimum value, $A_{dB\_90}$. Note that Figure 1 plots the gain in rectangular coordinates instead of the more traditional polar coordinates. This fact is also key to finding the signal's AoA and emitter distance. It will allow reaching a solution using straightforward algebra and trigonometry.

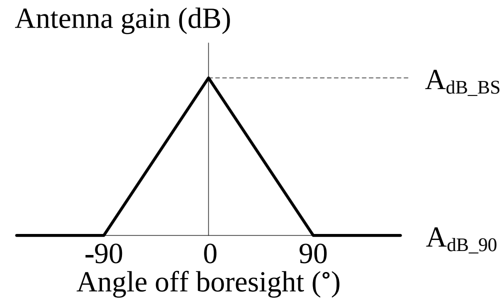



*Figure 1: RWR antenna with linear (in dB) gain*

In this scenario, four of these antennas are arranged on a notional fighter aircraft. They are spaced in 90° increments, and the array of four is offset 45° from the aircraft nose as shown in Figure 2.

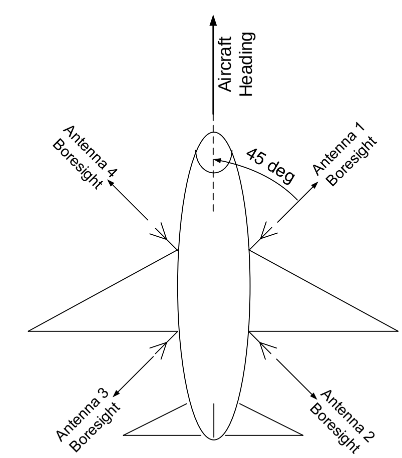



*Figure 2: Antenna configuration*

Figure 3 shows the antenna gain plot with the addition of an incident radar signal. The signal arrives at some angle θ off of the antenna's boresight. The receiver detects a signal with power $P1_{dBm}$. The problem, at this point, is that there is not enough information to find θ. Without θ there is no way to know how much gain the antenna has, which prevents calculation of the incident signal's actual power. More information is needed.

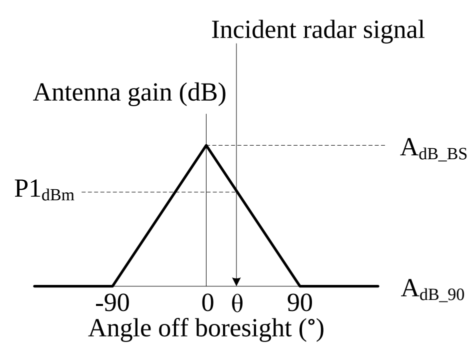



*Figure 3: RWR antenna with incident radar signal*

That information is obtained by adding a second antenna, oriented perpendicularly to the first, as shown in Figure 4. These two antennas are matched, so their gain characteristics are essentially the same. Their gains can be plotted together on rectangular axes, as shown in Figure 5. The first antenna, designated antenna 1, has its gain plotted in green using a solid line. The second antenna, antenna 2, has its gain plotted in blue using a dashed line. The gains overlap, with each antenna's gain diminishing to its minimum value at the adjacent antenna's boresight.

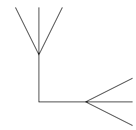


*Figure 4: Perpendicular antennas*

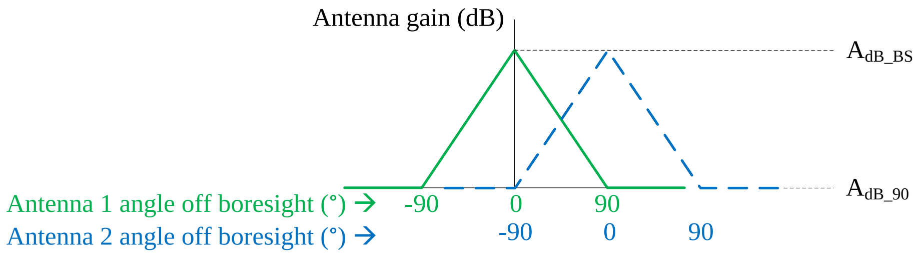



*Figure 5: Matched antenna gain patterns*

Figure 6 depicts the matched antenna gain patterns with the incident radar signal added. This gives two different received powers: $P1_{dBm}$ at receiver 1 (connected to antenna 1), and $P2_{dBm}$ at receiver/antenna 2. Look carefully at Figure 6. Note that the difference between the two received powers varies linearly with the angle off of boresight, from a maximum of $A_{dB\_BS}$ at 0° (antenna 1's boresight) to a minimum of 0 dB at 45°. (Beyond that the arrival angle is closer to antenna 2, so all of the following calculations would be reversed and worked with respect to antenna 2 instead of Antenna 1.) It is this relationship that allows translating received powers to AoA. Also note the symmetry regarding the angle of arrival. Theta (θ) is both the angular displacement from antenna 1's boresight and from antenna 2's -90° point. Likewise, if the $P2_{dBm}$ line is extended to the right, it intersects the antenna 1 gain plot at (90° - θ) from its boresight. Thus, moving from the $P2_{dBm}$ point to the $P1_{dBm}$ point along the antenna 1 gain plot is a horizontal displacement of (90° - 2θ) and a vertical displacement of ($P1_{dBm}$ – $P2_{dBm}$) in dB.

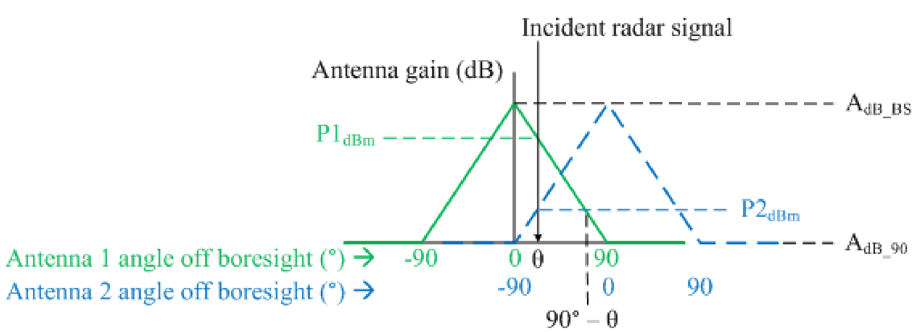



*Figure 6: Antenna pair with incident radar signal*

This is depicted graphically by the red triangle in Figure 7. Moreover, the slope of the gain plot

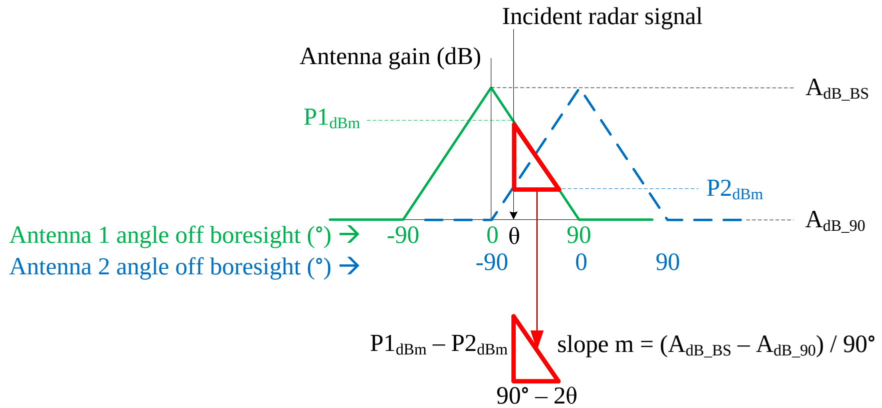



*Figure 7: Incident radar signal power/angle triangle*

is known. It is given by Equation (1):

$$m = \frac{A_{dB\ BS} - A_{dB\ 90}}{90°} \tag{1}$$

Thus, the angle θ is the only unknown:

$$m = \frac{A_{dB\ BS} - A_{dB\ 90}}{90°} = \frac{P1_{dBm} - P2_{dBm}}{90° - 2\theta} \tag{2}$$

Solving for theta:

$$\theta = 45°\left(1 - \frac{P1_{dBm} - P2_{dBm}}{A_{dB\ BS} - A_{dB\ 90}}\right) \tag{3}$$

In a similar manner, the antenna's gain at the angle of arrival can be calculated:

$$A_\theta = A_{dB\ BS} - m\theta \tag{4}$$

$$A_\theta = A_{dB\ BS} - \left(\frac{A_{dB\ BS} - A_{dB\ 90}}{90°}\right)\theta \tag{5}$$

$$A_\theta = A_{dB\ BS} - \left(\frac{\theta}{90°}\right)(A_{dB\ BS} - A_{dB\ 90}) \tag{6}$$

This gain can be subtracted from the power at Receiver 1 to get the power incident to the antenna:

$$P1ant_{dBm} = P1_{dBm} - A_\theta \tag{7}$$

The value from Equation (7) can then be used with the appropriate propagation loss equation to determine an approximate distance to the emitter.

There is one more calculation needed to fully characterize the direction to the emitter. Although θ specifies the direction with respect to the antenna's boresight, a more useful number is a line of bearing from the aircraft to the emitter. (This value does not change immediately when the aircraft's attitude changes.) Two additional pieces of information are required to perform this calculation: 1) the antenna's pointing angle relative to the nose of the aircraft, and 2) the aircraft's heading (assuming the aircraft is "heading" in the direction of its nose). The heading can generally be acquired from the aircraft's navigation system. The antenna pointing angle is dictated by the antennas' placement on the airframe.

As illustrated in Figure 2, one common configuration for aircraft RWR systems using this technique is an array of four antennas mounted on the airframe as follows: front right (starboard), back right, back left (port), and front left.[1] Figure 8 plots the four antennas' boresights and gain patterns relative to the airframe. The patterns for antenna 1 and antenna 2 are the same as above (solid green and dashed blue lines, respectively). The pattern for antenna 3 (left rear) is in the dotted purple line, and the pattern for antenna 4 (left front) is in the maroon

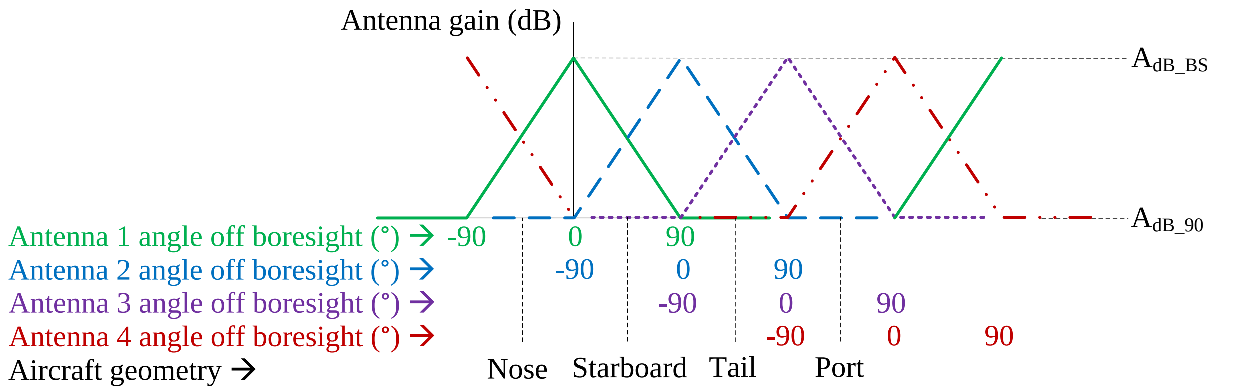



*Figure 8: Antenna placement and pointing relative to airframe*

dash-double dot line. Also note the addition of the aircraft geometry points: nose, starboard, tail, and port. By using Figure 8, the signal's AoA relative to the nose of the aircraft can be determined, then it can be adjusted to compensate for the aircraft's heading, giving a line of bearing to the threat emitter.

The following is a summary of the steps to find a threat emitter with this type of RWR system:

1. Determine quadrant for AoA (between two antennas with strongest signal).
2. Determine reference antenna (antenna with stronger signal).
3. Use P1, P2, and the gain slope to find AoA, θ, relative to the reference antenna's boresight.
4. Find bearing to the emitter by adjusting θ to account for the antenna's location on the airframe and for aircraft heading.
5. Use θ and the gain slope to find antenna gain at the angle of arrival.
6. Find the incoming power at the antenna by adjusting the power level at the receiver to account for antenna gain.
7. Use the appropriate propagation loss model to calculate distance to the emitter.

## Homework Assignment and Assessment

The emitter location technique described herein is covered by one homework assignment in the Military RF Electronic Applications course referenced previously.[2] The course is a 300-level elective course for juniors and seniors in a 4-year EET curriculum, and has been taught three times: spring 2011, spring 2013, and spring 2015. The assignment has been used all three times, although its form has been modified. In the initial course offering the assignment was a traditional paper homework assignment as shown in Figure 9. Due to a hard drive crash and bad CD-ROM backup, no assessment data is available for the first iteration of the course.

ECET 499 Intro to Military Electronic Apps
Homework #15: Emitter Location Problem
Due: 21 Apr 11

Consider the scenario depicted in Figure 1. A fighter aircraft is flying at a speed of 420 knots, an altitude of 17,200 meters, and a heading of 344°. Its radar warning receiver uses four directional antennas with gain characteristics as shown in the figure (linearly decreasing dB gain as signal direction moves away from boresight). The antennas are oriented at 90° with respect to each other, and the antenna set is rotated 45° with respect to the nose of the aircraft, as shown in the figure. Each antenna has the same gain characteristics: 12 dB at boresight and -6 dB at ±90°.

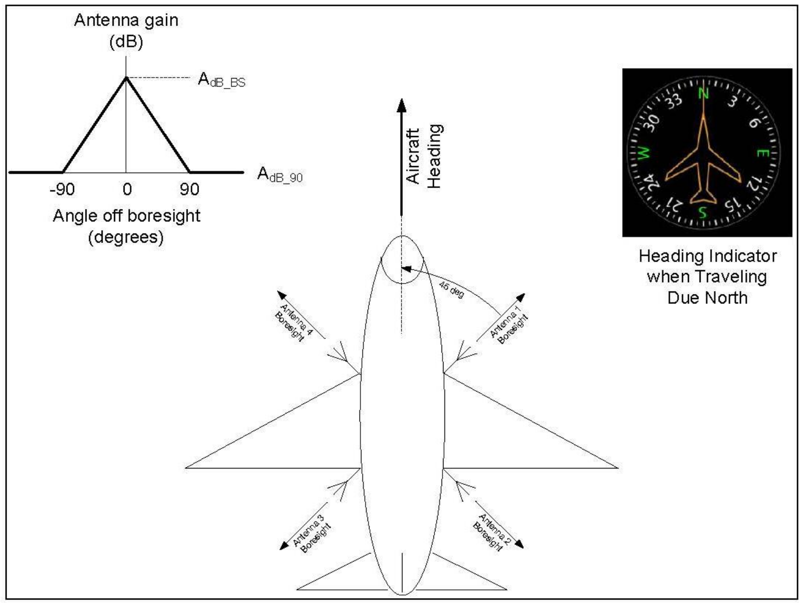



**Figure 1: RWR emitter location scenario**

Antenna 1 detects a radar signal with a strength of -71 dBm, and antenna 2 detects the same signal with a strength of -64 dBm.

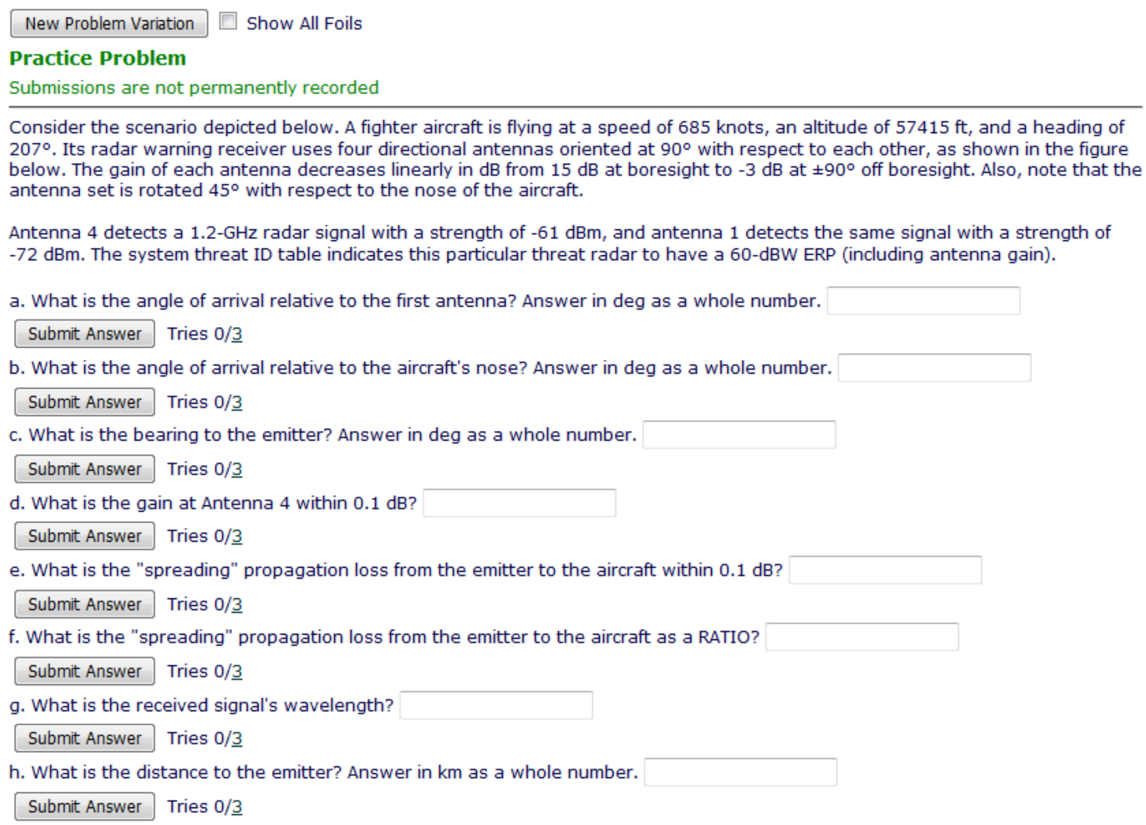
1. (35 pts) What is the bearing to the emitter? Answer in deg as a whole number.
2. (35 pts) What is the total received power? Answer in dBm as a whole number.
3. (30 pts) How far away is the emitter? Answer in km as a whole number.

*Figure 9: Spring 2011 RWR homework assignment*

For the spring 2013 course offering, the paper homework was converted to an electronic homework/quiz problem in Blackboard Vista™. Six students took the course, which was graded with the following scale: A—90.00-100%, B—80.00-89.99%, C—70.00-79.99%, D—60.00-69.99%, and F—below 60%. For the overall course, one student earned an A, three earned B's, and two C's. There were no D's or F's, although two other students dropped the course very early in the semester. For this particular assignment the average score was 85%.

When Blackboard transitioned from Vista to Learn, support for the features required to implement assignments like this was dropped, so the author migrated the assignment to LON-CAPA, a very powerful and flexible, but also free, open-source, distributed learning content management system.[3]

LON-CAPA provides the capability to implement multi-part problems, so the author modified the homework/quiz assignment in the spring 2015 course offering. A practice version of the problem was created that "walks" the student through the process of solving the problem. A screenshot of this version is shown in Figure 10.



New Problem Variation ☐ Show All Foils

**Practice Problem**

Submissions are not permanently recorded

Consider the scenario depicted below. A fighter aircraft is flying at a speed of 685 knots, an altitude of 57415 ft, and a heading of 207°. Its radar warning receiver uses four directional antennas oriented at 90° with respect to each other, as shown in the figure below. The gain of each antenna decreases linearly in dB from 15 dB at boresight to -3 dB at ±90° off boresight. Also, note that the antenna set is rotated 45° with respect to the nose of the aircraft.

Antenna 4 detects a 1.2-GHz radar signal with a strength of -61 dBm, and antenna 1 detects the same signal with a strength of -72 dBm. The system threat ID table indicates this particular threat radar to have a 60-dBW ERP (including antenna gain).

a. What is the angle of arrival relative to the first antenna? Answer in deg as a whole number.

Submit Answer Tries 0/3

b. What is the angle of arrival relative to the aircraft's nose? Answer in deg as a whole number.

Submit Answer Tries 0/3

c. What is the bearing to the emitter? Answer in deg as a whole number.

Submit Answer Tries 0/3

d. What is the gain at Antenna 4 within 0.1 dB?

Submit Answer Tries 0/3

e. What is the "spreading" propagation loss from the emitter to the aircraft within 0.1 dB?

Submit Answer Tries 0/3

f. What is the "spreading" propagation loss from the emitter to the aircraft as a RATIO?

Submit Answer Tries 0/3

g. What is the received signal's wavelength?

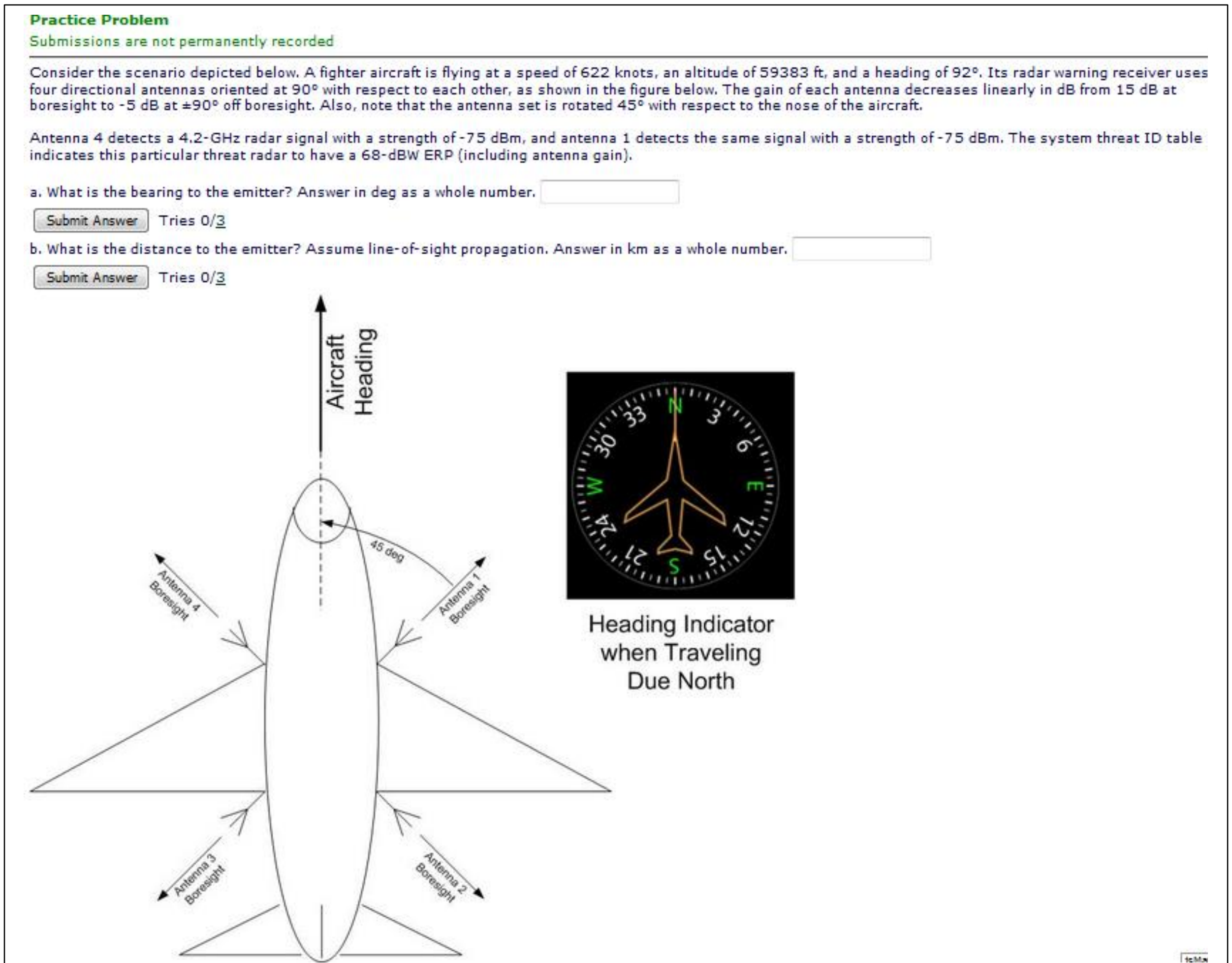
Submit Answer Tries 0/3

h. What is the distance to the emitter? Answer in km as a whole number.

Submit Answer Tries 0/3

*Figure 10: Spring 2015 multi-part practice problem*

The practice version is intended to help the students learn the process for solving the problem. There is also a "single-part" version of the practice problem, as shown in Figure 11. This is the version used in the graded quiz.



**Practice Problem**

Submissions are not permanently recorded

Consider the scenario depicted below. A fighter aircraft is flying at a speed of 622 knots, an altitude of 59383 ft, and a heading of 92°. Its radar warning receiver uses four directional antennas oriented at 90° with respect to each other, as shown in the figure below. The gain of each antenna decreases linearly in dB from 15 dB at boresight to -5 dB at ±90° off boresight. Also, note that the antenna set is rotated 45° with respect to the nose of the aircraft.

Antenna 4 detects a 4.2-GHz radar signal with a strength of -75 dBm, and antenna 1 detects the same signal with a strength of -75 dBm. The system threat ID table indicates this particular threat radar to have a 68-dBW ERP (including antenna gain).

a. What is the bearing to the emitter? Answer in deg as a whole number.

Submit Answer Tries 0/3

b. What is the distance to the emitter? Assume line-of-sight propagation. Answer in km as a whole number.

Submit Answer Tries 0/3



*Figure 11: Spring 2015 practice and graded problem*

Seven students took the spring 2015 course, which used the same grading scale as earlier offerings. Two students earned A's, three B's, and two C's. For this quiz assignment, the average score was 74%.

This problem relates to course objectives number 3 and 5:

3. Analyze and predict RF propagation characteristics under various conditions.
5. Apply a variety of EW techniques and characterize their effectiveness in different operating environments.

The problem was not used on the final exam in 2013, but the first portion of the problem, determining bearing to the emitter, was added to the 2015 final exam. The results were rather disappointing. The average score on the problem was 45%. It was worth six points out of 100 for the final exam. One student earned 6/6, four students 3/6, one student 1/6, and one student zero points. A careful review of the exams revealed that five of the seven students calculated θ correctly, but not a single student used the antenna gain plot, which was provided on the final exam, to aid calculation of the bearing to the signal.

## Conclusion

This paper explored a technique for emitter location, and how well the students in the course applied the technique. The notional scenario was a radar warning receiver on a fighter aircraft using an array of four matched antennas and receivers to geo-locate a threat emitter. The data from the last two offerings of the course indicated an adequate ability to apply the technique, with average scores of 85% and 74%, respectively.

On the other hand, when part of the problem was added to the final exam in the last iteration of the course, student performance was quite poor, 45%. A careful review of the students' exams revealed that most of them calculated the angle of arrival with respect to the antenna's boresight correctly, but only one student was able to successfully translate that to an accurate bearing to the threat emitter, and none of the students used a graphical aid to augment their calculations.

Two changes appear to be in order. First, the antenna gain plot could be reinserted into the figure used in the online problem. That would allow students to print and use it to aid in their calculations. Second, this technique should be emphasized a bit more during the course review, with particular emphasis on use of graphical aids.

---

[1] Adamy, D. (2001). *EW101: A First Course in Electronic Warfare*. Norwood, MA: Artech House.
[2] Harding, G.L. (2014). An Undergraduate Course in Military RF Electronic Applications, aka Electronic Warfare. *2014 Annual Conference Proceedings,* American Society for Engineering Education.
[3] The Learning Online Network with CAPA. Retrieved January 30, 2016, from http://lon-capa.org/.