An Algorithm for the Digital Demodulation of an Interferometer

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Abstract

As mechanical technology proceeds into the microscopic realm and sub-wavelength motion becomes a concern, interferometers, which are highly sensitive, non-invasive measurement tools, are finding increasing applications. The scope of this paper extends to the measurement of systems experiencing vibration, including applications in transducer calibration, acoustic sensing, and microelectromechanical systems (MEMS) characterization. We are investigating an interferometric measurement system based on a phase-generated carrier modulation scheme. Demodulation of the photodetected signal is performed in real-time using digital signal processing techniques. Our demodulation approach operates in the frequency domain, using the Goertzel algorithm to extract specific harmonics of the reference and target signals. We report a computationally efficient algorithm to demodulate the interferometer. We show this demodulation algorithm to be independent of system gain constant, the mean optical path length difference between the signal and reference paths, and the reference modulation depth.

Introduction

Interferometry, a highly accurate measurement technique with high sensitivity, has gained popularity as nanotechnology progresses and the detection of small displacements becomes a critical issue[1, 2]. The rapid expansion of the microelectromechanical systems (MEMS) industry and the integration of MEMS into practical technology in the fields of telecommunications, displays, and sensors[3] have led to an increasing demand for robust characterization techniques capable of *in situ* characterization of MEMS structures. Interferometry is well suited to such characterization due to its wide measurement dynamic range, its fine resolution, and its non-invasive qualities.

An optical interferometer detects the displacement of an object from a reference location by observing the interference pattern between two beams of light. An obstacle to be overcome in the measurement process is the stabilization of the interferometer at quadrature, its point of maximum sensitivity. An ideal interferometer has no need for active stabilization, but due to environmental conditions, such as air currents and temperature variations, feedback is needed to provide stabilization. Active phase tracking of the interferometer may be accomplished by employing a phase-generated carrier[4,5]. We have developed an interferometer according to the Michelson configuration, and we have incorporated a feedback controller as shown in Figure 1 to stabilize the interferometer's mean optical path length difference at quadrature[6]. The stabilized interferometer signal must then be demodulated to determine the vibration amplitude of the target.

Digital signal processing (DSP) is a useful method of analyzing and manipulating an interferometer signal. The architecture and instruction set of digital signal processors are optimal for the high-speed, mathematically intensive calculations involved in demodulation. The advantages of DSP include programmability, computational sophistication, flexibility, and low-cost. In this paper, a novel method of demodulating a stabilized interferometer using DSP is described. First, a theoretical foundation of interferometers is presented. The need for quadrature stabilization is motivated in this section. Next, we describe our demodulation algorithm, which is based on a phase-generated carrier modulation scheme. This demodulation technique results in the measurement of sub-wavelength vibration amplitudes. A brief discussion of the DSP implementation is provided. Finally, we summarize our demodulation approach and discuss its advantages.

Interferometer Theory

Calibrating an interferometer for optimal sensitivity is the primary step toward successful interferometric measurement. The relevant system parameters are defined as follows:

λ = 632.8 nm = wavelength of light,
K = system gain constant,
Φ_E = mean optical path length difference,

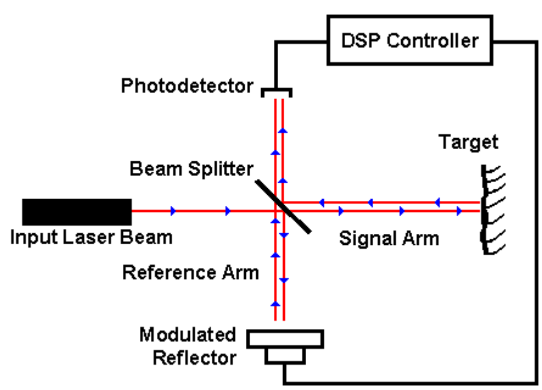

Figure 1. Michelson interferometer with feedback controller.

$J_n(\bullet)$ = nth order Bessel function of the first kind,
$k = 2\pi/\lambda$ = propagation constant in air and fiber,
b = reference amplitude,
a = target amplitude,
ω_r and ω_t = reference and target angular frequencies, respectively, and
t = time.

Assuming a target is vibrating through a displacement δ according to

$$\delta = a\sin(\omega_t t + \phi_t), \tag{1}$$

the ac component of the optical power output of a Michelson interferometer[6] is given by

$$P_{oAC} \propto K\cos(\Phi_E + 2\delta k). \tag{2}$$

Equation (2) may be expanded using a double angle trigonometric identity, yielding

$$P_{oAC} \propto K\{\cos(\Phi_E)\cos[2ak\sin(\omega_t t + \phi_t)] - \sin(\Phi_E)\sin[2ak\sin(\omega_t t + \phi_t)]\}. \tag{3}$$

The harmonics of this signal may be extracted using the Fourier-Bessel expansion[6] such that

$$\begin{aligned} P_{oAC} \propto K\{&\cos(\Phi_E)J_0(2ak) \\ &- 2\sin(\Phi_E)J_1(2ak)\sin(\omega_t t + \phi_t) \\ &+ 2\cos(\Phi_E)J_2(2ak)\cos(2\omega_t t + 2\phi_t) \\ &- 2\sin(\Phi_E)\sum_{m=2}^{\infty} J_{2m-1}(2ak)\sin[(2m-1)(\omega_t t + \phi_t)] \\ &+ 2\cos(\Phi_E)\sum_{n=2}^{\infty} J_{2n}(2ak)\cos[2n(\omega_t t + \phi_t)]\Big\}. \end{aligned} \tag{4}$$

Not only do the frequency components contain the vibration amplitude as a multiplicative factor, but they also suggest an equilibrium optical path length difference Φ_E. This equilibrium condition physically results in the greatest sensitivity in optical power caused by small deviations from equilibrium. The sensitivity of the optical power signal to changes in target amplitude may be determined by differentiating equation (2), resulting in

$$\frac{dP_o}{d\delta} \propto K'\sin(\Phi_E + 2\delta k). \tag{5}$$

Neglecting the vibration term $2\delta k$, we deduce that the interferometer is most sensitive at odd multiples of $\pi/2$. This is consistent with our intuition, since the slope of the optical power curve depicted in Figure 2 is greatest at odd multiples of $\pi/2$. Therefore, the peak-to-peak modulation of the target must be at least π radians to ensure that quadrature is found.

Ideally, a mean optical path length difference of $\pi/2$ may be achieved by precisely adjusting the separation between the target and the light source until the largest signal is detected. Environmental fluctuations, however, cause the interferometer to drift from quadrature. Stabilization of the interferometer at quadrature is important to maximize the sensitivity of the optical power output to vibrations.

Digital Demodulation Algorithm

Our interferometer may be applied to the measurement of a vibrating structure, which oscillates at a known frequency. Since our demodulation algorithm is based on a phase-generated carrier,

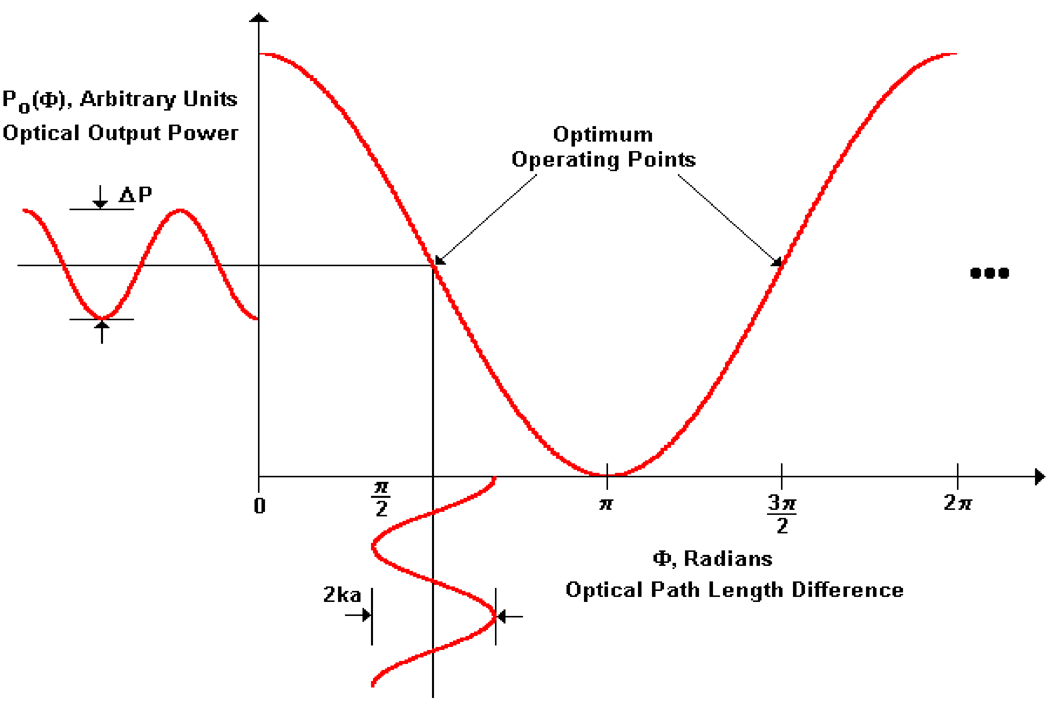

Figure 2. Interferometer optical power output as a function of the optical path length difference between the signal and reference paths. Note that oscillations about a mean optical path length difference of $\pi/2$ results in the greatest signal magnitude.

let us modify equation (2) for the interferometer's optical power by adding a term for a reference modulating through $2bk = \pi$ radians. The result is

$$P_{oAC} \propto K \cos\left[\Phi_E + 2ak \sin(\omega_t t + \phi_t) + 2bk \sin(\omega_r t + \phi_r)\right]. \tag{6}$$

A truncated Fourier-Bessel expansion of this equation reveals the new harmonic content of this signal to be

$$\begin{aligned} P_{oAC} \propto K\{ & \cos(\Phi_E) J_0(2ak) J_0(2bk) \\ & - 2\sin(\Phi_E) J_0(2ak) J_1(2bk) \sin(\omega_r t + \phi_r) \\ & + 2\cos(\Phi_E) J_0(2ak) J_2(2bk) \sin(2\omega_r t + 2\phi_r) \\ & - 2\sin(\Phi_E) J_1(2ak) J_0(2bk) \sin(\omega_t t + \phi_t) \\ & - 2\cos(\Phi_E) J_1(2ak) J_1(2bk) \cos(\omega_t t + \omega_r t + \phi_t + \phi_r) \\ & - 2\cos(\Phi_E) J_1(2ak) J_1(2bk) \cos(-\omega_t t + \omega_r t - \phi_t + \phi_r) \\ & + 2\cos(\Phi_E) J_2(2ak) J_0(2bk) \sin(2\omega_t t + 2\phi_t) \\ & - 2\sin(\Phi_E) J_2(2ak) J_1(2bk) \sin(2\omega_t t + \omega_r t + 2\phi_t + \phi_r) \\ & - 2\sin(\Phi_E) J_2(2ak) J_1(2bk) \sin(-2\omega_t t + \omega_r t - 2\phi_t + \phi_r) \ldots\}. \end{aligned} \tag{7}$$

Two well-chosen frequency components are sufficient for demodulation. These components are

$$\begin{aligned} & S_C = K * -2\sin(\Phi_E) J_0(2ak) J_1(2bk) \sin(\omega_r t + \phi_r) \text{ and} \\ & S_D = K * -2\sin(\Phi_E) J_2(2ak) J_1(2bk) \sin(2\omega_t t + \omega_r t + 2\phi_t + \phi_r). \end{aligned} \tag{8}$$

The angular frequencies of the signals S_C and S_D are ω_r and $2\omega_t + \omega_r$, respectively. By taking the ratio of the magnitudes of these signals, the terms $\sin(\Phi_E)$, $J_1(2bk)$, and K are strategically removed. In this manner, the resultant signal is independent of the equilibrium optical path length difference, Φ_E, the reference modulation amplitude, b, and the system gain constant, K. The resultant ratio is

$$\frac{|S_C|}{|S_D|} = \frac{|J_0(2ak)|}{|J_2(2ak)|}. \tag{9}$$

A root-finding algorithm, such as bracketing and bisection[7], may be used to extract the argument $2ak$ of the Bessel function ratio. Division of this result by $2k$ reveals the desired vibration amplitude, a.

The demodulation algorithm described above is depicted in the flow chart of Figure 3. This flow chart also shows that two additional signals at other frequency harmonics are required for stabilization of the interferometer[6].

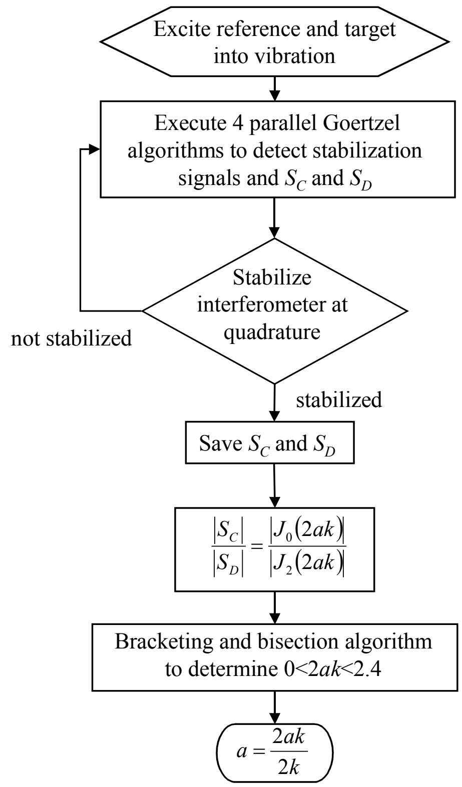

Figure 3. Flow chart for the DSP algorithm to stabilize and demodulate an optical interferometer.

Figure 4 plots the zero-to-peak vibration amplitude a versus the Bessel function ratio $|J_0(2ak)|/|J_2(2ak)|$, indicating a maximum detectable amplitude. Because the Bessel function ratio exhibits recurring poles and zeros with increasing values of a, amplitude certainty can only be guaranteed below the first zero. The maximum unambiguously detectable amplitude at the optical wavelength of λ = 632.8 nm occurs at $a_{\max} \approx$ 120 nm. This is known as the distance ambiguity function. The theoretical minimum detectable displacement due to quantum shot noise for this phase-generated carrier demodulation scheme is $a_{\min}$ = 5 pm[5].

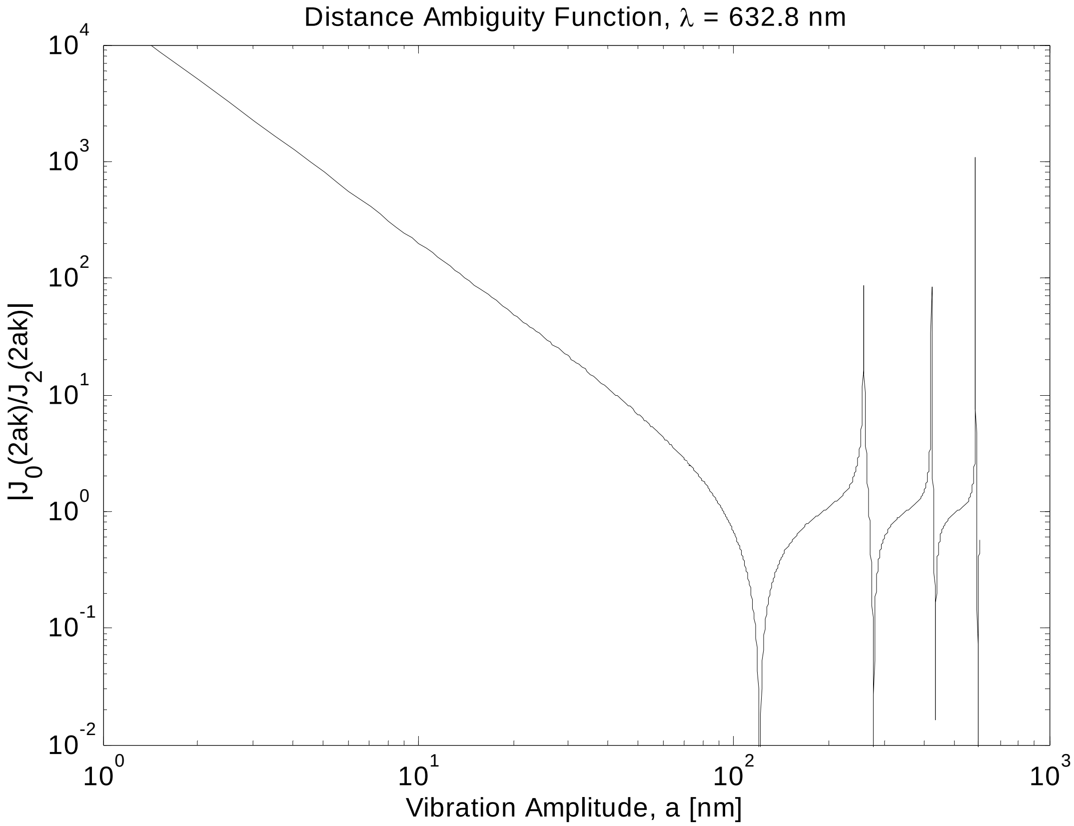

Figure 4. The distance ambiguity function of vibration amplitude, *a*, versus $\left|J_0(2ak)\right|/\left|J_2(2ak)\right|$ at a wavelength λ = 632.8 nm. The wavelength determines the maximum amplitude that can be unambiguously demodulated, which occurs at the first zero; for the case λ = 632.8 nm, $a_{max} \approx$ 120 nm.

DSP Implementation

The TMS320C31 32-bit floating-point digital signal processor, provided with the Digital Signal Processor Starter's Kit (DSK) available from Texas Instruments, Inc., shall be used to stabilize and demodulate the interferometer. The device parameters are identical to those specified for the stabilization algorithm[6]. A photodetector transforms the optical power signal into a voltage signal, which is fed to the analog input of the DSK. The DSK measures the magnitudes of the requisite frequency components of the interferometer signal and generates a feedback signal to adjust the mean position of the modulated reference according to the stabilization algorithm[6]. Once stabilization has been achieved, the DSK demodulates the interferometer signal using signals S_C and S_D defined by equation (8). The calculated target vibration amplitude is then sent to a computer interface.

Summary and Conclusions

We have reported the development of a DSP algorithm to demodulate an optical interferometer for the detection of sub-wavelength vibration amplitudes. While time-domain interferometric demodulators capable of measuring large displacements are currently available[8], our demodulation algorithm operates in the frequency domain. The algorithm manipulates specific frequency components of the interferometer output signal to demodulate the interferometer. This demodulation scheme is mathematically independent of the mean optical path length difference, the reference modulation depth, and the system gain constant.

Utilizing a laser source with wavelength $\lambda = 632.8$ nm, the technique manifests a theoretical dynamic range from 5 pm to 120 nm. Future research includes experimental demonstration of this demodulation scheme and the investigation of a means of resolving the distance ambiguity function to extend the maximum detectable amplitude.

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