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An Analysis of STEM Students' Integral and Area Under the Curve Knowledge

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Abstract. STEM students' ability to combine a concept's possible image that can be structured mentally, and its definition based on their integral knowledge is important particularly for area and integral related engineering applications. Detailed oriented application to the solution under certain conditions is also important; for instance, the relationship between integral calculation and area under the curve needs to be treated carefully in calculations. The data analyzed in this work was collected from 24 STEM students at a mid-sized Northeastern university that either enrolled or completed the second 4-credit course in the United States during 2020 and 2021 years. The participants completed a questionnaire and had gone through video recorded interviews to explain their written questionnaire responses by following an Institutional Review Board (IRB) process attained for the research. Action-Process-Object-Schema (APOS) theory is used for evaluation of the research question, along with the concept image and concept definition approach of Dreyfus et. al [1]. The written responses alone were not sufficient neither for APOS classification nor for concept image and concept definition application in which case video recorded interviews helped with this classification of the participants. The results indicated the need for designing more interactive calculus assignments, exercises, and quizzes for concepts that require better mental construction of such concepts by involving participants actively as a part of the questions asked in assignments, exercises, and quizzes.

1. Introduction

In pedagogy, researchers needed to observe students' comprehension of the function concept. The pedagogical approach to the function concept in the undergraduate curriculum was not explored until the 1970s. The concept image and concept definition of functions in mathematical education were defined in [1] with a geometric approach and again in [2] (pg. 153); however, the most extensive research in the undergraduate curriculum was done in [1], in which they defined the concept image and concept definition of functions based on their research of undergraduate students. In this work, concept image and concept definition of functions' definite integral will be used to understand undergraduate and graduate STEM majors' ability to relate the area between a function and the input axis to the definite integral of functions.

Action-Process-Object-Schema (APOS) theory is used in mathematics and engineering education to evaluate conceptual knowledge of undergraduate students' conceptual classification in topics such as function, limit, derivative, and integral. APOS theory is particularly useful in measuring students' knowledge of a specific concept by determining how much students know about the prerequisite concepts taking place in this subject. Students' cognitive improvement of concepts in APOS theory is questioned with in-depth questions. For instance, APOS theory can be applied to understand students' definite integral knowledge by determining their conceptual knowledge in the prerequisite topics such as function and derivative.

The conducted research received Institutional Review Board (IRB) approval at a university located on the Northeastern side of the United States. The participants are 24 undergraduate engineering students from different disciplines and backgrounds. The quantitative data collected consisted of written responses of the research participants to the integral question that require knowledge of different calculus concepts. The collected qualitative data consisted of the transcription of the participants' video-recorded follow-up interviews. Participants' APOS classification as well as how much their concept image is matching with the concept definition knowledge for the stated question are evaluated in this work. The collected data was analyzed by using additional questionnaire data as a part of the written and interview responses. The results indicated that the participants had the knowledge of integral concept from area analysis perspective, however they did not respond to the question correctly.

The rest of the article is organized as follows: Section 2 contains the relevant research literature results to concept image and concept definition as well as APOS theory. Analysis of the collected data is outlined in Section 3 by using these two pedagogical methods. Last section is devoted to a summary of the research presented in this work and suggestions to educators for potential improvements that relate to anti-derivatives.

2. APOS Theory and Concept Image and Concept Definition

APOS theory was applied recently on calculus-based applications on a variety of concepts to understand and improve STEM majors' calculus sub-concept through suggestions to the educators [3-18]. These applications included sub-concepts such as functions, derivatives, limits, Riemann sums, and the use of technology for advancing calculus education. APOS theory application in the pedagogical literature for understanding students' mental construction of integral concept from a geometric perspective is limited [3]. One of the studies that relate to this interest was conducted in [19] by observing students' ability to approximate integral calculations by sampling points during the process of writing a code for integral calculations. It is pointed out in [19] that integration needs to be thought from two perspectives: *as the culmination of a limiting process and the* *application of this process over an interval of variable length, as producing a correspondence.* The importance of designing a curriculum with instructional emphasis in algebra and pre-calculus by helping students to develop images of arithmetic operations in analytically defined functions as operations on functions was also pointed out. This approach may help students to develop a better understanding of the concepts with a better mental construction of concepts and sub-concepts taking place in many calculus questions.

Development of Action, Object and Process (APO) idea in mathematics education for the undergraduate curriculum was initiated in [19] through a study on students' conceptual view of the function similar to the concept image and concept definition application in mathematics [20]. APO is extended to Action, Process, Object and Schema theory (called APOS theory) in [21] to understand students' function knowledge and this theory is explained as the combined knowledge of a student in a specific subject based on Piaget`s philosophy. APOS theory was designed in [22] as follows:

- *An action is a transformation of objects perceived by the individual as essentially external and as requiring, either explicitly or from memory, step-by-step instructions on how to perform the operation...*
- *When an action is repeated and the individual reflects upon it, he or she can make an internal mental construction called a process which the individual can think of as performing the same kind of action, but no longer with the need of external stimuli...*
- *An object is constructed from a process when the individual becomes aware of the process as a totality and realizes that transformations can act on it...*
- *A schema is an ... individuals' collection of actions, processes, objects, and other schemas which are linked by some general principles to form a framework in individual's mind...*

APOS theory on area calculations are explained in [23] as follows:
In Calculus: *Actions are needed to construct an estimate of the definite integral as the area under a curve: for example, in dividing an interval into specific subintervals of a given size, constructing a rectangle under the curve for each subinterval, calculating the area of each rectangle, and calculating the sum of the areas of the rectangles.*
…The area under the curve for a function on a closed interval is the limit of Riemann sums—an Action applied to the Riemann sum Process. In order to determine the existence of this limit and/or to calculate its value, the student needs to encapsulate the Riemann sum Process into an Object.
APOS theory was recently used to better understand and improve STEM undergraduate and graduate students' ability to respond to a variety of calculus questions [3-18]. Evaluation of the results indicated a variety of APOS classification of the participants depending on the research question for analysis of integral, series, function, limit, derivative, and asymptote knowledge. Application of APOS theory in other areas of interest included vector space concept-related

observations in [24], mean, standard deviation, and the central limit theorem related conceptual understanding in [25], and observing students' ability to construct and develop two-variable functions in [26] and [27].

The development of the individual schemas can also be accomplished by using the triad classification in APOS theory; a progression of three stages proposed in [28]. Triad stages Intra, Inter, and Trans are used in [5] to investigate how STEM students’ ability to relate integral to area under the curve. The APOS theory classification is determined to be insufficient therefore they included the schema development idea. The following triad classification was used by the researcher:

- ***Intra Stage:*** *Students' classified in this category if they were able to recognize the connection between the area concept and the definite integral of the given function but did not necessarily remember other details.*
- ***Inter Stage:*** *Students qualified to be in the intra stage are also classified to be in the inter stage if they recognize the need of an absolute value to find the area and explain the concept image properly.*
- ***Trans Stage:*** *Inter stage students are qualified to be at the Trans stage if they were able to explain the area and integral connection through approximation of the integral by using rectangles.*

The corresponding analysis indicated students to be classified in the intra stage if they knew some of the derivative rules and apply the chain rule [5], however did not know the relationship between these rules. Inter stage classification was based on the ability to begin collecting all different relevant concepts and recognize that they are related, and the trans stage classification required to construct and apply a variety of several concepts correct simultaneously.

In this work the following classification is applied for the research question’s APOS analysis:

- **Action:** Students of this category recognized the connection between area and integral at a basic level and use the corresponding justification.
- **Process:** Participants fulfilled the above-mentioned Action stage and were expected to make this a process through the connection between the definition and the image.
- **Object:** An object is constructed as a result of matching the concept image and concept definition at a basic level for one particular aspect of the expected solution (e.g. area above the curve).
- **Schema:** This stage required the participant to structure a correct solution by considering all possible connections between the concept image and concept definition by recognizing the details on the expected outcomes.

3. Research Data Collection Protocol & Research Question

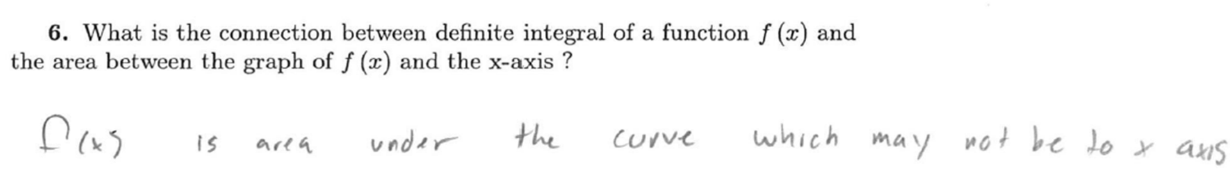
Institutional Review Board (IRB) approval was attained to collect data from engineering and mathematics undergraduate and graduate students who were either enrolled or completed a calculus 2 course at a mid-sized Northeastern university in United States. The collected data in this work included 24 STEM students continuing their education during 2020 and 2021 academic years. The participants completed a questionnaire and had gone through video-recorded interviews to explain their written questionnaire responses. Each participant was compensated for their participation in the research upon completing a questionnaire in 80 minutes and interviewing for about 40 minutes; the follow-up interviews were conducted to have a better understanding of students' written questionnaire responses that had a major role in classification analysis of the responses. Post-interviews were conducted by the principal investigator (PI) of the research and designed to have a better understanding of the pre-interview responses of the participants and all the interviews were video recorded and transcribed for analysis purposes. The following research question was provided to the participants as a part of the written questionnaire.

Question: What is the connection between definite integral of a function f(x) and the area between the graph of f(x) and the x-axis?

The research question was evaluated by using a combination of concept image and concept definition as well as APOS theory based on the corresponding APOS classification to be stated in the following section that also contains the qualitative and quantitative response analysis of the participants. The concept definition relates to the absolute value of the definite integral that is desired to be determined to calculate the area while concept image requires to make a connection to this definition. Noting the generic nature of the research question, the participants were expected to recognize details in their responses to the question that did not appear to be reflected in their written responses. The video-recorded oral interviews played a major role in methodological classification of the collected data.

4. Analysis of Collected Data

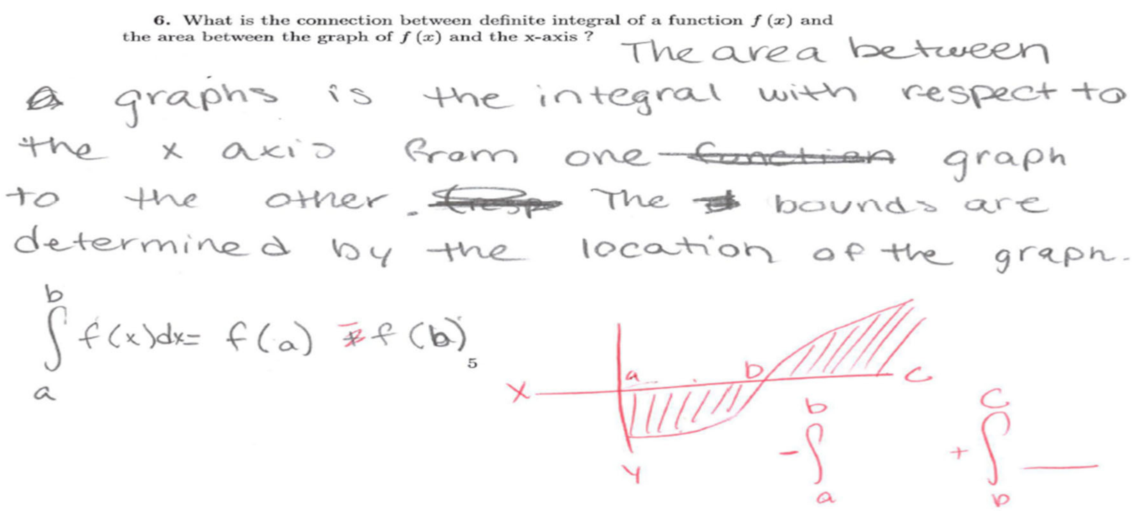
In this section we display the responses of some of the participants with a variety of approaches to the solution of the question. Given that the question may seem to require a response of "yes" or "no", the participants' initial reaction to the question was a "yes" with some form of justification to explain that the integral is the same as the area under the curve as stated by RP 16 below in Figure 1.


6. What is the connection between definite integral of a function $f(x)$ and the area between the graph of $f(x)$ and the x-axis ?

f(x) is area under the curve which may not be to x axis

Figure 1. Response of RP 16 to the research question with verbal justification.

One important aspect of this response is the words used to express the answer. The meaning behind the expression "… which may not be to x-axis" in this figure is subject to further explanation required by the student that doesn't necessarily support the outcome even if the answer was correct. This particularly indicates the importance of the justification that is not recognized by all participants. Student RP 4 in Figure 2 below further explained the connection between a definite integral and the area under the curve.



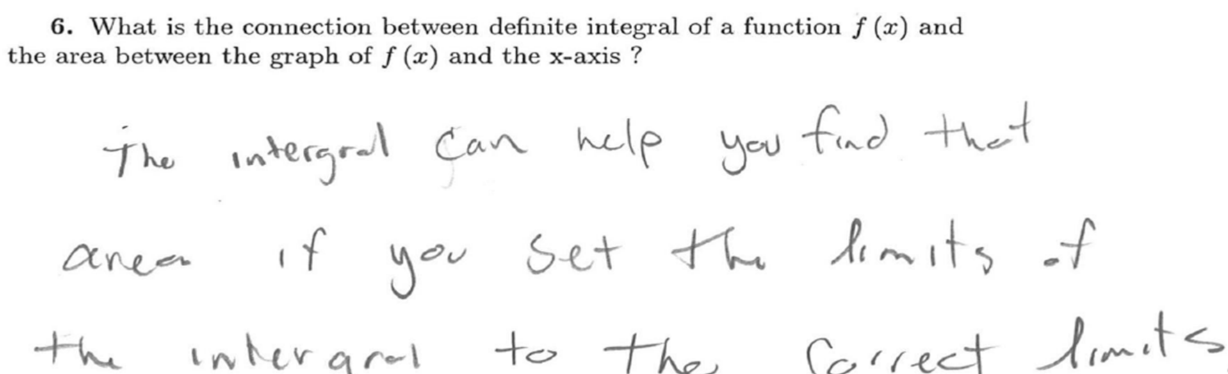
6. What is the connection between definite integral of a function $f(x)$ and the area between the graph of $f(x)$ and the x-axis ?

The area between graphs is the integral with respect to the x axis from one ~~function~~ graph to the other. ~~resp~~ The bounds are determined by the location of the graph.

$\int_a^b f(x)dx = f(a) \neq f(b)$

Figure 2. Response of RP 4 with a figure drawn to justify the response during the interview.

RP 4 recognized the mistake made in the response by changing the written response of definite integral during the interview. This also highlights the importance of some of the further details on the design of the question to require additional information to understand STEM students' conceptual understanding. This may require structuring questions that would help the participant realize step-by-step explained solutions. Similarly, response of RP 9 displayed in Figure 3 below "… the correct limits" is not supporting the correct justification to the question.



6. What is the connection between definite integral of a function $f(x)$ and the area between the graph of $f(x)$ and the x-axis ?

The intergral can help you find that area if you set the limits of the intergral to the correct limits.

Figure 3. Response of RP 9 with the justification of needing correct integral limits for integral to be the same as area of the curve.

The response of RP 6 in Figure 4 indicates the concept image and concept definition that supports participants' responses. This participant drew the graph of a curve above the x-axis and indicated that the area is same as the integral calculation. Noting that this is a partial solution, this participant was able to recognize the possibility of the curve to be below the x-axis and the need for the absolute value of the integral during the oral interview.

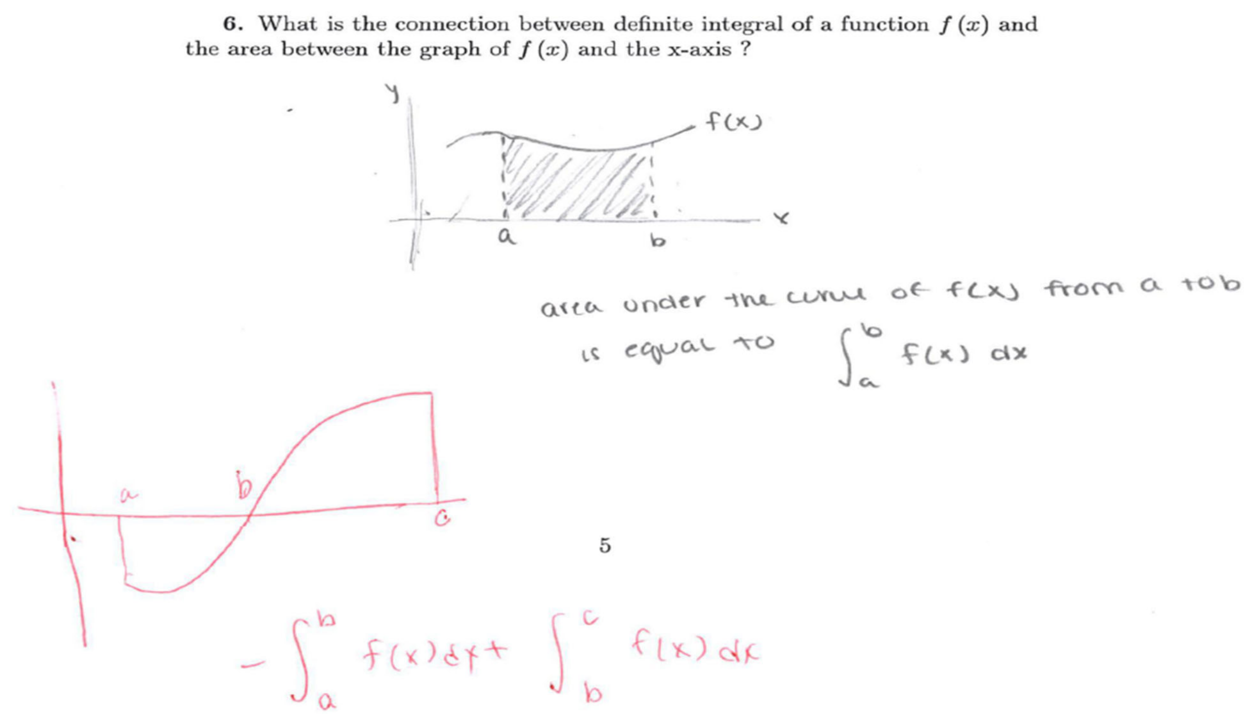


Figure 4. Response of RP 6 with a figure drawn to justify the response during the interview.

Some of the participants such as RP 18 explained the answer by stating "…a part of the area…" in Figure 5. During the oral interview this participant's justification didn't change.

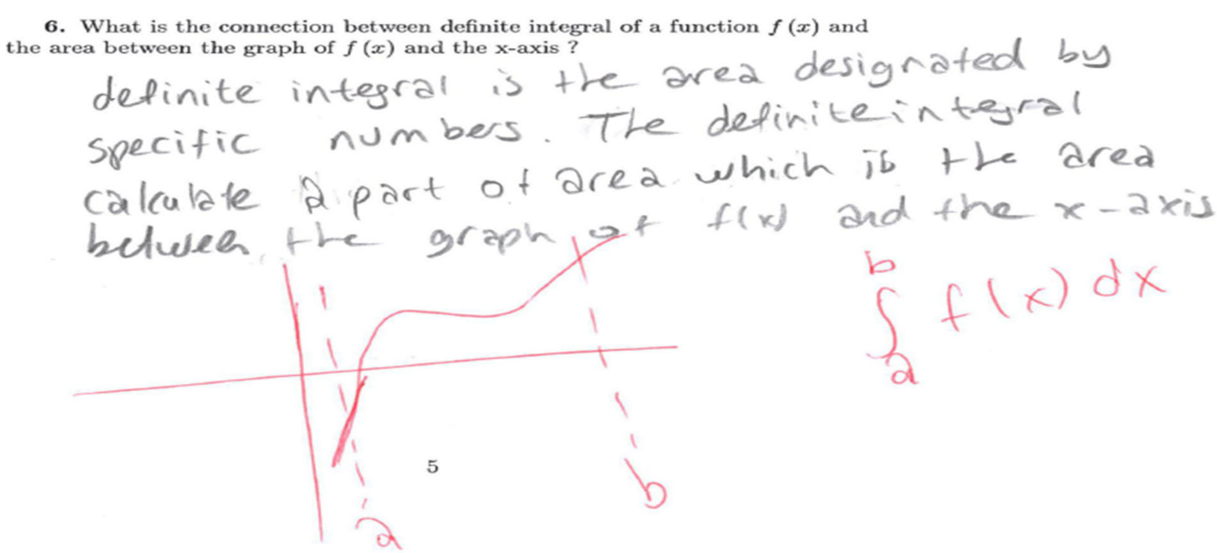


Figure 5. Pre- and post-response (in red marking) of RP 18 to the research question.

Using the pre- and post-responses of the participants, Table 1 contains the APOS classification of the participants using the APOS definitions introduced in Section 2. In this APOS classification 54.17% Action classification of the participants is due to the basic level responses for matching integral and area calculations. Process classification indicated participants better justification of the integral and area connection through the modulation of the integral as a result of absolute value utilization. Object indicates students' ability to make the Process applicable to integral and absolute value with the corresponding x-axis interval understanding. Schema level classification reflects all of Action, Process, and Object levels fulfilled and correct response is attained as a result of the response given.

	Action	Process	Object	Schema
APOS Participant Classification (RP)	1, 3, 7. 8, 9, 11-17, 21, 23, 24, 26	2, 12, 17, 19, 25	4, 10, 18, 20	5, 6
APOS Classification Percentage	54.17%	20.83%	16.67%	8.33%

Table 1. APOS classification and the corresponding percentages of this classification.

5. Conclusion & Future Work

In this work, upon receiving Institutional Review Board (IRB) approval at a university located on the Northeastern side of the United States, 24 STEM students' responses to a research question were collected qualitatively and quantitatively. Collected data was analyzed using APOS theory and concept image and concept definition based on the relationship between definite integral and area. The collected qualitative data consisted of the transcription of the participants' video-recorded follow-up interviews. Participants were compensated for providing written responses and participating oral interviews. The written results remained at very basic level of conceptual understanding for using the two theories to analyze the data however the oral interviews helped with the classification and better understanding of the participants responses. Through observations, there were several factors that played important roles in the responses of the participants:

- The verbal statements used by the participants needs to be emphasized by the educators for the students to convey their message
- The concept image and concept definition matching need to be established as a part of integral education; much better examples emphasizing the corresponding match in STEM education is necessary through geometric demonstrations using real-life examples.
- Instructors need to pay attention to the development of the right language developed by the students that takes place in the conceptual coverage.

- Questions that are designed to cover the concepts step-by-step for better investigation of students' concept understanding. For instance, several curves provided to the research question designed in this research can help the students realize that the integral-area connection is absolute value dependent.

The research question also pointed out the difficulty of participants to respond to a question that required the knowledge of both integral and absolute value in relation to the absolute value. The written responses collected for the research question reflected the depth of students' knowledge based on the integral knowledge without using the absolute value associated with it. If this question were asked on an exam or an assignment, even the most successful students would appear to fail providing a perfect answer. The detail-oriented thinking appears to be the main obstacle; however, this detailed thinking reflects the participants' education; Therefore, it is important for STEM educators to provide notes, exercises, and solved questions for students to develop a deep understanding of the concepts. The mismatch between the concept image of area between the curve and the input axis and the corresponding definition needs to be emphasized. The APOS classification of the participants also supports this outcome. We invite other researchers to conduct similar studies to expand on the approach followed in this study. The coupling of APOS theory with concept image and concept definition idea appeared to be very useful in research data analysis.

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