AC 2012-3053: AN ANALYSIS OF THE EFFECTIVENESS OF USING EXCLUSIVELY WORKSHOP-STYLE INSTRUCTION IN THE COLLEGE ALGEBRA CLASSROOM, FOCUSED ON ENGINEERING AND ENGINEERING TECHNOLOGY UNDERGRADUATES

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An analysis of the effectiveness of using exclusively workshop-style instruction in the College Algebra classroom

Introduction

Over the past few decades, evidence has grown for the effectiveness of collaborative learning in Science, Technology, Engineering, and Mathematics (STEM) fields. Hundreds, if not thousands, of studies have been conducted under various conditions – different subject matter, different levels, varying proportions of class time spent on collaborative learning versus traditional lecture-style instruction – and the clear trend in the studies indicates an improvement in the performance and the attitudes of students who participate in collaborative learning environments.

So compelling is the evidence in favor of collaborative learning that it has gained the support of such organizations as the American Association for the Advancement of Science[2], the National Science Foundation[17], and the Mathematical Association of America[15]. The accreditation board for engineering and technology (ABET) now includes collaborative learning activities in their accreditation criteria[1].

Despite general agreement over the effectiveness of collaborative learning, the question of how best to balance collaborative learning with traditional lectures remains open. Many universities now offer courses in which traditional lectures are supplemented with collaborative learning workshops once or twice a week. Students taking Calculus I and Calculus II as part of the Emerging Scholars Program at the University of Texas at Austin (http://cns.utexas.edu/community/emerging-scholars/) or the MathExcel Program at the University of Kentucky (http://www.ms.uky.edu/~mathexcel/) spend six hours each week in collaborative workshops in addition to their traditional three large lecture hours. In the Merit Workshop program at the University of Illinois at Urbana-Champaign (http://www.merit.uiuc.edu/), calculus students spend two or three hours per week in a large lecture and four hours per week in workshops; precalculus students spend three hours in a large lecture and attend a two-hour workshop. The University of Illinois also offers Calculus II in a workshop-only format.

Southern Polytechnic State University (SPSU) is a special-purpose institution in the University System of Georgia, with close to 5000 students enrolled. Many of the students are nontraditional. The school's mission is to offer both traditional and nontraditional students bachelors and masters degrees and continuing professional development in the sciences, engineering, engineering technology, applied liberal arts, business, and professional programs.

Despite the technical focus of the degrees at SPSU, typically around 40% of incoming students are placed into College Algebra by the university's mathematics placement exam. Retaining these underprepared students pursuing engineering or engineering technology degrees at institutions similar to this one and providing them with a strong mathematical background is an essential component of increasing graduates in engineering fields.

Unfortunately, like many universities, SPSU has a low success rate in College Algebra. Students are required to earn a C or better to move on to Precalculus. Between 2005 and 2009, among all students beginning the course in the fall semester, the average percentage of students receiving a C or better was 53.8%. Spring success rates have been even lower.

In Fall 2009, in an effort to improve this success rate, SPSU offered a section of College Algebra in which all learning was done collaboratively in workshops; there was no lecture component to the course. In this paper, we compare the performance and the attitudes of the students in this collaborative learning section to that of a traditional lecture section taught by the same instructor, the first author. In some cases we break out the analysis for engineering majors specifically. Statistical analysis at the end of the semester indicated very little difference in the performance of the two sections. Given the additional resources required for the workshop-only format, this outcome does not support the continued offering of the workshop-only approach. However, there is some evidence indicating that a combined lecture and workshop approach may be superior to a workshop-only approach at this level.

The rest of the paper is organized as follows: In the next section, we review the literature related to collaborative learning. We continue by describing in detail the methods used in the study, which is followed by a section on the results of the study. The final section includes some additional observations and conclusions and describes future work.

Literature Review

Several meta-analyses and reviews of the literature have been published that found significant empirical support for cooperative and collaborative learning. (There is some dispute as to the definitions of "cooperative" and "collaborative" in this context. We use both interchangeably and in their broadest sense, to indicate an environment in which students work together in small groups.) Johnson et al.[12] analyzed 168 studies comparing cooperative, competitive and individualistic learning approaches among college students. They concluded that on average, college students who would score at the 50th percentile level when learning competitively will score in the 69th percentile when learning cooperatively. Students who would score at the 53rd percentile level when learning individualistically will score at the 70th percentile when learning cooperatively. In addition, they found improvements in "social support", positive attitudes toward learning, and self-esteem in the cooperative environment. Similarly, in earlier work reviewing 90 years of research, Johnson et al.[11] found that cooperation improved learning outcomes in all areas, relative to individual work.

Springer et al.[22] performed a meta-analysis of 39 different studies between 1980 and 1997. In this study of close to 4000 mathematics, science, engineering, and technology students, they found that small-group learning helped improve students' attitudes toward learning, as well as their determination to understand the material. The study also compared the effect of small, medium and high amounts of group time. They found the greatest achievement-related effects under a medium amount of group work, and a high amount of group work achieved the greatest favorable effects on students' attitudes. In addition, the positive effect of small-group learning on students' achievement was significantly greater for groups composed primarily or exclusively of African Americans and Latinos as compared with predominantly white and relatively heterogeneous groups. The most significant attitude improvements related to group work were found in groups of women.

Another review of the research, compiled by Prince[19], found broad empirical support for collaborative learning, both for enhanced academic achievement and positive attitudinal outcomes. Additional reviews of the literature include: Davidson and Kroll[5] (1991), Sharan[20], and Slavin[21].

More recent literature exists citing specific examples of the academic benefits of cooperative and collaborative education in the STEM classroom. Some examples include Calculus [8,16,26], Geometry[9,27], Real Analysis[7], Statistics[13] , Electrical Engineering[4], and General Chemistry[14]. Other studies found no significant difference in learning outcomes, but an improvement in students' attitudes, such as confidence level and completion rates[6], team skills[24,18] and retention of under-represented groups[10,3].

While a significant amount of published research exists on collaborative learning, there is an absence of that quantitatively analyzing the benefit of fully workshop-style teaching in college level mathematics. The closest paper is a study of high school mathematics in Greece where students were split into groups with reading guides to read and understand sections during class. However, at the end of each section, the instructor summarized the key points and cleared up any errors he found. This method was also used for only selected topics in the course. A formal study was not completed, but the author observed positive attitudinal differences from the students' participation in the project[25].

Methodology

Structure of the traditional College Algebra class

The traditional section of College Algebra in this study met for five fifty-minute classes per week. The instructor typically lectured three days per week, one day was reserved for student questions, and one day was spent working on worksheets with practice problems under the guidance of an undergraduate assistant. The instructor periodically incorporated group work and other active learning techniques into the lecture, and student questions were frequent, but a majority of class time was spent with the students taking notes as the instructor lectured and worked example problems.

Traditional College Algebra sections at SPSU have an enrollment capacity of 38 students. The section in this study started with 33 students, of whom 26 completed the course. Two students officially withdrew before the withdrawal deadline at midsemester, and five students stopped attending later in the semester, as is typical of the course.

Structure of the workshop-style class

The workshop section of College Algebra met with the instructor three times per week for 105-minute sessions. Twenty-three students registered, and four officially withdrew before midsemester; 16 students took the last exam. In order to facilitate the group work, the class was held in a room with round tables that seated four students.

Prior to attending class, students were expected to read a section from the textbook and complete a brief study guide on topics to be covered that day. Attendance, participation, and preparedness accounted for ten percent of their grade in the course.

Each day, the students were given a worksheet to complete. (The appendix includes both a sample study guide and a sample worksheet; additional examples are available from the authors upon request.) The class was divided into groups of three or four, and the groups worked together on the assignment. The worksheet led them through the material for the day, sometimes relying on prior knowledge to build up to more difficult problems, sometimes relying on their reading of the textbook before coming to class, and sometimes pointing them to relevant examples in the book.

The instructor avoided answering questions directly during class, instead helping the groups to discover the answers to their questions on their own or directing them to another group that could help them. Particularly difficult problems were sometimes completed on the board by a member of a group that had solved it successfully. Students were informed that the instructor would answer questions during office hours.

The instructor also questioned groups as they worked, most often to ensure that all of the group members understood the solutions to the problems. When the instructor spotted mistakes, these were brought to the attention of the group in various ways, either by asking the group a question that led them to find their mistake, by encouraging them to compare their solution with that of another group, or by making a statement like, "I see a mistake somewhere in question 3. See if your group can find it."

Approximately every two weeks, the groups were shuffled. The methods for choosing groups varied. Sometimes the groups were randomly assigned; sometimes students chose their groups. Most often, groups were assigned by the instructor to mix ability levels (as determined by exam grades and instructor observation) and minimize personality conflicts.

Aspects common to both sections

The textbook for both courses was the eighth edition of Sullivan's *Algebra and Trigonometry*[23]. Weekly, the instructor collected and graded three to four homework problems from the textbook. More comprehensive homework sets were assigned and strongly recommended, and they were collected on exam day for extra credit. The students could earn up to 12 points of extra credit on each exam from these assignments.

Students had two primary methods for receiving help outside of class. The instructor held at least five office hours per week, which students from both sections were invited to attend. Also, SPSU's tutoring center provides tutoring in College Algebra (and other courses) free of charge. The center is open during the day on weekdays and two evenings per week.

Demographic		Workshop	Traditional
Gender			
	Male	72.7%	93.8%
	Female	27.3%	6.3%
Student Status			
	Freshman	68.2%	81.3%
	Continuing Student	9.1%	12.5%
	Transfer Student	22.7%	6.3%
Student Enrollment			
	Full-time	90.9%	100%
	Part-time	9.1%	0%
Hours per week working			
	0-10 hours	45.5%	65.6%
	11-20 hours	31.8%	21.9%
	21-30 hours	0%	9.4%
	31-39 hours	0%	0%
	40+	22.7%	3.1%
Number of students repeating College Algebra			
		9.1%	12.5%
Major			
	E/ET	45.5%	46.9%
	Other	54.5%	53.1%
Total students completing the survey			
		22	32

Table 1: Self-reported demographics

Demographics of the College Algebra sections

The students took a survey within the first few weeks of class that contained questions about their background. The survey was adapted from a student pre-survey used on a project entitled, "Developing statistical indicators of the quality of undergraduate mathematics education," headquartered at the Office for Mathematics, Science and Technology Education, at the University of Illinois at Urbana-Champaign (see http://mste.illinois.edu/indicators/). Twenty-two students in the workshop class and 32 students in the traditional class took the survey. Table 1 summarizes the demographics of students who completed the survey. Note that all statistics are self-reported.

In addition, the students' high school GPA, math SAT scores, and scores on the mathematics placement test given to incoming freshmen were obtained from the institution. There was no statistically significant difference between the two sections in any of those three areas. Since this was the first college mathematics course for most of the students, these three statistics were the best measure of prerequisite knowledge available.

The Research Method

There were four midsemester exams. In order to effectively compare their performance, the two sections in the study were given very similar exams. Each exam consisted primarily of open-ended questions, and partial credit was awarded.

To compare the attitudes towards mathematics of the students in the two sections, a series of surveys was administered throughout the semester. The same pre-survey that provided demographic information about the students also evaluated their attitudes upon entering the course. The questions concerning their attitudes were asked again at the end of the semester, along with a few additional questions. Both surveys are included in the appendix.

Interviews were also conducted by an undergraduate student assistant at the beginning and end of the semester. Almost all of the workshop students participated, and a subset of the students in the traditional section were interviewed. The interview allowed for more open-ended questions than the surveys. To encourage them to be candid, students were informed that their responses to both the surveys and the interviews would remain anonymous until after their grades were awarded.

Data Analysis

The data from all of the surveys throughout the semester was entered into Excel and Minitab for analysis. In addition to reporting summary statistics in the results below, we often use the mean of the Likert scale responses to make comparisons between questions and between demographics. There is some disagreement about which tests of the equivalence of sample means on results from a Likert scale are appropriate. For this reason, we chose to use the conservative Mann-Whitney U-test on unpaired data and the Wilcoxon Matched-Pairs Signed Ranks test when we had paired data.

Results

Assessments

The results on individual exams were consistently similar between the two sections. There were a few questions on which there was a statistically significant difference in performance between the two sections, but most of those differences can be explained by incidental circumstances. There was no obvious pattern to the questions on which the workshop students performed better or worse. The same holds true when looking at the subset of the classes that were engineering or engineering technology majors.

Final grades in the course are displayed in Figure 1. As noted above, a student needs a C or higher in the course to progress to Precalculus. This success rate was 51% for the workshop section and 58% for the traditional section. For engineering/engineering technology majors, the success rate was 60% for both sections. Neither is statistically significant when compared to the university average of 54%. (Note also that the denominator includes students who withdrew from the course. If the students who officially withdrew are excluded, the C-or-higher rate is 63% for the workshop, 61% for the traditional course, 67% for E/ET in the workshop course and remains 60% for E/ET in the traditional course.)

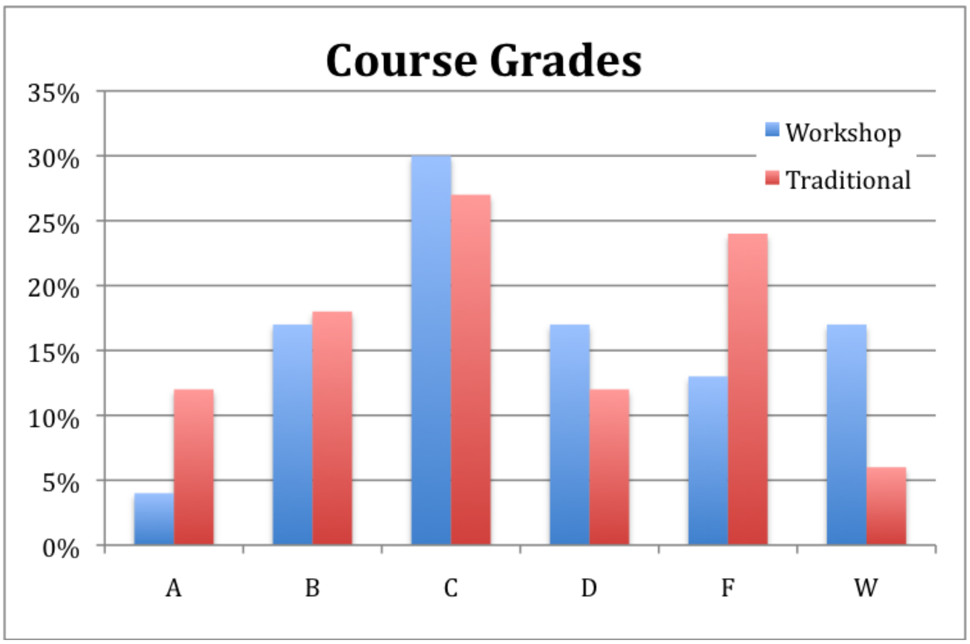

Figure 1: Course grades for both sections.

Student Effort

We measure student effort with two different statistics: homework average and extra credit average. As mentioned above, each section was assigned three to four problems each week that were collected and graded. The sections were given the same assignments. Both sections performed very poorly on the homework, despite being able to seek help from others on the assignment. However, this poor performance was particularly pronounced in the workshop section. The homework average for the traditional section was a 66.7 (65.8 among E/ET), whereas the workshop section averaged only a 45.9 (41.9 among E/ET) on their homework. This is statistically significant at a significance level of 0.05 for the whole class and the E/ET subset. A larger percentage of workshop students also regularly failed to turn in the assignment.

As previously noted, both sections were given the opportunity to turn in a more comprehensive homework assignment for up to 12 points of extra credit on each exam. Again, neither section performed well on this assignment; in both sections, on all four exams, fewer than half of the students submitted the extra credit assignment. The number of workshop students submitting the assignment, however, was particularly low. As a result, the average score on the four extra credit assignments for the traditional section was 3.1 (3.3 E/ET), but the average for the workshop section was only 1.9 (1.8 E/ET). These differences are not statistically significant, however.

Perceptions

As mentioned above, based on the self-reporting of demographics in the pre-survey and the institute record of GPA, SAT scores, and placement test results, the two classes were very similar. In the perceptions portion of the pre-survey, the traditional students agreed more strongly than the workshop students that the textbook is helpful (p-value from U-test 0.05) and the workshop students agreed more strongly with the statement "I am looking forward to taking more mathematics" (p-value 0.05).

Ironically, in the only other statistically significant differences on the pre-survey, the workshop students disagreed more with the statement "In order to learn mathematics, working with a partner is helpful" than the traditional students (p-value 0.06). They also agreed more that lectures are helpful (p-value 0.002). The interviews at the beginning of the semester offer some insight into this surprising statistic. Only 13 out of 23 of the students in the workshop section indicated that they had registered for the course because of its collaborative learning format, and even several of those 13 showed signs of doubt that the format would be successful. The remaining 10 students reported registering for scheduling convenience or by accident.

Analysis was then run on the differences in perception over the course of the semester to see what, if any, affect the different class types had. Paired data was available on 15 workshop students and 22 traditional students. (P-values from a 2-tailed Wilcoxen Matched-Pairs Signed Ranks test are in parentheses following each statement.) Results below are all in aggregate. There was not a statistically significant change in perceptions of mathematics over the course of the semester for the E/ET students versus the rest of the class under the two different teaching methods.

At the end of the semester, the workshop students agreed more with the statement, "In order for me to learn mathematics, lectures are helpful" (0.007), when compared to their perceptions at the beginning of the semester. They disagreed more with "It is important to know mathematics in order to get a good job" (0.06). Likewise, when compared to the pre-survey, in their responses to the post-survey the traditional students were less likely to agree that "In order for me to learn mathematics, the textbook is helpful" (0.03) and "New discoveries in mathematics are constantly being made" (0.10). They were more likely to agree with, "It scares me to have to take mathematics" (0.04) and "Mathematics is harder for me than for most persons" (0.02).

At the end of the semester, workshop students were also asked to compare their experience in the workshop-style class to their typical experience in a traditional lecture class. Forty percent of the students agreed with the statement "I learned new concepts in the workshop section more easily", 67% agreed with "The workshops helped me to better understand concepts I had learned in previous math classes" and 60% agreed with "I left workshop sessions more prepared to complete problems on my own".

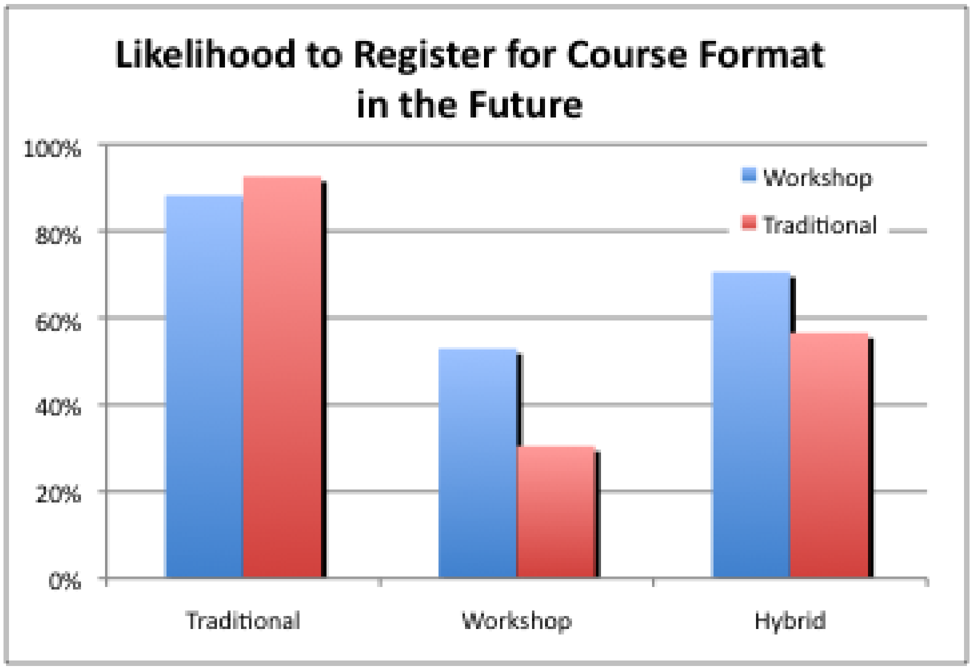

Figure 2: Reported likelihood of registering for various course formats

Both classes were also surveyed at the end of the semester about their likelihood to take the two formats, or a hybrid format (two 50-minute lectures per week and two 75-minute workshops per week) in the future. Results are in Figure 2. SPSU did offer hybrid sections of both College Algebra and Precalculus in the following semester, and ten workshop students registered for a hybrid section, whereas only two of the traditional students registered for a hybrid section. (The workshop-only format was not offered.)

Conclusions and Future Research

The results of this experiment indicate that a workshop-only approach in College Algebra does not significantly improve student outcomes or attitudes. As the workshop-style format requires more faculty hours and smaller class sizes, the added cost of offering the workshop-only section was not justified.

However, survey data and feedback from students offer clues that a hybrid (combined lecture and workshop) approach may be a successful format for College Algebra at this institution. The increase in the workshop students' perception that lectures are important is consistent with comments made throughout the semester by the workshop students indicating that although they liked working in groups, they needed someone to tell them how to get started on problems.

The students' disagreement with the statement "I learned new concepts in the workshop section more easily" and agreement with the statement "The workshops helped me to better understand concepts I had learned in previous math classes" is another indication that the students felt ill-equipped to tackle entirely new concepts on their own, but believe that working in groups is helpful when they have some prior knowledge. A hybrid approach would allow them to obtain this prior knowledge in a lecture environment, then immediately reinforce the new concepts in a workshop.

Responses from the exit interviews of the workshop students further support this conclusion. One student expressed a strong preference for at least one lecture per week, but also stated, "Groups do help, because when you know something and you can show that knowledge to someone else, you are … churning it through your mind over and over." Another student said of working with a partner, "If… you both understand it okay, you can usually work together to get a good understanding about it." Of the fifteen students interviewed, only two of them indicated a negative attitude toward working in groups. Most of the negative feedback from the students concerning the workshop-only approach related to not being able to ask questions and not having concepts and examples demonstrated to them by an instructor.

After completing this experiment, we do not believe that this preference for a hybrid approach is a surprising outcome. The workshop-only format was initiated in part because of the hypothesis that students would remember much of the material in College Algebra from a previous course. Although many of the students indicated frequently that portions of the material looked familiar, usually from a high school math course, this did not seem to help the students. With few exceptions, students were not noticeably drawing upon this prior exposure to the material to solve the problems. Since College Algebra students are at the beginning of their mathematical careers, it is not surprising that they lack the mathematical sophistication needed to teach themselves the material using only the textbook.

The workshop students appeared to avoid the dip in mathematical confidence experienced by the students in the traditional section. Whereas students in the traditional section agreed significantly more with the statements "It scares me to have to take mathematics" and "Mathematics is harder for me than for most persons" at the end of the semester than beginning of the semester, the workshop students had no statistically significant change in their agreement with these statements. This lends mild support to the positive attitudinal outcomes from collaborative learning that other studies have found.

It is interesting to note that the students in the workshop seemed to perform approximately as well as the traditional students with considerably less effort. Based on homework averages and extra credit averages, it appears that many workshop students were relying mostly or entirely on the time spent in class to prepare them for exams. While this poor effort is clearly not a desirable outcome, it may indicate that the time spent in workshops is more beneficial than the time spent in lectures. The workshop students' agreement with the statement "I left workshop sessions more prepared to complete problems on my own" lends support to this hypothesis. Exit interviews also indicated that almost all of the workshop students believed their time spent in class helped to prepare them for the exams.

We believe that the observational and statistical evidence gathered in the experiment indicates that a lecture-free format is not optimal for this level of student. However, we also feel that it supports the need to continue to offer courses with some workshop component. Future work will focus on finding the right balance between lectures and workshops. In addition, it may be worthwhile to investigate the effects of grouping the engineering students into their own College Algebra sections for workshop-style instruction. Creating more application-driven worksheets in order to motivate and reinforce ideas in a way that is more meaningful to them may result in greater student effort that could increase the success of a workshop-only environment.

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Appendix 1 – Study Guide

Name____

Preflight – Section 6.4

1. Write the definition of the logarithmic function to the base a.

2. What function is $f(x) = \log_2 x$ the inverse of?

3. Remembering back to 6.2: If the domain of f is D and the range is R, what is the (a) domain and the (b) range of f^{-1}?

4. Use the previous question to find the domain and the range of $f(x)=\log_2 x$.

5. What is the inverse of $f(x)=e^x$?

6. What base is implied in $f(x)=\log x$?

7. Draw a quick sketch of $f(x)=\log_2 x$. Label at least 3 key points. What are the intercepts?

8. Draw a quick sketch of $f(x)=\log_{1/2} x$. Label at least 3 key points. What are the intercepts?

Section 6.4
Math 1111 Worksheet 28

Question 1. *A few from 6.3.* Solve:

(a) $(\frac{1}{4})^{4x} = 8^x$

(b) $(e^2)^x = e^x \cdot (\frac{1}{e})^3$

(c) $(e^4)^x \cdot e^{x^2} = e^{12}$

Question 2. Recall that we decided $f(x) = a^x$ is a one-to-one function, and this means it has an inverse. To find the inverse, as usual, we swap x for y in $y = a^x$, and get:

$$x = a^y.$$

Now we want to solve for y. PROBLEM! How do we get y out of the exponent? Using things we know so far, we *can't*. We need a new function! That's where the logarithmic function to the base a from your preflight comes in. The mantra my high school precalculus teacher taught me that I still use when working with logarithmic functions is the following:

"The log of a number equals an exponent."

Use that mantra to change the following logarithmic expressions into exponential expressions.

(a) $\log_2 4 = x$

(b) $\log_a 20 = 3$

(c) $\log_{10} b = 3$

Question 3. Going the other way: Change the exponential expressions into logarithmic expressions.

(a) $2^5 = y$

(b) $e^x = 7$

(c) $a^2 = 16$

Question 4. Remember: Just like 2^5 is a fancy way of writing a number (in particular, 32), $\log_2 32$ is also a fancy way of writing a number. (Notice there are no variables!) It's just harder to interpret what that number might be. If we change the logarithmic expression to exponential form, however:

$$\log_2 32 = y \text{ means } 2^y = 32$$

...we get an exponential equation that's easy to solve: $2^y = 32$ and $32 = 2^5$, so $2^y = 2^5$, and therefore $y = 5$. Use that technique to find the values of the following logarithms:

(a) $\log_5 125$

(b) $\log_3 \frac{1}{9}$

Question 5. On your preflight, you investigated the domain and the range of $\log_2 x$. Note that for any a, the domain of $\log_a x$ is all POSITIVE reals. This means the argument (that is, what you "plug in" to the log function) must always be positive. Use this idea to find the domain of the following functions.

(a) $f(x) = \log_3 2x + 1$

(b) $g(x) = \log_{21} x^2$

(c) $h(x) = \log_{57} x^2 - 2x - 3$

Question 6. In the expression $\log_a x = y$, there are a lot of restrictions on the various "letters."

(a) What are the restrictions on a (if any)? (Think back to the base of the exponential function.)

(b) What are the restrictions on x (if any)?

(c) What are the restrictions on y (if any)?

Question 7. Also on your preflight, you graphed $f(x) = \log_2 x$ and $f(x) = \log_{1/2} x$. Graph them again with your group members to make you had them correct. Label three key points, decide where the functions are increasing and decreasing, and find the domain and the range. What are the intercepts?

Question 8. Since e^x is such an important exponential function, we need a handy notation for its inverse. The function $f(x) = \log_e x$ is usually written $f(x) = \ln x$, and it's called the *natural logarithm*. With that in mind:

(a) Sketch the graph of the function $g(x) = \frac{1}{2}\ln(x + 2)$ using transformations.

(b) Use your picture to graph $g^{-1}(x)$.

(c) Find g^{-1} algebraically. (Hint: Change to exponential form.)

(d) Find the domain and the range of g and g^{-1}.

(e) Now do the very same thing with the function $h(x) = -3\log(x - 1)$.

Question 9. So far, we have just one technique to handle exponential equations: Make the bases match so we can equate the exponents. Similarly, in this section we're going to learn just one technique for handling logarithmic equations: Get rid of the logarithm by changing it into exponential form. See Example 8, then solve the following equations. MAKE SURE YOU CHECK YOUR ANSWER, because the solutions you fund may be outside the domain of the log function!!!

(a) $\log_3(3x - 2) = 2$

(b) $\ln e^{-5x} = 15$

(c) $\log_x 9 = 2$

Appendix 3

Pre-Survey

Which section are you registered for? _______Workshop (9am) _______Traditional (1pm)

Background information

(1) Please indicate the year of your birth: ____________

(2) Gender:

__________ Female

__________ Male

(3) Please indicate your ethnicity:

__________ Hispanic or Latino/a

__________ Non-hispanic or Latino/a

(4) Please indicate your race:

__________ American Indian or Alaska Native

__________ Asian

__________ Black or African American

__________ Native Hawaiian or Other Pacific Islander

__________ White

__________ Other

(5) Please indicate your intended major:

__________ Mathematics

__________ Engineering (of any type)

__________ Physical or Life Science

__________ Architecture

__________ Information Technology or Computer Science

__________ Construction or Surveying and Mapping

__________ Management

__________ Humanities, Liberal Arts, or Social Science (English, History, Psychology, Sociology, etc.)

__________ Undecided

__________ Other

(5) Please indicate the student status that best describes you:

___________ Freshman

___________ Continuing Student

___________ Transfer Student

___________ Other

(6) Please indicate the enrollment status that best describes you:

___________ Full-time

___________ Part-time (more than one course)

___________ Single course-taker

(7) For the current semester, when are your courses scheduled mainly:

___________ Daytime

___________ Evening

___________ Other

(8) How many hours per week do you anticipate you will be working in a job during this semester?

___________ 0 - 10 hours

___________ 11 - 20 hours

___________ 21 - 30 hours

___________ 31 - 39 hours

___________ Full-time (40 or more)

Mathematical Background

(9) Please indicate how many full years of mathematics you took during grades 9-12:

___________ 1 year

___________ 2 years

___________ 3 years

___________ 4 years

(10) Please provide the following information on the last mathematics course (excluding statistics) you have completed prior to this course:

(a) How many years ago was that course taken?

 ___________ 0-1 year ago

 ___________ 2-3 years ago

 ___________ 4-10 years ago

 ___________ more than 10 years ago

(b)Where was that course taken?

__________ high school

__________ community college

__________ four-year college or university

__________ other

(c) What was the name of the course? ______________________________________

(d) Final letter grade in that course: _____________

(11) (a) Is this the first time you are taking MATH 1111 at SPSU?

__________ Yes __________ No

(b) If you are repeating MATH 1111, why?

__________ Received a D the first time.

__________ Failed the first time.

__________ Withdrew from the course due to a failing grade.

__________ Withdrew from the course for other reasons.

__________ Received at least a C in the course, but am repeating it for other reasons

(c) If you are repeating the course, how many times have you taken it? ________
How many times have you completed it? _________

(12) What was your GPA in high school? __________

(13) What was your score on the math portion of the SAT (if you took it)?__________

(14) What was your score on the math portion of the ACT (if you took it)? __________

(15) Approximately how much time did you spend studying for the Mathematics Placement Test?

__________ None

__________ 1-2 hours

__________ 3-5 hours

__________ More than 5 hours

(16) Which of the following best describes how do you feel about your placement into MATH 1111?

__________ I should have been placed into a more advanced class.

__________ MATH 1111 is the right class for me right now.

__________ I think MATH 1111 is going to be too difficult for me.

Opinions about Mathematics:

The remainder of the survey asks a variety of questions about what you have done in previous mathematics courses, and your opinions about mathematics in general.

Please circle your responses to the following:

(17) In order for me to learn mathematics, working with a partner is helpful:

Strongly disagree Disagree Neutral Agree Strongly agree

(18) In order for me to learn mathematics, lectures are helpful:

Strongly disagree Disagree Neutral Agree Strongly agree

(19) In order for me to learn mathematics, the textbook is helpful:

Strongly disagree Disagree Neutral Agree Strongly agree

(20) I prefer to do my mathematics homework alone.

Strongly disagree Disagree Neutral Agree Strongly agree

(21) I study for the majority of my math tests with other students.

Strongly disagree Disagree Neutral Agree Strongly agree

(22) It scares me to have to take mathematics.

Strongly disagree Disagree Neutral Agree Strongly agree

(23) It is important to know mathematics in order to get a good job.

Strongly disagree Disagree Neutral Agree Strongly agree

(24) Trial and error can often be used to solve a mathematics problem.

Strongly disagree Disagree Neutral Agree Strongly agree

(25) Learning mathematics involves mostly memorizing.

Strongly disagree Disagree Neutral Agree Strongly agree

(26) I am looking forward to taking more mathematics.

Strongly disagree Disagree Neutral Agree Strongly agree

(27) Mathematics is a good field for creative people.

Strongly disagree Disagree Neutral Agree Strongly agree

(28) No matter how hard I try, I still do not do well in mathematics.

Strongly disagree Disagree Neutral Agree Strongly agree

(29) Mathematics is harder for me than for most persons.

Strongly disagree Disagree Neutral Agree Strongly agree

(30) Mathematics is useful in solving everyday problems.

Strongly disagree Disagree Neutral Agree Strongly agree

(31) Most of mathematics has practical use on the job.

Strongly disagree Disagree Neutral Agree Strongly agree

(32) There is little place for originality in solving mathematics problems.

Strongly disagree Disagree Neutral Agree Strongly agree

(33) Estimating is an important mathematics skill.

Strongly disagree Disagree Neutral Agree Strongly agree

(34) There are many different ways to solve most mathematics problems.

Strongly disagree Disagree Neutral Agree Strongly agree

(35) If I had my choice, this would be my last mathematics course.

Strongly disagree Disagree Neutral Agree Strongly agree

(36) New discoveries in mathematics are constantly being made.

Strongly disagree Disagree Neutral Agree Strongly agree

(37) I expect to use the mathematics that I will learn in this course in my future career.

Strongly disagree Disagree Neutral Agree Strongly agree

Appendix 4

Exit Survey - Workshop

(1) On average, approximately how many hours **PER WEEK** did you spend on this course? (Please don't include class time.)
____ 0-1 ____ 2-3 ____ 4-5 ____ 6-7 ____ 8-9 ____ 10 or more

(2) How many MATH 1111 classes did you miss this semester (to the best of your recollection)?
____ 0-1 ____ 2-3 ____ 4-5 ____ 6-7 ____ 8-9 ____ 10 or more

(3) Approximately what portion of the assigned homework (that is, the "Extra Credit" assignment) would you estimate you completed **over the course of the semester**?
___ less than 25 % ___ between 25% and 50% ___ between 50% and 75% ___ more than 75%

(4) Check ALL of the following that you believe would have helped you to earn a higher grade this semester.

____ Attending class more regularly
____ Completing more homework
____ Studying with other people
____ Visiting the tutoring center
____ Signing up for the lecture section instead
____ Putting more effort into the preflights
____ Attending office hours
____ Having class later in the day

<u>Opinions about Mathematics:</u>
The next part of the survey asks your opinions about mathematics in general.
Please circle your responses to the following:

(17) In order for me to learn mathematics, working with a partner is helpful:
Strongly disagree Disagree Neutral Agree Strongly agree
(18) In order for me to learn mathematics, lectures are helpful:
Strongly disagree Disagree Neutral Agree Strongly agree
(19) In order for me to learn mathematics, the textbook is helpful:
Strongly disagree Disagree Neutral Agree Strongly agree
(20) I prefer to do my mathematics homework alone.
Strongly disagree Disagree Neutral Agree Strongly agree
(21) I study for the majority of my math tests with other students.
Strongly disagree Disagree Neutral Agree Strongly agree
(22) It scares me to have to take mathematics.
Strongly disagree Disagree Neutral Agree Strongly agree
(23) It is important to know mathematics in order to get a good job.
Strongly disagree Disagree Neutral Agree Strongly agree
(24) Trial and error can often be used to solve a mathematics problem.
Strongly disagree Disagree Neutral Agree Strongly agree

(25) Learning mathematics involves mostly memorizing.

Strongly disagree   Disagree   Neutral   Agree   Strongly agree

(26) I am looking forward to taking more mathematics.

Strongly disagree   Disagree   Neutral   Agree   Strongly agree

(27) Mathematics is a good field for creative people.

Strongly disagree   Disagree   Neutral   Agree   Strongly agree

(28) No matter how hard I try, I still do not do well in mathematics.

Strongly disagree   Disagree   Neutral   Agree   Strongly agree

(29) Mathematics is harder for me than for most persons.

Strongly disagree   Disagree   Neutral   Agree   Strongly agree

(30) Mathematics is useful in solving everyday problems.

Strongly disagree   Disagree   Neutral   Agree   Strongly agree

(31) Most of mathematics has practical use on the job.

Strongly disagree   Disagree   Neutral   Agree   Strongly agree

(32) There is little place for originality in solving mathematics problems.

Strongly disagree   Disagree   Neutral   Agree   Strongly agree

(33) Estimating is an important mathematics skill.

Strongly disagree   Disagree   Neutral   Agree   Strongly agree

(34) There are many different ways to solve most mathematics problems.

Strongly disagree   Disagree   Neutral   Agree   Strongly agree

(35) If I had my choice, this would be my last mathematics course.

Strongly disagree   Disagree   Neutral   Agree   Strongly agree

(36) New discoveries in mathematics are constantly being made.

Strongly disagree   Disagree   Neutral   Agree   Strongly agree

(37) I expect to use the mathematics that I will learn in this course in my future career.

Strongly disagree   Disagree   Neutral   Agree   Strongly agree

Opinions about the workshops

To answer the following questions, please compare your experience in the workshop-style class to your typical experience in a traditional lecture class.

(38) I learned new concepts in the workshop section more easily.

Strongly disagree   Disagree   Neutral   Agree   Strongly agree

(39) The workshops helped me to better understand concepts I had learned in previous math classes.

Strongly disagree   Disagree   Neutral   Agree   Strongly agree

(40) I left workshop sessions more prepared to complete problems on my own.

Strongly disagree   Disagree   Neutral   Agree   Strongly agree

Please use the space below to include any thoughts you would like to share about your experience in the workshop course this semester.