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An Elective in Rocketry

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An Elective Course in Rocketry

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Abstract

A course in rocketry is offered as a 1-hour elective. The objective is for students to design, analyze, construct and launch a rocket. The rockets must be designed to not exceed the maximum-allowable altitude for the launch site as specified by the FAA. Launches are done under the auspices of the National Association of Rocketry (NAR). For approximately the first 2/3 of the semester, physics of trajectory, stability analysis and construction methods are covered via lectures. In the second 1/3, shop time is scheduled for construction. Generally, launches are conducted the following semester when the field is available.

Course Description

The course meets once a week for 50 minutes. Students are required to have had the first series of physics and calculus, though not necessarily differential equations. Material covered includes trajectory analysis by solution of the differential equation of Newton's Second Law of motion, accounting for change of mass and drag variation with velocity. The equation is solved both analytically by means of simplifications and by use of finite differencing where no approximations are required. The principle of stability based on center-of-mass and center-of-pressure is covered, as well as basic rocket design.

Students are required to perform a trajectory and stability analysis, and produce a design, specified in an engineering drawing. Each student is allotted a budget of $125. The design is required to include an altimeter as payload and can include other instruments at the discretion of individuals. Rockets are designed so as not to exceed maximum allowable launch-site altitude, 5280 ft. for the site we use. Status presentations are made at mid-term and final designs are presented at end-of-term along with a final written report. A parts list must be included in the report for purchase by the instructor. The course is graded on a satisfactory/unsatisfactory basis. Actual rocket construction, though begun during the semester after completion of the lecture-part of the course, is typically completed in the following semester on students' own time. University shop equipment is used under supervision.

Summary

This course has been offered over a period of several years. Students are typically junior and senior physics and engineering majors with the necessary mathematics and physics requisites. Some sophomores also take the course. One interesting aspect of the course is the evolution of designs over the years, when first offered, rocket designs typically employed a 3" diameter body tube and have gradually increased to 4" and even 5.5" diameters. Also more multi-thruster designs are done as opposed to a single thruster. Generally, two-stage designs are not attempted though are certainly permitted.

Generally speaking, the course has been favorably received by students. Their overall assessment is that it provides a "real world" experience, an actual engineering application of physics and analysis.

This paper is organized such that anyone interested in pursuing the technical area as a course offering has the analysis and references available. Any use of the material herein does not require the author's permission.

Trajectory Analysis

Analysis of the rocket trajectory, in particular, the altitude it attains is of primary importance, namely to insure it does not exceed the allowable altitude for the launch field. The trajectory analysis makes use of Newton's Second Law,

$$\sum F = \frac{d}{dt}(MV)$$

or

$$T - D - Mg\cos\theta = \frac{d}{dt}(MV) \tag{1}$$

Here, T = thrust, D = drag, M = mass, V = velocity, θ = flight angle, g = acceleration of gravity and t = time.

The preceding equation can be solved analytically by first applying the simplifying assumption,

$$M\frac{dV}{dt} > V\frac{dM}{dt}$$

Furthermore, if drag and trajectory angle θ are ignored, the classic rocket equation can be obtained by integration,

$$V_B = -u_{eff}\ln MR - gt_B \tag{2}$$

where V_B = velocity at motor burnout,
u_{eff} = effective velocity = motor total impulse/propellant mass,
MR = propellant mass/initial rocket mass,
t_B = motor burn time.

A more complicated version of this equation is obtained when drag is considered (consult author for details).

Newton's Second Law can be solved without simplifying assumptions by employing finite-difference time-stepping where M, V, D, and T all vary in time. Time variation of thrust is obtained from motor test data provided by the manufacturers (AeroTech for example).

Basic time-stepping is employed to solve the differential equation (1) by first writing it in finite-difference form,

$$\frac{(MV)_{i+1} - (MV)_i}{t_{i+1} - t_i} = T_i - D_i - M_i g\cos\theta_i$$

where, $i = 0, 1, 2, \ldots, n$
with the initial conditions, $t_0 = 0,\ V_0 = 0$
θ_0 = initial launch angle,
$M_0 = M_I =$ initial rocket mass,
$T_0 =$ initial motor thrust.

Solving for (MV) at the new time step, $i+1$,

$$(MV)_{i+1} = (MV)_i + \left[T_i - D_i - M_i g\cos\theta_i\right]\frac{\Delta t_i}{M_i}$$

and the velocity,

$$V_{i+1} = \left\{(MV)_i + \left[T_i - D_i - M_i g\cos\theta_i\right]\Delta t_i\right\}\Big/M_{i+1} \tag{3}$$

where $\Delta t_i = t_{i+1} - t_i =$ time increment,
$M_{i+1} = M_i - \dot{m}_p\Delta t_i$,
$\dot{m}_p = M_P/t_B \rightarrow$ assumed constant (usually not sufficient info to do otherwise).

Thrust, T_i, is obtained from a thrust vs. time curve per the example given in Appendix A or an average thrust is used calculated from total impulse and motor burn time.

Drag force is given by,

$$D_i = \tfrac{1}{2} C_D \rho A V_i^2 \tag{4}$$

where ρ = air density (assumed about constant over the trajectory),
C_D = drag coefficient, assumed constant,
A = frontal area.

Drag coefficient as a function of angle-of-attack, α, and velocity is given in Appendix B.

The altitude gained in each time increment Δt_i is,

$$\Delta h_{i+1} = \tfrac{1}{2}\left(V_{i+1} + V_i\right)\Delta t_i \tag{5}$$

where the average velocity is used over the time increment.

For the glide portion, i.e. unpowered portion, of the flight, eq. (2) reduces to,

$$V_{i+1} = V_i - \left[D_i + M_i g \cos\theta_i\right]\frac{\Delta t_i}{M_i} \tag{6}$$

The total altitude gain is,

$$H = \sum_{i=1}^{n} \Delta h_i \tag{7}$$

An example of results from a student team on their trajectory analysis is given in the following table, where H_B = altitude at end of motor burn, H_A = altitude gained during free-flight and H_{TOT} = final altitude. In the analysis, a constant coefficient-of-drag of 0.5 was assumed. A single AeroTech G-80T motor is used with an average thrust = 80.35 N.

Table 1. Trajectory Analysis

Mass of Rocket	0.354	kg	ρ(air density)	1.2	kg/m^3	Coefficient of Drag C_D	0.5	
Mass of Propellant	0.0625	kg	dm/dt	0.03676	kg/s	Trust T (constant)	80.35	N
$A_{gravity}$	9.8	m/s^2	t(burn)	1.7	s	Mass ratio	0.85	
θ	0		Frontal area A	0.0034211	m^2	Total Impulse	136.6	N-s

Time step analysis

$V_{i+1} = V_i + [T_i - D_i - M_i g\cos\theta_i](\Delta t/M_i)$

$D_i = 1/2 * C_D \rho A V_i$^2

$M_i+1 = M_i - dm/dt * \Delta t_i$

$dm/dt = M_P/t_B$

H_B	H_A	H_{tot}	
230.71	483.50	714.21	m

V_{burn} 231.4032 m/s

A drawing of the rocket for this case is given in Appendix C.

A stability analysis of the rocket design is also required to insure appropriate location of center-of-pressure, cp, with respect to center-of-mass, cm, to insure stable flight. Development of the analysis for proper location of cp with respect to cm is given in Appendix D. Determination of center-of pressure employs the Barrowman equations, Ref. 4, that are programmed in a spreadsheet. Use of the spreadsheet requires input of rocket body, nosecone and fin geometry.

Conclusion

The course is offered each semester to accommodate as many students and their schedules as possible. It also includes some enrollees who are not physics or engineering majors, indicating that it does have some universal appeal. One outgrowth of the course is the intent to encourage participation in rocket competitions through the Engineering Club, a request made by current students. Overall, the course has been well received over the years, has been successful and will continue to be offered.

References

1. Penko, P.F., Rocketry Class Notes, PHY-263, Spring Semester 2018.
2. Stine, G.H., Stine, W., Handbook of Model Rocketry, John Wiley & Sons, 2004.
3. Sutton, S.P, Biblarz, O., Rocket Propulsion Elements, John Wiley & Sons, 2010.
4. Barrowman, J.S., National Aeronautics and Space Administration, The Practical Calculation of the Aerodynamic Characteristics of Slender Finned Vehicles, March, 1967.

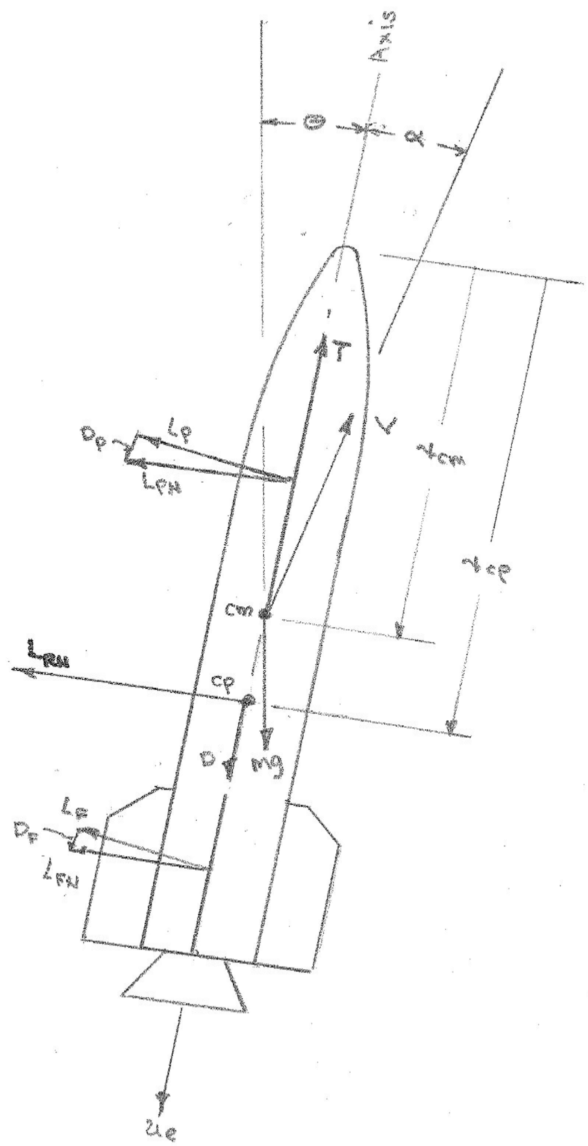
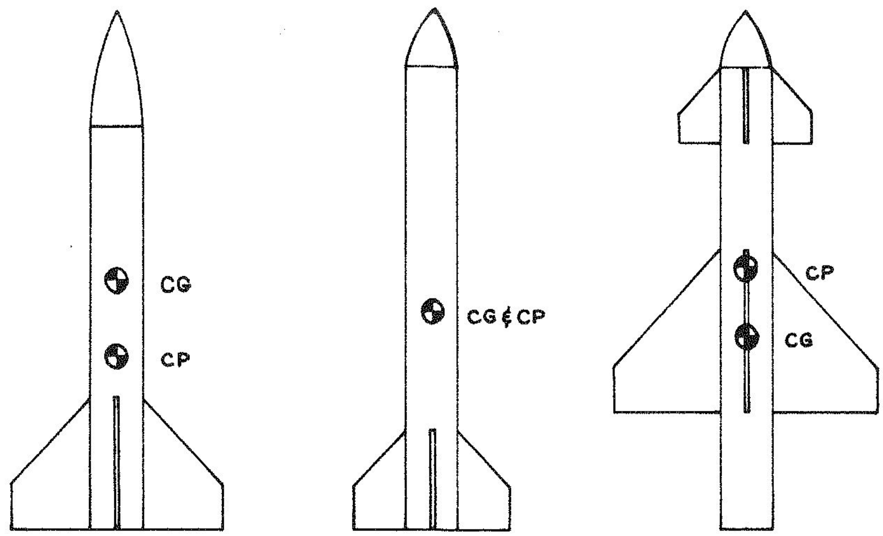


Fig. 1. Vector diagram for stability analysis.



Unstable Neutral Stable

Note: *CG & CM* are synonymous.

Fig. 2. The three conditions of stability. From Handbook of Model Rocketry

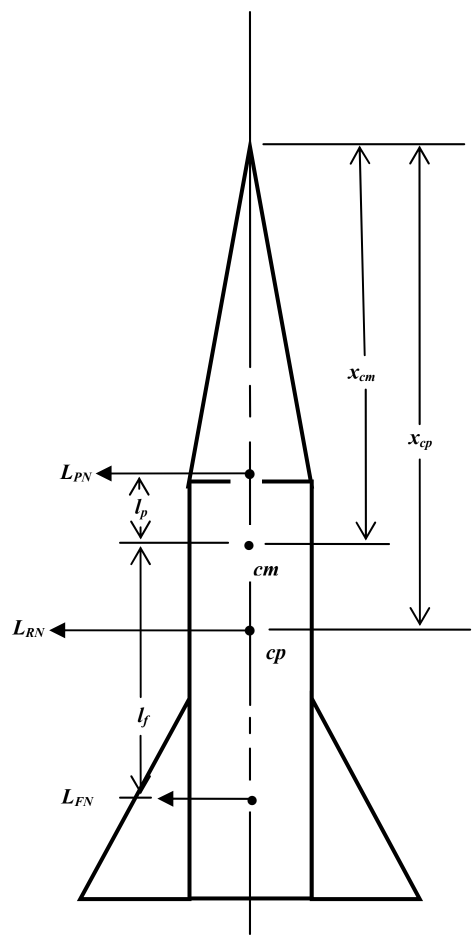
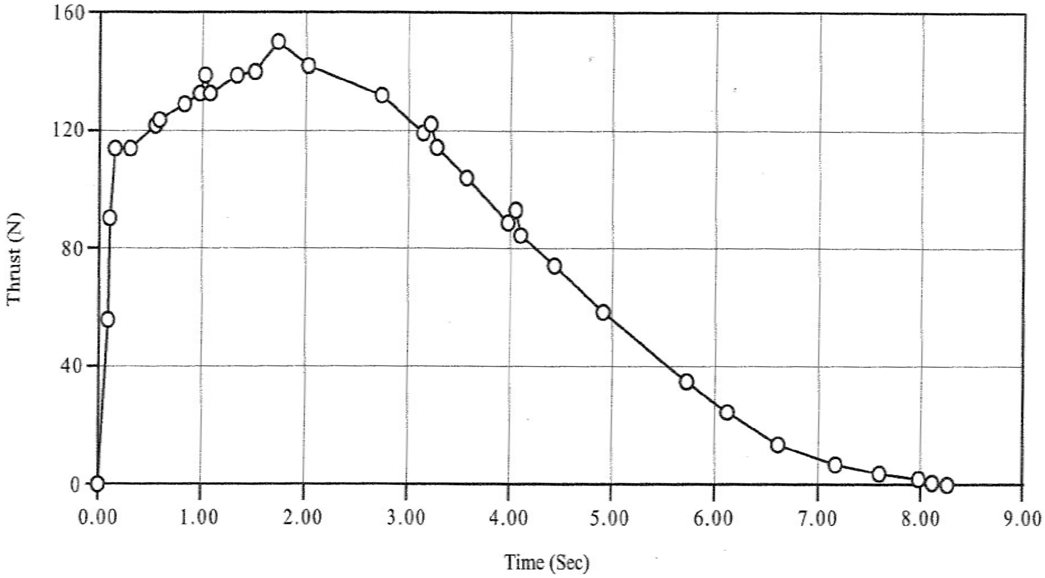


Fig. 3. Definition of dimensions for stability analysis.

Appendix A

AeroTech I65 Thrust Curve



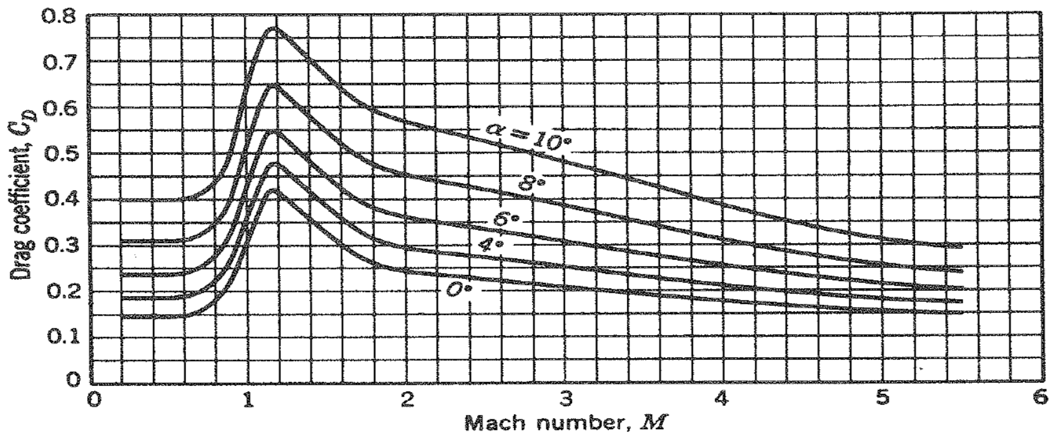
Thrust Data

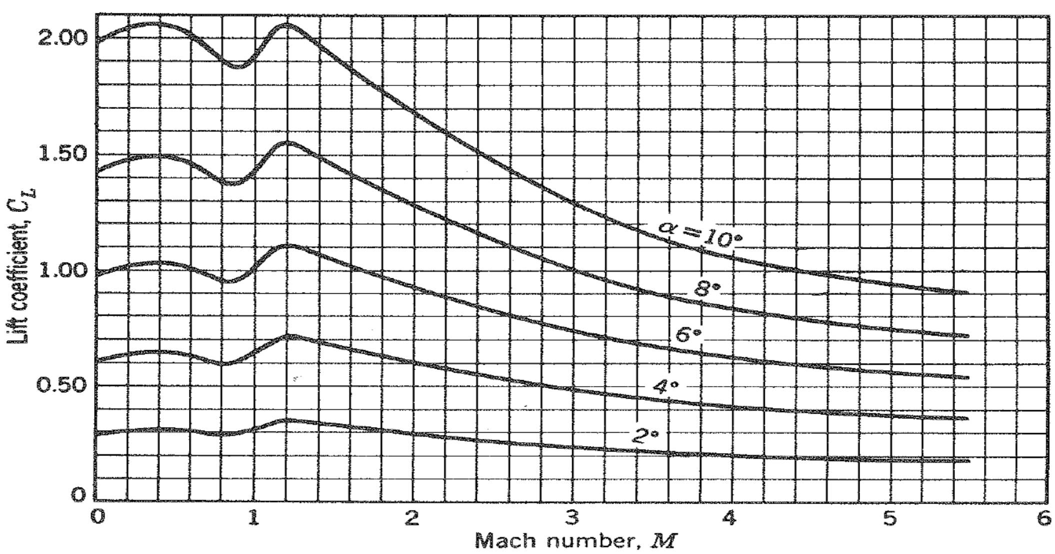
Time	Thrust
0.086	55.739
0.099	90.267
0.148	113.911
0.296	113.911
0.542	121.767
0.579	123.588
0.825	128.953
0.973	132.593
1.022	138.725
1.071	132.593
1.330	138.629
1.502	139.875
1.724	150.070
2.019	141.886
2.733	131.923
3.140	119.181
3.213	122.151
3.275	114.295
3.570	103.948
3.977	88.791
4.051	93.036
4.100	84.547
4.432	74.229
4.912	58.440
5.725	34.758
6.119	24.430
6.611	13.489
7.166	6.787
7.596	3.729
7.978	1.886
8.114	0.665
8.260	0.000

Appendix B

Lift & Drag Coefficients

- Following are lift and drag coefficients for the V-2 rocket as a function of Mach Number*, M, and angle-of-attack, α,





From Rocket Propulsion Elements, Sutton & Biblarz, 8th Edition, 2010.

* Note: $M = V/S$

V = vehicle velocity, m/s

$S = \sqrt{\gamma RT}$ speed of sound, m/s

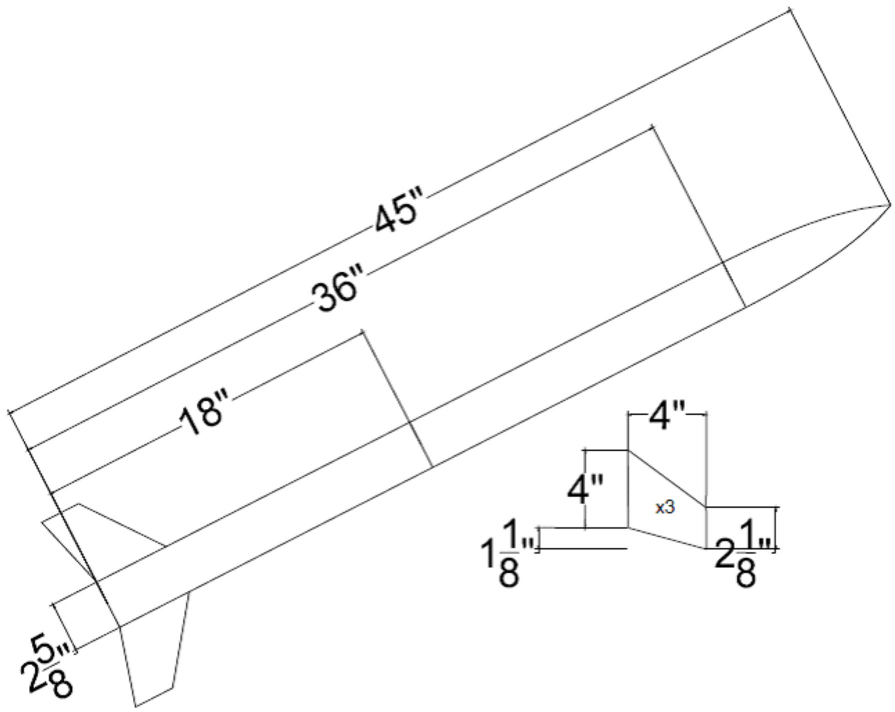
$\gamma = 1.4$, ratio of specific heats, dimensionless

$R = 8310$ J/kg$_{mol}$-K

T = temperature, K

Appendix C

<u>Rocket Dimensions</u>



Appendix D

Stability Analysis

Induced lift and drag on the rocket body, where lift is normal to the direction of motion (velocity vector, $\vec{V}$) and drag is coincident with the direction of motion, are illustrated in Fig. 1. Lift, L, and drag, D, are related to the velocity, V, by,

$$D = \tfrac{1}{2} C_D \rho A V^2$$

$$L = \tfrac{1}{2} C_L \rho A V^2$$

where C_D = the drag coefficient as before,
C_L = the lift coefficient.
ρ = air density.

The resultant lift force, $\vec{L}_{RN}$, from the lift that occurs when the rocket axis is at an angle-of-attack, α, acts through the center-of-pressure, x_{cp}, and normal to the rocket axis, where,

$$\vec{L}_{RN} = \vec{L}_{PN} + \vec{L}_{FN}$$

Here $\vec{L}_{PN}$ = lift from pressure distribution over rocket body normal to rocket axis,
$\vec{L}_{FN}$ = lift from fins normal to rocket axis,
$\vec{L}_{RN}$ = resultant lift acting through the center-of-pressure, x_{cp}, normal to rocket axis.

The location of x_{cp} is such that $\vec{L}_{RN}$ produces the same moment about the center-of-mass, x_{cm}, as the forces $\vec{L}_{PN}$ and $\vec{L}_{FN}$, i.e.,

$$\vec{L}_{RN} \times \vec{l}_R = \vec{L}_{PN} \times \vec{l}_P + \vec{L}_{FN} \times \vec{l}_F \qquad \text{(vector cross product)} \qquad (8)$$

where $\vec{l}_P$ = displacement from the body-lift vector to *cm*

$\vec{l}_F$ = displacement from the fin-lift vector to *cm*

$\vec{l}_R$ = displacement from the resultant lift vector to *cm*.

With respect to the center-of-mass, x_{cm}, the resultant force acting through the center-of-pressure, x_{cp}, will give a moment,

$$\vec{M} = \vec{L}_{RN} \times \vec{l}_R$$

where $l_R = x_{cp} - x_{cm}$ (scalar magnitude)

If x_{cp} and x_{cm} are <u>not</u> coincident, a pitching moment acts on the vehicle and the main stabilizing force is $\vec{L}_{FN}$, the force of fin lift. For stability, the relation between the lift from pressure distribution on the body of the rocket, $\vec{L}_{PN}$, and lift from the fins, $\vec{L}_{FN}$, i.e. where the fin-lift moment counteracts the body-lift moment is,

$$\left|\vec{L}_{FN} \times \vec{l}_F\right| \geq \left|\vec{L}_{PN} \times \vec{l}_P\right| \qquad (9)$$

Dropping vector notation and using magnitudes of force and distance, and the right-hand-rule for moment sign, i.e. scalar notation,

$$-L_{RN}\left(x_{cp} - x_{cm}\right) = L_{PN} l_P - L_{FN} l_F$$

or

$$\left(x_{cp} - x_{cm}\right) = -\frac{L_{PN} l_P - L_{FN} l_F}{L_{RN}} = \frac{L_{FN} l_F - L_{PN} l_P}{L_{RN}}$$

But from (8),

$$\left(x_{cp} - x_{cm}\right) \geq 0$$

therefore,

$$x_{cp} \geq x_{cm} \qquad (10)$$

This states that the location of center-of-pressure, x_{cp}, should be <u>aft</u> (behind) or below, the center-of-mass, x_{cm}, or (ideally) <u>coincident</u> with the center-of-mass for stable flight (see Fig. 2). In this case, the lift produced by the fins will always overcome the moment from body lift.

The Barrowman equations (Ref. 4) are employed to determine center-of-pressure, x_{cp}. Center-of-mass can be determined by a simple balance test or by,

$$x_{cm} = \sum_{1}^{n} x_{cmn} m_n \Big/ \sum_{1}^{n} m_n \qquad (11)$$

where x_{cmn} = center-of-mass for component n;

m_n = mass of component n.