AC 2012-5126: AN EXERCISE FOR IMPROVING THE MODELING ABILITIES OF STUDENTS IN AN OPERATIONS RESEARCH COURSE

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An Exercise for Improving the Modeling Abilities of Students in an Operations Research Course

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Abstract

An exercise for improving skills of Operations Research (OR) students in formulating optimization problems is developed and implemented. The pilot experiment, as described by Chelst and Edwards (2005), is called the Lego® furniture. In this experiment, a furniture company has two types of resources available: small and large pieces; and produces two types of products: tables and chairs. Industrial Engineering students at the bachelor and master level were given the Lego® furniture problem to formulate as an optimization problem defined by the decision variables, the objective function, and a set of constraints. This exercise serves as an introduction to the OR course. It complements lectures, seminars, and case studies used in this type of courses. The exercise is intended to produce formulation-of-the-problem attitude among the students. The results presented in this work show an improvement in student modeling abilities as well as high student satisfaction with the described experiment. Master level students, already exposed to the concepts of modeling and optimization, were slightly less satisfied than bachelor level students that have never been exposed to the same concepts. Both, master and bachelor level students showed an improvement in their modeling abilities.

Justification

Morse and Kimball (1951)\(^1\) defined Operations Research (OR) as "a scientific method of providing executive departments with a quantitative basis for decisions regarding the operations under their control". OR follows a scientific approach to analyze problems and to support any decision making process. OR professionals seek to understand the structure of complex situations, use this understanding to develop appropriate mathematical models, and then use them to improve the system performance. Hillier and Lieberman (2001)\(^2\) summarized the usual phases of an OR study as the following:

1. Define the problem and collect the data
2. Formulate mathematic model to represent the problem
3. Develop a procedure to find the solutions to the problem from the model
4. Test and refine the model
5. Prepare for the application of the model
6. Implement the application

Step 2 consists of defining a set of decision variables, an objective function, and a set of constraints; all of them following a linear structure in order to represent the relationships that best describe an optimization problem. In general, the result of step 2 is a mathematical model, known as a linear optimization problem.

OR Modelers are aware of the fact that very powerful algorithms exist to solve these linear optimization problems so that they often try to formulate real-life problems by using the linear optimization approach in order to ensure they can be solved efficiently. Many real problems can be formulated as linear optimization problems and many others can be well approximated by using linearization. Hence, training aimed to develop the capabilities in modeling is a primary concern in the OR courses.

Hosein et al. (2006) and Albritton et al. (2003) have identified formulation as one of the main topics being covered in linear optimization courses. Hosein et al. (2006) identified formulation as the common topic among hard-pure (e.g. mathematics), hard-applied (e.g. engineering), and soft-applied (e.g. business) disciplines teaching linear optimization courses with an intensity varying between 3.2 and 3.5 hours. However, their research did not present the methodology used to present the topic of formulation.

Most of the research on teaching linear optimization has been devoted to develop new and innovative ways to teach the Simplex method, one of the two main methods used to solve linear optimization problems. This initial research tries to improve the capabilities of the students to model real-life problems as linear optimization problems and to help them develop a formulation-of-the-problem attitude.

Next, the curricular context of the OR course will be presented followed by a detailed description of the experiments.

**Curricular Context**

The Operations Research course (EN471) is one of the core courses in the Bachelor of Science in Industrial Engineering (BSIE) being offered in the fall semester of the junior year. At the time the undergraduate students take the OR course; they have gone through all the Math and Physics classes.

A parallel OR course (EN571) is also a core course for the Master of Science in Systems and Industrial Engineering (MSISE). The OR course gives the basic mathematical and modeling tools required for the master students to successfully complete the other 4 MSISE core courses as well as the basics for those students interested in pursuing a Thesis in the optimization field.
Moreover, the OR course plays an introductory role into IE for the MSISE students with a background different than IE.

The OR course not only gives the basic analytical tools required in the other core courses but also motivates the students to learn how to solve the rich real-world problems that IEs are required to tackle in their profession. Therefore, one of the most important outcomes of this course is to encourage students to formulate real problems as linear optimization problems and obtain and analyze an optimal solution to them. While formulating the problems as linear ones, the students will understand the information requirements as well the constraints and their effect on the solution. Additionally, this process will help students to convert statements in ordinary language into mathematical models, and connect the solutions of those mathematical models to solutions of real problems.

Table 1 enumerates the general topics discussed in both courses (EN471 and EN571) and presents the outcomes by topic.

<table>
<thead>
<tr>
<th>Topic</th>
<th>Suggested Number of Hours</th>
<th>Outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction to Model Building</td>
<td>3</td>
<td>1, 2</td>
</tr>
<tr>
<td>Linear Programming</td>
<td>6</td>
<td>1, 2, 3</td>
</tr>
<tr>
<td>The Simplex Method</td>
<td>9</td>
<td>1, 2</td>
</tr>
<tr>
<td>Special Problem Types</td>
<td>9</td>
<td>1, 2</td>
</tr>
<tr>
<td>Integer Programming</td>
<td>9</td>
<td>1, 2, 3</td>
</tr>
<tr>
<td>Network Analysis</td>
<td>3</td>
<td>1, 2</td>
</tr>
<tr>
<td>Exams and discussions</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Total hours</td>
<td>45</td>
<td></td>
</tr>
</tbody>
</table>

The outcomes are defined accordingly to the general engineering outcomes (a – k) established by the Accreditation Board for Engineering and Technology (ABET). The outcomes are:

1. Basic knowledge of the tools of operations research (a, e, k)
2. Insight into the modeling and application of solutions to operations research problems (g, i)
3. Basic knowledge of some software used for operations research modeling (e, k)

In the topics “Introduction to Model Building” and “Linear Programming” (9 hours total), the Lego® experiment is presented to the students in addition to the discussion of three case studies showing real-world implementation of linear optimization models. Later on, 9 typical linear problems are discussed in detail. Initially, the general model for production mix is discussed and it is linked to the Lego® experiment. Next, detailed formulation for the diet problem, the work scheduling problem, the short term financial problem, the blending problem, the multi-period
inventory model, the multi-period financial problem, and the multi-period work scheduling problem are discussed and examples are developed and solved in class. The problems are taken from Winston and Venkataramanan (2003)\textsuperscript{5}, which is one of the recommended textbook for the OR course. Along with the Lego® experiment, the graphical solution of a two-variable linear optimization problem is discussed with the students. The experiment details are presented next.

**The Lego® Furniture Experiment**

The experiment presented in this research is based on the work of Chelst and Edwards (2005)\textsuperscript{6}. They define a set of basic problems representing a simplified version of the real world problems that both manufacturing and services companies deal with frequently. Problems such as optimizing the product mix in order to maximize profit, finding the optimal cutting pattern, the minimum cost diet, and scheduling workforce in the service industry are just a small set of those representative problems.

In the Lego® experiment, a furniture company has two types of resources available (eight small and six large Lego® pieces) and produces two types of products: tables and chairs. A table is made of two large and two small pieces and produces a profit of $16. A chair is made of one large and two small pieces and produces a profit of $10. Figure 1 depicts the structure of both products.

![Figure 1. Products and resources for the Lego® Furniture Experiment](image)

The students were presented with two set of questions: The first set asked them about the product mix the company has to manufacture, the relationships with its level of resources, and the economical goal of the company. This set of questions guided the students to formulate the problem in a mathematical way by using the linear optimization framework. The second set of questions asked students about the nature of the information (deterministic versus stochastic and static versus dynamic) and the data requirements. This set of questions was intended to give
students a connection point with a real problem and make them aware of the assumptions required to formulate the linear optimization problem.

A set of resources is given to group of students (no more than three students per group). Students started by trying to make as many tables as the available resources allow since tables produce the highest profit. Eventually they found a solution with 3 tables and a profit of $48. They realized, later on, that this solution did not allow them to produce chairs. Another possible solution students tried was to produce as many chairs as possible since a chair uses fewer resources so that it is possible to produce more chairs to upset their lower level of profit. They found a solution with 4 chairs and a profit of $40. Eventually, they were able to discover that a mix of both products (2 tables and 2 chairs) produced the highest profit ($52) and used all the resources available. Figure 2 presents the three feasible solutions.

Figure 2. Three feasible solutions: tables only (top), chairs only (middle), and the mix of products maximizing profit (bottom)
In order to wrap up the experiment, the mathematical model was jointly developed in class and the use of the language of linear optimization was introduced. Students defined the decision variables, the objective function, and the set of constraints. Discussion about the nature of the value of decision variables in the optimal solution (integer versus real values) was introduced at this point. The mathematical model is presented below. Figure 3 shows the students working on the Lego® experiment in class.

![Students working on the Lego® experiment](image)

Figure 3. Students working on the Lego® experiment

**Decision variables**

Let \( T \) be the number of tables to produce.

Let \( C \) be the number of chairs to produce.

**Objective Function**

The furniture company wants to maximize profit so that the objective function is:

\[
\text{Max } P = 16T + 10C
\]

**Constraints**

Since the company has limited amount of resources, a set of constraints needs to be defined:
Because only 6 large pieces are available and tables require 2 while chairs require only one of those large pieces, one constraint can be formulated as $2T + 1C \leq 6$.

Because only 8 small pieces are available and both tables and chairs require 2 of those small pieces, one constraint can be formulated as $2T + 2C \leq 8$.

The final mathematical model is then

$$\text{Max } P = 16T + 10C$$
Subject to:

$$2T + 1C \leq 6$$
$$2T + 2C \leq 8$$
$$T \geq 0 \text{ and } C \geq 0.$$  

Next, the graphical method to solve a two-variable linear optimization problem was introduced. Students are able to link the three solutions they tried before as the corner points in the feasible region. They started to form a rationale of the corner principle (For details see Hillier and Lieberman, 2001) and were reminded of some basic algebra and geometry concepts used in the procedures to solve linear optimization problems. Figure 4 presents the graphical solution to the Lego® furniture problem.

![Figure 4. Graphical solution to the Lego® furniture problem](image)

### Results

In order to test if a formulation-of-the-problem attitude was developed for the students of the OR course after the Lego® experiment, a control (Fall 2009) and a two experimental (Fall 2010 and Fall 2011) groups were evaluated. The control group was not given the Lego® experiment.
Both groups were evaluated at the midterm exam by using three standard formulation problems: a minimum-cost diet problem, a multiproduct inventory control problem, and a staff scheduling problem. The evaluation scale was from 0 to 100, where 100 means the student provided the right mathematical formulation for the three problems and 0 means that there was no mathematical formulation provided for any of the problems. The graphs in Figure 5 show average grades for the BSIE and MSISE students in the control and the experimental groups.

The formulation of the problems makes 30% of the total grade of the midterm and each of the three problems has the same value, 10%. The statements of the three problems are presented in Appendix 1.

When hypothesis tests for the difference among the control and the two experimental groups were run for both BSIE and MSISE students, no difference was found in the BSIE students’ grades (Student’s t (6, 95%), p = 0.76 between the control group and the Fall 2010 experimental group and Student’s t (8, 95%), p = 0.42 between the control group and the Fall 2011 experimental group). On the other hand, for the MSISE students, a higher grade was found for the Fall 2010 experimental group (Student’s t (14, 95%), p = 0.05). However, no difference was found for the Fall 2011 experimental group (Student’s t (16, 95%), p = 0.44). At this point, it is important to notice that while the BSIE students come from about the same population since they have gone through the same math courses, the MSISE students come from a diverse background and a lot of them have had a course on OR before.

When comparing MSISE students with engineering background and previous exposure to an OR course (100% in the control group and 67% and 73% in the Fall 2010 and Fall 2011 experimental groups respectively) the same results were found: a higher grade for the Fall 2010 experimental group (Student’s t (12, 95%), p = 0.02) and no statistical difference for the Fall 2011 experimental group (Student’s t (13, 95%), p = 0.37).
The experimental groups completed a survey after the Lego® furniture experiment was completed and the typical linear problems were discussed. The survey is intended to evaluate the students’ level of satisfaction with the experiment as well as the impact of the experiment on the development of their formulation-of-the-problem attitude and it is presented in Appendix 2.

Among the students without a previous exposure to an OR course, 87% of them agree or strongly agree that “The Lego® exercise increased my chances to understand how to develop a model for maximization problems with constrained resources by using linear programming.” Among the students with previous background in OR, 25% of them were unsure on this question. About 74% of the students in the experimental groups agree or strongly agree that “This initial experience in modeling helped me deal with different and more complicated models for constrained linear optimization”. Also, 70% of the students in the experimental groups agree or strongly agree that the experiment helped them to increase their understanding of how to develop maximization problems with constrained resources by using linear programming. Finally, 78% of the students in the experimental groups agree or strongly agree that “This initial experience in modeling helped me to develop the skills required by engineers in the workplace.”

Conclusions

Students are receptive to OR whenever instructors are successful in connecting them with the power and wide applicability of modeling and optimization. Initial experiences with OR are important to show IE students the usefulness of the optimization tools when dealing with problems in their workplace.

The Lego® experiment helped students understand the usefulness of linear optimization models as they are applied to real-world problems. Furthermore, the experiment links and deepens basic knowledge of mathematics used to solve those real-world problems.

Most of the students in the experimental groups reported that the experiment helped them develop skills required to deal with the type of problems IEs deal frequently. Hence, the experiment helped them develop an idea about optimizing the product mix in order to maximize profit, which is then connected with basic mathematical knowledge and extended to other optimization problems.

Results of the survey completed by the experimental groups suggested that students were made aware of the importance of developing a formulation-of-the-problem mind set in order to transform problem statements in ordinary language into linear optimization problems.

Even though Figure 5 suggests there is an overall improvement in the BSIE students’ modeling abilities when compared with the control group, this could not be proved statistically maybe due to the small sample sizes. For the MSISE students an overall increase in the experimental groups can be observed, however, the Fall 2011 experimental group’s grades are not statistically different from the control group. Even when analyzing just MSISE students with either math or
engineering background the same trend is observed: overall increase in grades, statistically better for the Fall 2010 experimental group and not statistically different for the Fall 2010 experimental group.

Bibliography

Appendix 1. Statements of the three formulation problems

II. (30 points). Develop a mathematical formulation for the next 3 problems

a. (10 points). Joyce and Marvin run a daycare for preschoolers. They are trying to decide what to feed the children for lunch. They would like to keep the cost down, but also need to meet the nutritional requirements of the children. They have already decided to go with peanut butter and jelly sandwiches, and some combination of graham crackers, milk, and orange juice. The nutritional content and the cost of each food are given in the table below.

<table>
<thead>
<tr>
<th>Food Item</th>
<th>Calories from Fat</th>
<th>Total Calories</th>
<th>Vitamin C (mg)</th>
<th>Protein (g)</th>
<th>Cost (cents)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bread (1 slice)</td>
<td>10</td>
<td>70</td>
<td>0</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Peanut butter (1 tbsp)</td>
<td>75</td>
<td>100</td>
<td>0</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Strawberry jelly (1 tbsp)</td>
<td>0</td>
<td>50</td>
<td>3</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>Graham cracker (1 cracker)</td>
<td>20</td>
<td>60</td>
<td>0</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>Milk (1 cup)</td>
<td>70</td>
<td>150</td>
<td>2</td>
<td>8</td>
<td>15</td>
</tr>
<tr>
<td>Orange juice (1 cup)</td>
<td>0</td>
<td>100</td>
<td>120</td>
<td>1</td>
<td>35</td>
</tr>
</tbody>
</table>

The nutritional requirements are as follows. Each child should receive between 400 and 600 calories. No more than 30 percent of the total calories should come from fat. Each child should consume at least 60 mg of vitamin C and 12 grams (g) of protein. Furthermore, for practical reasons, each child needs exactly 2 slices of bread (to make the sandwich), at least twice as much peanut butter as jelly, and at least 1 cup of liquid (milk and/or juice). The problem is to determine the minimum cost diet that meets the above nutritional requirements.

b. (10 points). Our university keeps an IT phone service line (2002) for use by all students, faculty, and staff. During working hours an operator must be available to answer the phone and schedule the technicians if a service is required. Mr. T, the director of the IT line, oversees the operation.

It is now the beginning of the fall semester, and Mr. T is confronted with the problem of assigning different working hours to his operators. Because all the operators are currently enrolled in classes, they are available to work only limited number of hours each day, as shown in the following table.
There are six operators (4 undergraduate students and 2 graduate students). They all have different wage rates because of differences in their experience with technology. In the table above, you can see their wage rates and the maximum numbers of hours each can work each day. Each operator is guaranteed certain number of hours per week that will maintain an adequate knowledge of the operation. This level is set at 8 hours per week for the undergraduate students (K. C., D. H., H. B., and S. C.) and 7 hours per week for graduate students (K. S. and N. K.). The IT line is open for operation from 8 a.m. to 10 p.m. Monday through Friday with at least one operator on duty during these hours. Because of tight budget, Mr. T has to minimize cost. He wishes to determine the number of hours he should assign to each operator on each day.

c. **(10 points)** A manufacturing company produces two types of products: A and B. The company has agreed to deliver the products on the schedule shown below.

<table>
<thead>
<tr>
<th>Date</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>March 31</td>
<td>5,000</td>
<td>2,000</td>
</tr>
<tr>
<td>April 30</td>
<td>8,000</td>
<td>4,000</td>
</tr>
</tbody>
</table>

The company has two assembly lines, 1 and 2, with the available production hours shown below.

<table>
<thead>
<tr>
<th>Month</th>
<th>Production Hours Available</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Line 1</td>
</tr>
<tr>
<td>March</td>
<td>800</td>
</tr>
<tr>
<td>April</td>
<td>400</td>
</tr>
</tbody>
</table>

The production rates for each assembly line and product combination, in terms of hours per product, are shown below. It takes 0.15 hour to manufacture 1 unit of product A on line 1 and so on.
<table>
<thead>
<tr>
<th>Product</th>
<th>Production Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Line 1</td>
</tr>
<tr>
<td>A</td>
<td>0.15</td>
</tr>
<tr>
<td>B</td>
<td>0.12</td>
</tr>
</tbody>
</table>

It cost $5 per hour of line time to produce any product. The inventory carrying cost per month for each product is 20 cents per unit (charged on each month’s ending inventory). Currently, there are 500 units of A and 750 units of B in inventory. Management would like at least 1000 units of each product in inventory at the end of April. Formulate an LP problem to determine the production schedule that minimizes the total cost incurred in meeting demands on time.
Appendix 2. Survey taken by the experimental groups

EN 471/571 Operations Research

Survey about Modeling Introduction

Instructions: For each question circle the option which describes best your current status.

Q1. Have you been previously exposed to any type of training or teaching regarding modeling and optimization by using linear programming?

Yes  No

Q2. I understand the standard framework for modeling maximization problems with constrained resources by using linear programming.

Strongly Disagree  Disagree  Unsure  Agree  Strongly Agree

Q3. The Lego® exercise increased my chances in understanding how to develop a model for maximization problems with constrained resources by using linear programming.

Strongly Disagree  Disagree  Unsure  Agree  Strongly Agree

Q4. The Lego® exercise added to my understanding on developing a standard model for maximizing profits with constrained resources by using linear programming.

Strongly Disagree  Disagree  Unsure  Agree  Strongly Agree

Q5. I am confident that I can develop a standard model for maximizing profits with constrained resources by using linear programming.

Strongly Disagree  Disagree  Unsure  Agree  Strongly Agree

Q6. This initial experience in modeling helped me deal with different and more complicated models for constrained linear optimization.

Strongly Disagree  Disagree  Unsure  Agree  Strongly Agree

Q7. This initial experience in modeling helped me develop the skills required by engineers in the workplace.

Strongly Disagree  Disagree  Unsure  Agree  Strongly Agree