An Experimental Approach for Evaluating Harmonic Frequencies of a Flexible Beam

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Abstract

This paper presents a simple experimental approach that can be used to evaluate lower-order harmonic frequencies of a flexible beam. The beam was simply excited by a tap at a specific point, and the beam vibration was detected by a piezoelectric accelerometer. The vibration data was acquired and analyzed in the frequency domain. With proper choices of sampling frequency and the locations of the sensor, the frequency modes of a flexible beam can be estimated via frequency domain analysis.

Introduction

The transverse vibration of a simply supported beam (Figure 1) has been thoroughly studied in a number of literatures, such as in [1] and [2] as well as the references included therein. The vibration study of a flexible beam highlights the relationship between the beam dynamic characteristics and its material properties. The fundamental frequency of the beam is given in [1] as

$$f_1 = K_1 \frac{EI}{L^4 \rho A} \quad (1)$$

where

E = beam modulus of elasticity
I = beam moment of inertia
ρ = beam mass density, and
A = beam cross-sectional area

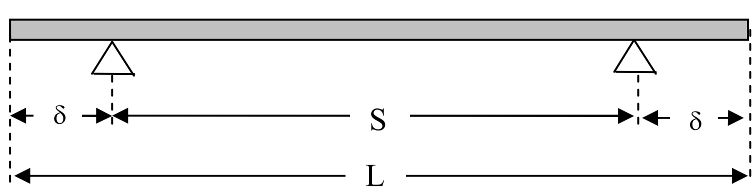

Figure 1: A simply supported beam with symmetric overhang (L - beam length, S - span between the two support points, δ - overhang length)

K_1 is the so-called transformed fundamental root of the frequency equation and can be numerically calculated through a specific iterative procedure as detailed in [1]. Note that the value of K_1 depends only on the physical dimensions - the ratio of span to length α:

$$\alpha = \frac{S}{L} \qquad (2)$$

As illustrated in [1], when the overhang length δ is very small, i.e. α is close to 1, then

$$K_1 \approx 2.467 \frac{L^4}{S^4} \qquad (3)$$

In this paper the special case of zero overhang (δ = 0) is considered, and the beam vibration patterns corresponding to the first three modes are shown in Figure 2. The theoretical values of the harmonic frequencies are given by [2]:

$$f_n = \frac{n^2 \pi}{2L^2} \sqrt{\frac{EI}{\rho A}} \qquad (4)$$

where n is an integer with values being 1, 2, 3, … and f_n representing the corresponding frequency of each vibration mode.

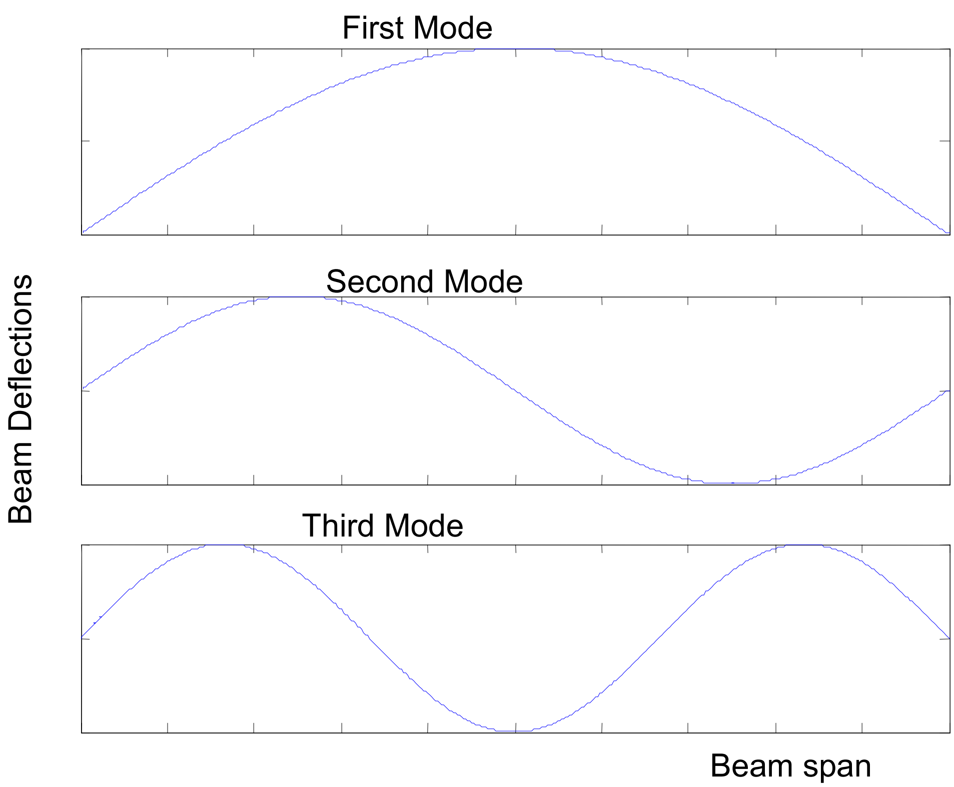

Figure 2: Modes of vibration (the curves representing the maximum deflection of the beam)

Obviously, if the value of the fundamental frequency f_1 is known, it is possible to determine the value of the beam's modulus of elasticity, *E*, and flexural stiffness, *EI,* as indicated in equation (4). This is an especially useful approach to adopt if the beam is made of non-standard material.

In this paper, the focus is on a simple experimental method that can be used to determine the low frequency modes, including the fundamental frequency f_1, of a flexible beam. With a minimum requirement on the instrumentation (a simple data acquisition device that captures the time domain data of the vibration), the values of low order harmonic frequencies can be determined via Fourier transformation of the corresponding time domain data, without the need for any special excitation device. After completing this kind of experiments, students should be able to convert time-domain data into frequency-domain by means of algorithms such as FFT (Fast Fourier Transform) which can be found in Excel, and be able to interpret the frequency-domain representation for the modes of vibrations.

Frequency Analysis Based on Impulse response

In the field of control system engineering and signal processing, it is a well-known fact that [4] a system output response completely reflects its dynamic properties when subjected to an impulse input, assuming that the system has a linear input-output property (see Figure 3). Take the example of a second-order under-damped system, by converting the impulse response (i.e. the output response to an impulse input) into frequency domain via Fourier transformation, the resonant frequency of the system can be determined by identifying the frequency where the peak occurs.

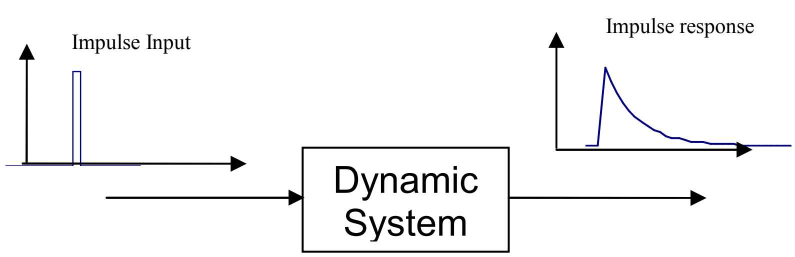

Figure 3: A dynamic system under Impulse input test

Adopting the same logic as above, if a flexible beam is excited by an impact which can be regarded as an impulse and the induced beam vibration can be recorded, in the ideal situation, most of the frequency modes of the beam will be revealed. The method of analysis used here is the discrete Fourier transform (DFT) and the method for efficient computation of DFT is the FFT (Fast Fourier Transform). This is the basis of the simple experimental method being proposed in this paper.

In comparison with the earlier work by the authors [3], the signal generator that was used to provide sinusoidal excitation in [3] is no longer needed here. Instead, a quick tap on the beam serves as the impulsive excitation. Note that, in order to avoid Rayleigh ripple effect, the tap on the beam was made with a rubber contact point. As a consequence, the instrumentation requirement for this experiment is at a minimum level, consisting of an accelerometer and a data acquisition (DAQ) system that is used to sample and record the time-domain data of the sensor output. The sampled data can be processed on-line if the DAQ system has the capability, or off-line by using standard software tools such as Microsoft Excel.

A Case Study

The system used in the case study consists of the following items/devices:

1. **An aluminum alloy beam**: This is a simply supported beam with zero overhang. It has the following parameters:
   Length L = 96 inches
   Cross-sectional area $A = 0.1972\ in^2$
   Density $\rho = 0.0926\ lb/in^2$
   Modulus of elasticity $E = 1.0x10^7$ PSI
   Moment of inertia $I = 0.0296\ in^4$

   Therefore the theoretical values of the low order harmonic frequencies, using equation (4), are
   Fundamental frequency: $f_1 = 13.8$ Hz
   Second mode: $f_2 = 4f_1 = 55.3$Hz
   Third mode: $f_3 = 9\ f_1 = 124.5$Hz
   Fourth mode: $f_4 = 16\ f_1 = 220.8$Hz
2. **A piezoelectric accelerometer:** This is one of the sensors manufactured by Piezotronics (model 353B34) with a sensitivity of 100.0 mV/g. It is attached to the beam during each test.
3. **A Tektronix TDS-380 digital oscilloscope**: This digital oscilloscope is used to display and record the output from the accelerometer. Each data file contains 1000 sampled data points.

In what follows, the results of two separate tests are presented.

Test 1: The sensor was placed at 1/8 of the beam length from the left end and the tap (excitation) on the beam occurred at ¼ of the beam length from the right end. The sensor output was sampled at a rate of *Fs* = 1000 samples/ sec.

Figure 4a illustrates the sensor output in time-domain and, clearly, it is not possible to directly obtain information about the various frequency modes of the beam. However, distinct frequency characteristics of the beam are show in Figure 4b that was obtained by processing the first 512 data points of Figure 4a using the FFT algorithm in Excel. Note that in order to properly visualize the frequency components in Figure 4b, the mean value of the time-domain data in Figure 4a was removed before applying the FFT algorithm.

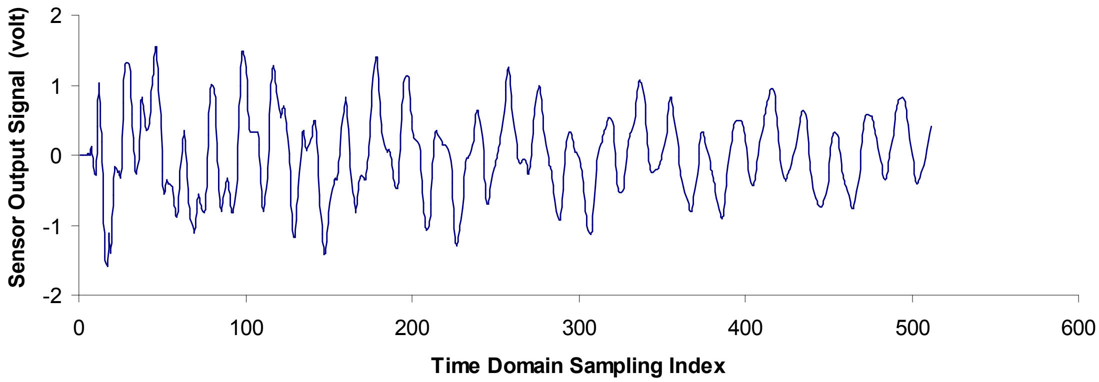

Figure 4a: Sensor output (time-domain) of Test 1

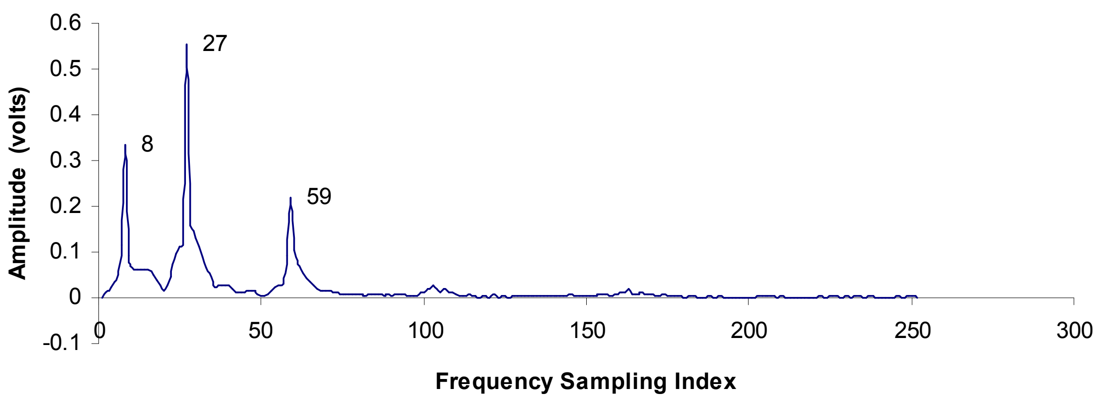

Figure 4b: Sensor output (frequency-domain) of Test 1

In Figure 4b, there are three peaks with their corresponding frequency point index number shown as $N_i = 8$, 27, and 59 respectively. These three peaks represent the three low frequency modes f_1, f_2, and f_3, and their values based on the test can be determined as follows:

$$f_i = \frac{Fs}{512} \cdot N_i \tag{5}$$

Where i = 1, 2, 3, … . The number 512 in the above equation is due to the fact that the FFT algorithm used to generating the frequency response of Figure 4b was based on the first 512 ($= 2^9$) time-domain data points. In Test 1, $N_1 = 8$, $N_2 = 27$, $N_3 = 59$, therefore,

$$f_1 = \frac{Fs}{512} \cdot N_1 = \frac{1000}{512} \cdot 8 \approx 15.6(Hz)$$
$$f_2 = \frac{Fs}{512} \cdot N_2 = \frac{1000}{512} \cdot 27 \approx 52.7(Hz)$$
$$f_3 = \frac{Fs}{512} \cdot N_3 = \frac{1000}{512} \cdot 59 \approx 115.2(Hz)$$

The ratio 512/F_s is referred to as the *frequency resolution* and it directly affects the accuracy of the frequency estimation. Compared with the theoretical values of the harmonic frequencies, the experimental values of f_1, f_1, and f_3 have errors of 13.0%, 4.7%, and 7.5% respectively. The sampling rate used in this test is most suited to the second mode. For the fundamental frequency, a lower sampling rate would provide a more accurate estimated value for f_1.

Test 2: In this test both the sensor and the excitation occurred at the middle of the beam. The sampling rate is Fs = 5000 samples/sec. The time and frequency domain response of the sensor is given in Figures 5a and 5b.

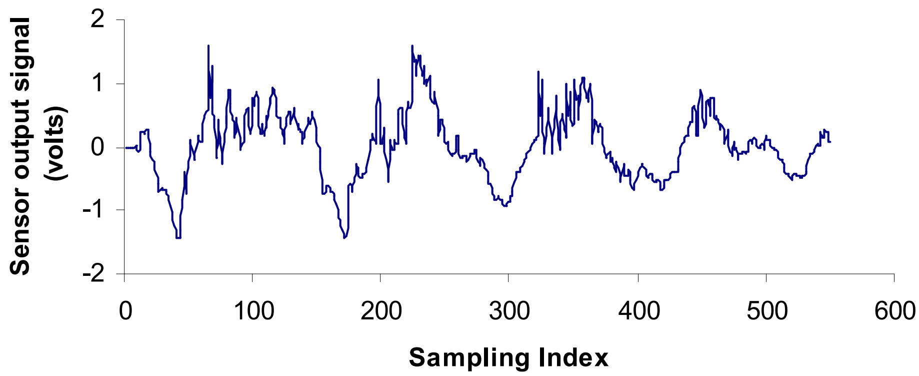

Figure 5a: Sensor output (time-domain) of Test 2

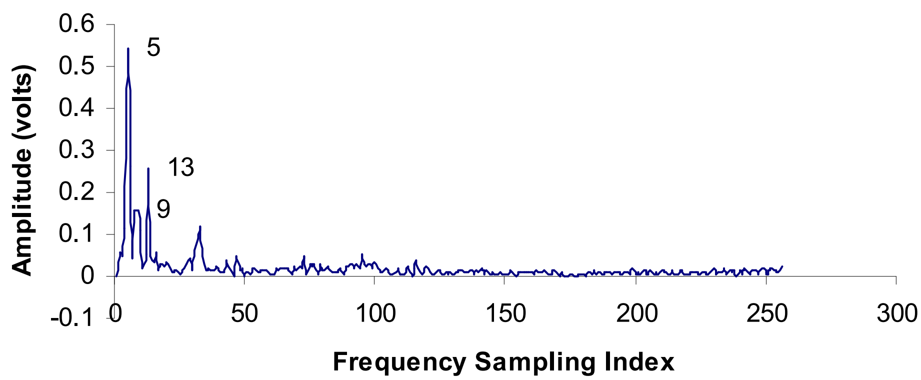

Figure 5b: Sensor output (frequency-domain) of Test 2

In Figure 5b, the three peaks occurred at the frequency points N_i = 5, 9, 13 (i = 1, 2, 3), therefore the corresponding frequencies are

$$f_1 = \frac{Fs}{512} \cdot N_1 = \frac{5000}{512} \cdot 5 \approx 48.8(Hz)$$
$$f_2 = \frac{Fs}{512} \cdot N_2 = \frac{5000}{512} \cdot 9 \approx 87.9(Hz)$$
$$f_3 = \frac{Fs}{512} \cdot N_3 = \frac{5000}{512} \cdot 13 \approx 127.0(Hz)$$

with the estimate of f_3 being the most close to its theoretical value, due to the higher sampling rate as compared with that in Test 1. The location of the sensor in this test facilitates the detection of the first and the third frequency modes as indicated in Figure 2.

Summary

The significance of the experimental method presented in this paper for determining the frequencies of low vibration modes of flexible beams is two-fold:

1. This method can be applied to complex flexible structures as well as simple flexible beams to obtain the estimates of harmonic frequencies.
2. The method involves a basic instrumentation system and a simple procedure, and it can be used to estimate second and third frequency modes as well as the fundamental frequency.

Some preliminary research work has been carried out using this method on a relatively complex flexible frame and the results obtained so far agree well with the theoretical values. It is anticipated that these results will be published in separate report in the near future.

Applying Fourier transformation to process vibration measurement data is a useful and important skill for students of structural engineering. There are a number of technical issues that deserve special attention. The accuracy of the frequency estimate depends on a number of factors, the main one being the sampling rate. As a rule of thumb, the sampling rate should be about 10-20 times the estimated frequency. For instance, if the second mode frequency being 55.3Hz, the sampling rate of 1000Hz would be a suitable value. On the other hand, this sampling rate would be a little too high to be used to estimate the fundamental frequency with a theoretical value of 13.8Hz as this means a smaller frequency resolution N_s/F_s ratio (as seen in equation (5), N_s =512 being the total time-domain data points used in the FFT computation) which is undesirable for determining the frequency values accurately. Without prior knowledge of the value of frequency to be estimated, it is advisable to obtain a more accurate result by iterating the values of the sampling rate. This also means that different sampling rates should be used to capture the sensor output data in order to determine different frequency modes more accurately.

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Bibliographic Information

[1] Murphy, J. F., "Transverse Vibration of a Simply Supported Beam with Symmetric Overhang of Arbitrary Length," *Journal of Testing and Evaluation,* JTEVA, Vol. 25, No. 5, September 1997, pp. 522-524.

[2] Gomez-Rivas, A., "Natural Frequencies of Transverse Vibration of Curved Beams", PhD Dissertation, the University of Texas, Austin, Texas, 1968.

[3] Feng, W., Gomez-Rivas, A., and Pincus, G., "Control and Signal Processing in a Structural Laboratory," Proc. of International Conference on Engineering Education, July 21-25, 2003, Valencia, Spain.

[4] Ogata, K., *Modern Control Engineering*, (4th Ed.), Prentice Hall, 2001.

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