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An Inexpensive Control System Experiment: Modeling, Simulation, and Laboratory Implementation of a PID Controller-Based System

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Abstract

This paper presents a classroom-proven control system experiment that conveys the fundamental concepts of designing a PID controller based closed-loop system. The laboratory experiment presented herein provides an opportunity for students to model, design, simulate, and implement a complete feedback control system in a very inexpensive way by using only a couple of quad op-amp ICs and a few discrete resistors and capacitors. Students are able to design various controller configurations (P, PD, PI, and PID) and investigate the effects of proportional, integral, and derivative gains on system performance including steady state error, damping ratio, overshoot, rise time, time constant, settling time, frequency of oscillation, and system stability. This laboratory experiment emphasizes developing an intuitive understanding of PID controller concepts grounded in theory and design, yet not too mathematical in nature to impede learning for many of the engineering technology students we serve.

Introduction

Teaching a control systems course can easily become too mathematical for many engineering technology students. This is mitigated primarily by using simulation and hands-on experiments in support of developing and strengthening basic control concepts such as system modeling, steady state and dynamic performance characterization, time domain and frequency domain analysis, specification based controller design, and system stability. A specific topic of significant importance is the conceptual, mathematical, practical, and intuitive understanding of proportion-integral-derivative (PID) controller design and its hardware implementation. This paper presents a very inexpensive and highly effective PID controller laboratory experiment whereby students analyze and mathematically model a mass-spring-damper system, followed by simulation and op-amp based implementation of the mechanical system. Once the control plant is implemented, the design of a PID controller is undertaken followed by simulation and op-amp based implementation of the closed-loop feedback control system. In this experiment, students are able to independently control the values of proportional, integral, and derivative gains and study their impact on steady state and dynamic performance of a control system. Dynamic performance characterization focuses on damping ratio, overshoot, frequency of oscillation, time constant, rise time, and settling time. Once the effects of individual gains are established, students can study various controller configurations (P, PD, PI, and PID) and eventually design an optimal PID controller by tuning in the parameters. The strength of this experiment lies in the fact that students are in charge of modeling, designing, simulating, implementing, and testing of this simple yet complete system. This experiment can be implemented very inexpensively (needs a couple of quad op-amp ICs and a few discrete resistors and capacitors) and yet provides an opportunity for students to investigate fundamental concepts relating system dynamics to controller configurations in a clear and concise manner. A strong feature of the proposed experiment is that students have full access to the system, allowing them to investigate controller-plant interaction at a fundamental level. Experiments on PID control is abundantly available in literature, however most of them use expensive plants[1-3] such as liquid-level system or motor drive system. However a major goal of the experiment presented herein is to develop a pedagogically sound controls experiment that is inexpensive and easily accessible by others.

Even though the standard textbooks[4-8] cover analysis and design of PID controllers in detail, engineering technology students often find the textbook presentations to be highly mathematical in nature. This impedes students' ability to achieve a clear and concise understanding of the role each of the parameters (K_P, K_I, and K_D) play in designing a PID controller in relation to steady state and dynamic performance of a system. This observation is applicable to teaching controls to non-electrical engineering students[9] as well. Engineering Technology students are generally interested in developing an intuitive understanding, grounded in theory, of PID controller's functionality. And the expectation is that this understanding should lead them to develop a skillset easily transferable to designing PID controllers in practical systems. This need analysis led to the development of this inexpensive but highly effective mass-spring-damper control system experiment covering analysis, modeling, design, simulation, laboratory implementation, and testing of a PID-controller based feedback control system.

The following sections present student outcomes for the proposed experiment, model development for the plant (mass-spring-damper system), step response simulation using MATLAB/SIMULINK, op-amp based plant implementation, PID controller design and simulation, op-amp based closed loop system implementation, and testing. Student outcomes assessment data for the laboratory experiment are also presented along with plans for further improvement to the experiment.

Student outcomes for the proposed experiment

After conducting the proposed control system experiment, students will develop:

- an improved understanding of various controller configurations (P/PD/PI/PID),
- an improved ability to design PID controllers for the end-of-semester course project,
- an ability to identify which gains (K_P, K_I, and K_D) to be increased and which gains to be decreased in a controller to improve system response, and
- an ability to prototype and test an op-amp based system model, given the system transfer function.

Open-loop mass-spring-damper system

A mass-spring-damper mechanical system[10] excited by an external force (f) is shown in Figure 1. This second-order system can be mathematically modeled as a position (x) control system with object mass (m), viscous friction coefficient (b), and spring constant (k) as parameters. Based on a free-body diagram, the system differential equation is expressed in (1). Using Laplace Transform, this time-domain equation leads to a position-to-external force transfer function given by equation (2). Once the transfer function is derived, a set of parameter values (m = 0.1 kg, b = 1 N*s/m and k = 2 N/m) is used to obtain the system transfer function shown in (3).

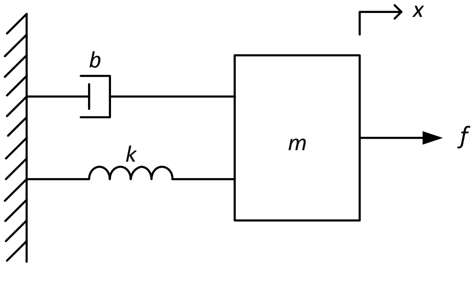


Figure 1: A mass-spring-damper system.

$$f(t) - b\frac{dx}{dt} - kx = m\frac{d^2x}{dt^2} \tag{1}$$

$$\frac{X(S)}{F(S)} = \frac{1}{mS^2+bS+k} \tag{2}$$

$$\frac{X(S)}{F(S)} = \frac{10}{S^2+10S+20} \tag{3}$$

Equation (3) can be used to observe that this second-order open-loop system is stable since both of its poles (-2.76 and -7.24) are located in the left-half of the S-plane. Additionally, it can be observed that the system's dc or steady-state gain is 0.5 and that it is an overdamped system with a damping ratio of 1.12. Using the significant pole location, the system settling time can also be estimated to be 1.45 s (= 4/2.76 s). All of these data can be easily verified by simulating the open-loop system transfer function in SIMULINK. The simulated unit step response is shown in Figure 2.

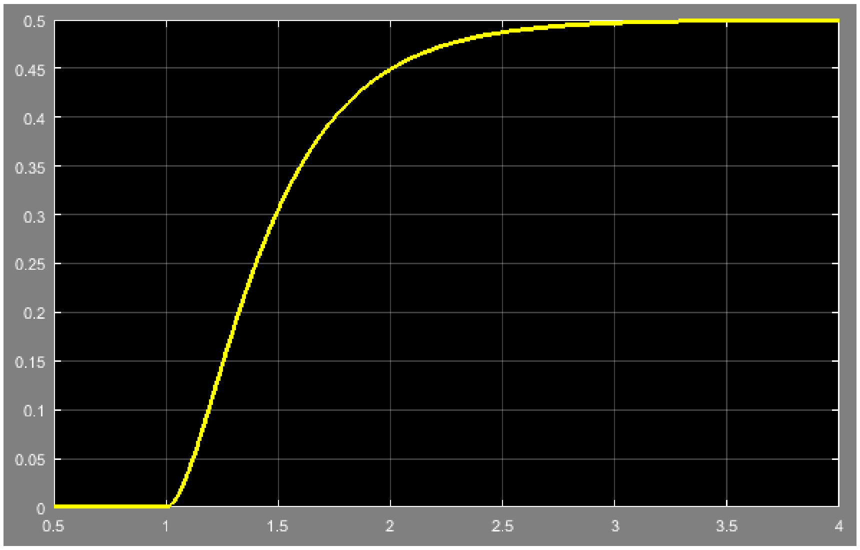


Figure 2: Simulated unit step response of the mass-spring-damper system.

Circuit implementation of the open-loop system

Circuit implementation of the mass-spring-damper open-loop system starts with simulating the system differential equation. The system simulation diagram for the given parameter set (m = 0.1 kg, b = 1 N*s/m and k = 2 N/m) is shown in Figure 3. The corresponding circuit implementation using op-amps is shown in Figure 4. This circuit implementation uses a single LM324A quad op-amp IC biased with ±12 VDC. The measured experimental step response of the plant is shown in Figure 5, and by comparing Figures 2 and 5 it can be observed that the circuit implementation shown in Figure 4 is an efficient and inexpensive way to model the mechanical system under consideration.

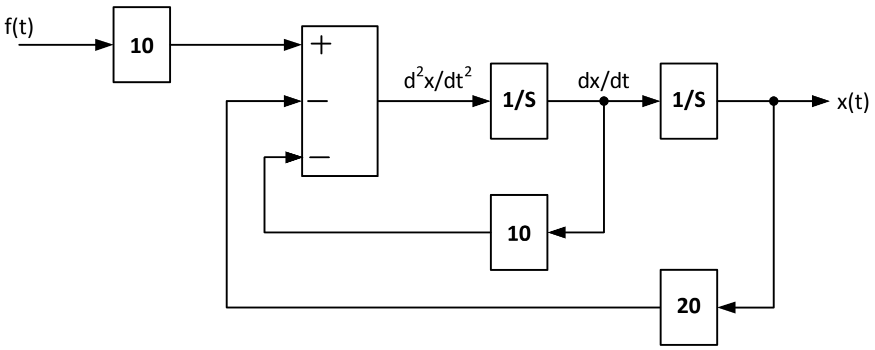
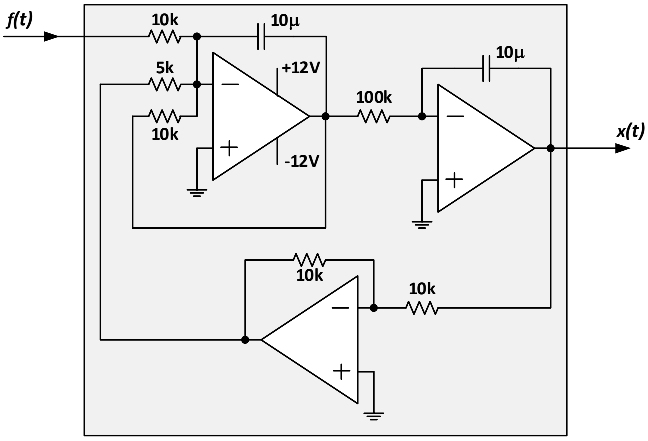


Figure 3: Simulating the differential equation representing the mass-spring-damper system.



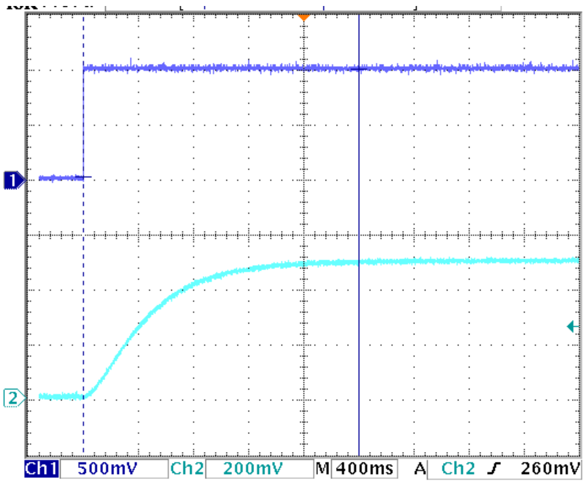


Figure 4: Circuit implementation of the mass-spring-damper system.

Figure 5: Experimental unit step response of the mass-spring-damper system.

Closed-loop control system design

Before starting the laboratory implementation of a PID-controller based feedback control system, students simulate the system using SIMULINK to study the effects of proportion, integral, and derivative gains on system performance in terms of steady-state error and dynamic performance characteristics such as overshoot, damping ratio, peak time, frequency of oscillation, settling time, and system stability. An example SIMULINK diagram and the associated time domain responses are shown for P, PD, PI, and PID controller configurations in Figures 6 and 7, respectively. Table-I summarizes the gains used for each of the controller configurations. Additionally, students are able to derive the associated closed loop transfer function and calculate the corresponding system poles and steady state error. Based on the system dominant poles students are also able to calculate the damping ratio, overshoot, peak time, time constant, and settling time. All of these calculated data are summarized in Table-I and can be compared to the simulation plots in Figure 7. It can be observed from Table I as well as Figure 7 that a simple P controller may not be a good choice if there is a tight requirement in terms of overshoot/undershoot, settling time, and steady state error. A PD controller can increase damping ratio and thereby reduce overshoot, oscillations, and settling time very effectively but it has no practical impact on the steady state error. A PI controller on the other hand can reduce

Table I: System transfer functions and calculated dynamic/steady state performance characteristics for various controller configurations.

Controller configuration	**Closed-loop transfer function**	**System poles**	**Damping ratio**	**Overshoot**	**Peak time**	**System time constant**	**Settling time**	**Steady-state error**
P ($K_P = 50$)	$\frac{500}{S^2+10S+520}$	-5 ± j22.25	0.22	49.2%	0.14 s	0.20 s	0.80 s	3.8%
PD ($K_P = 50$, $K_D = 2$)	$\frac{20(S+25)}{S^2+30S+520}$	-15 ± j17.18	0.66	6.3%	0.18 s	0.07 s	0.28 s	3.8%
PI ($K_P = 50$, $K_I = 50$)	$\frac{500(S+1)}{S^3+10S^2+520S+500}$	-0.98, -4.51 ± j22.15	0.20	52.7%	0.14 s	0.22 s	0.88 s	0%
PID ($K_P = 50$, $K_D = 2$, $K_I = 50$)	$\frac{20(S^2+25S+25)}{S^3+30S^2+520S+500}$	-1.02, -14.49 ± j16.75	0.654	6.6%	0.19 s	0.07 s	0.28 s	0%

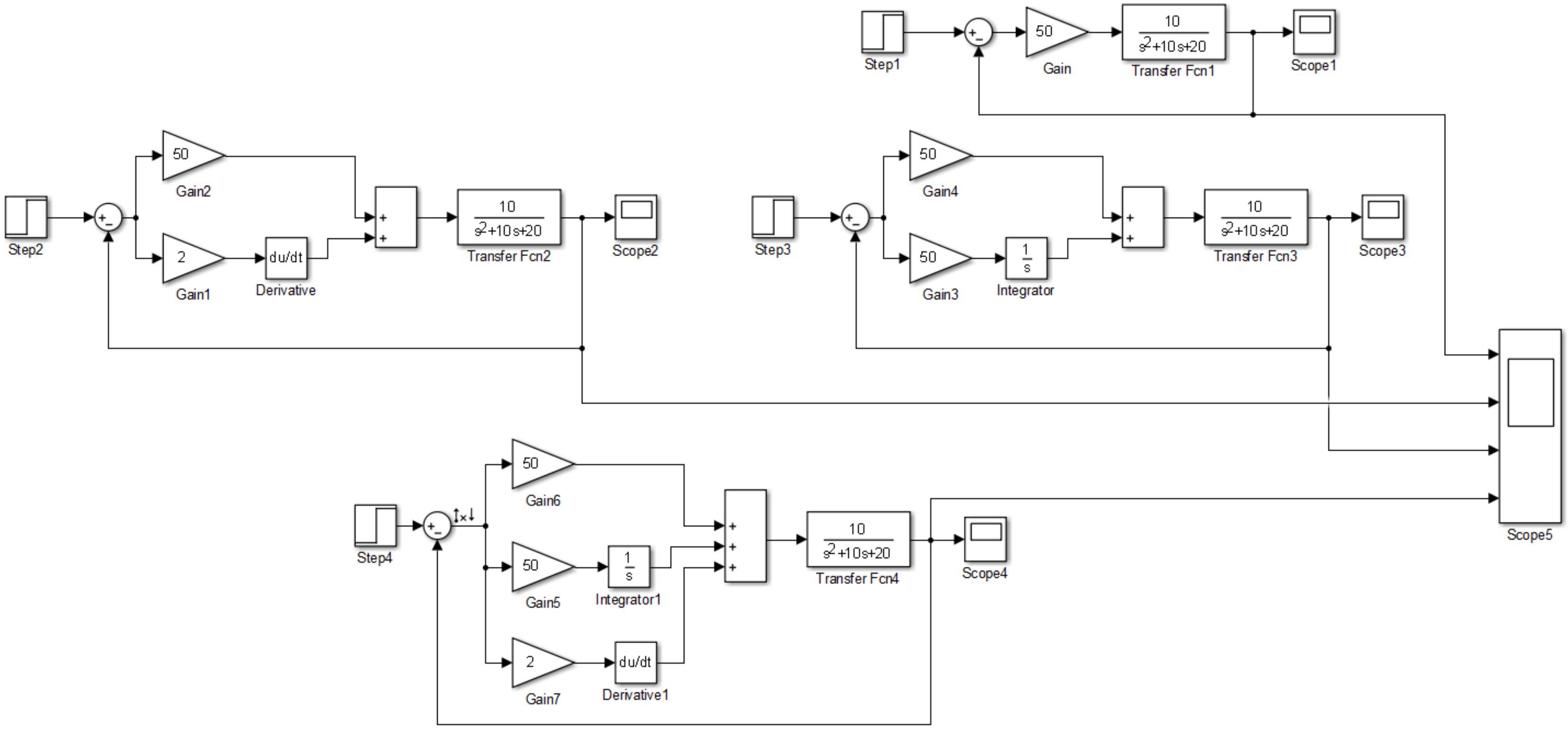


Figure 6: SIMULINK diagram of the closed loop system for various controller configurations (P, PD, PI, and PID).

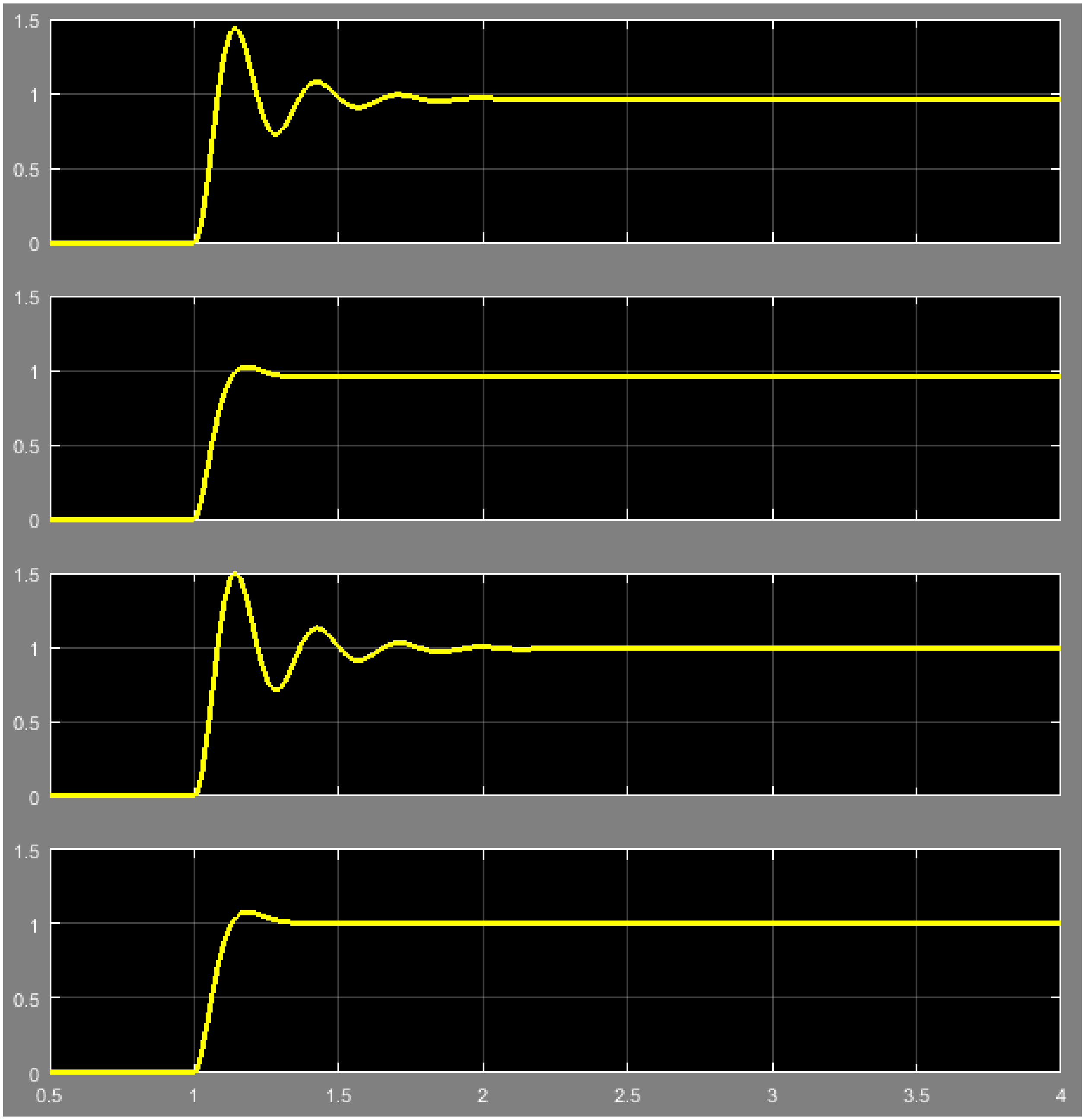


Figure 7: SIMULINK simulation plots of the unit step response of the mass-spring-damper system for various controller configurations.
{Top to bottom: P controller ($K_P = 50$); PD controller ($K_P = 50$, $K_D = 2$); PI controller ($K_P = 50$, $K_I = 50$); and PID controller ($K_P = 50$, $K_D = 2$, $K_I = 50$)}

the steady state error significantly but it has little impact on overshoot, oscillations, and settling time (compared to a P controller). By extension, a PID controller is expected to provide an improved response in terms of both steady state and dynamic performance. The bottom plot of Figure 7 clearly shows a significant improvement in steady state error with a reduced overshoot (under 10%). This system is still underdamped but the damping ratio is much higher resulting in almost no oscillations and much improved settling time.

Circuit implementation of the closed loop feedback control system

The error signal can be generated by using an op-amp based difference amplifier circuit and the PID controller can be implemented using an inverting amplifier circuit for proportional gain, an integrator circuit for the integral gain, and a differentiator circuit for the derivative gain. Finally, an op-amp based summing circuit is used to combine the P, I and D circuit outputs to create a PID controller. A complete circuit schematic of the mass-spring-damper feedback control system including the plant, error generation, and the PID controller is shown in Figure 8 in the next page. Three 100 kΩ potentiometers are used to achieve variable values for the proportional, integral, and derivative gains. Based on the resistor and capacitor values shown in the circuit schematic, the proportional gain (K_P) can be varied between 10 and 110, the integral gain (K_I) can be varied between 9.1 and 100, and the derivative gain (K_D) can be varied between 0.2 and 2.2. All three of these gain ranges are appropriate for the mass-spring-damper system under study, however a different set of gain values can be easily achieved just by using a different set of discrete resistor and capacitor values. In summary, the complete feedback control system can be implemented using just two quad op-amp ICs (e.g., LM324A), three 100 kΩ potentiometers, a few 1 kΩ/10 kΩ/100 kΩ discrete resistors, and a few 1 μF/10 μF discrete capacitors. This implementation approach lets students clearly see and manipulate each block of a typical feedback control system.

Testing the closed loop control system

Once the designed circuit representing the PID-controller based mass-spring-damper system is prototyped, it can be tested for step response by connecting a low frequency square-wave signal (e.g., 0.5 Hz) as the reference input while monitoring both the reference input and the controlled output using a two-channel oscilloscope to evaluate the system steady-state and dynamic performance. Figure 9 shows the time-domain output responses of the system due to a unit step reference input. For a P-controller, as expected the output response [Figure 9(a)] has overshoot, oscillations, and steady-state error. And by adding a derivative component, the output response [Figure 9(b)] due to a PD-controller reduces the overshoot and oscillations, but the steady-state error remains unchanged. By adding an integral component to the proportional controller, the PI-controller provides an output response [Figure 9(c)] with much improved steady-state error but the overshoot and oscillations remain practically unchanged, compared to the response for a P-controller. Next, the system performance was studied for three different sets of PID controller parameters. As shown in Figure 9(d), a PID controller reaps benefits of both PI and PD controllers and provides an output response with reduced overshoot, very little oscillation and much improved steady state error. Then the K_I value was reduced and K_D value was increased, resulting in an output response [Figure 9(e)] with even less overshoot and almost no oscillations while the steady state error stays practically unchanged, compared to the previous PID controller parameter set. For the final PID parameter set, K_P was reduced significantly, K_I was further reduced, and K_D was increased in order to eliminate the overshoot and oscillations while keeping the steady state error practically unchanged. As shown in Figure 9(f), this resulted in a nearly-optimal response since the settling time is practically unchanged with a very slight increase in rise time.

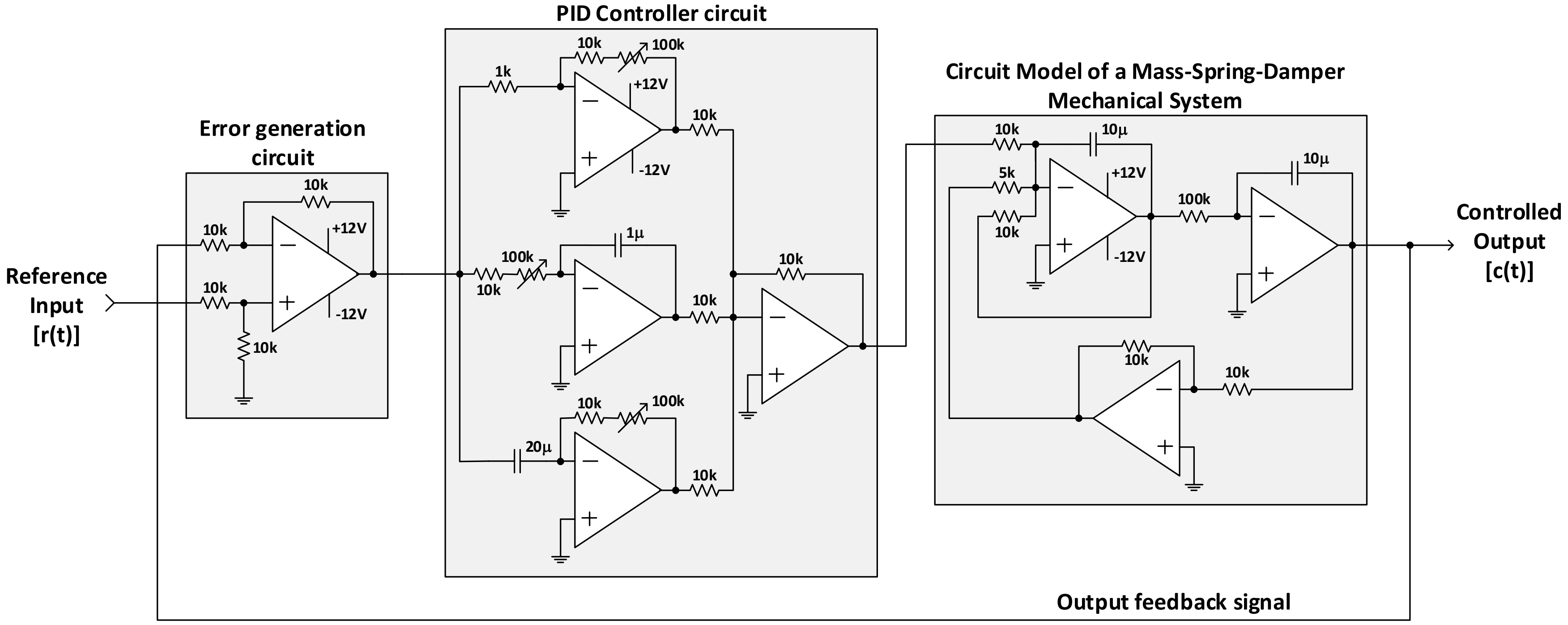
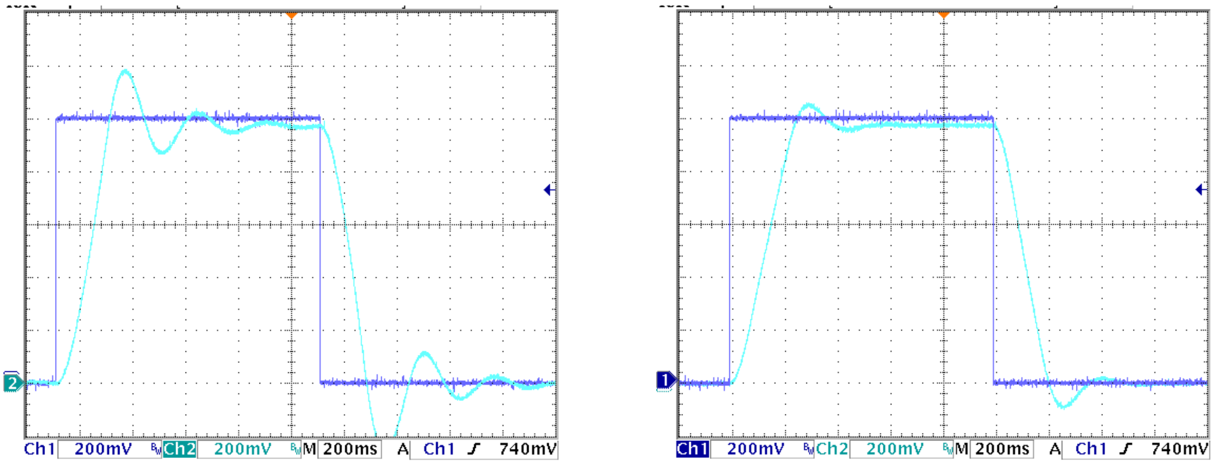
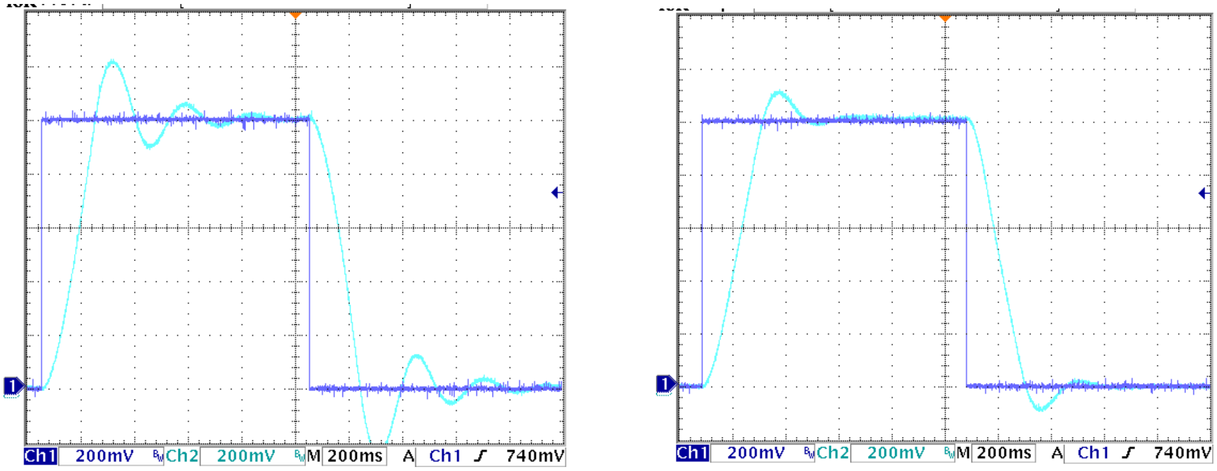


Figure 8: Circuit implementation of the mass-spring-damper closed-loop feedback control system with PID tuning capability.



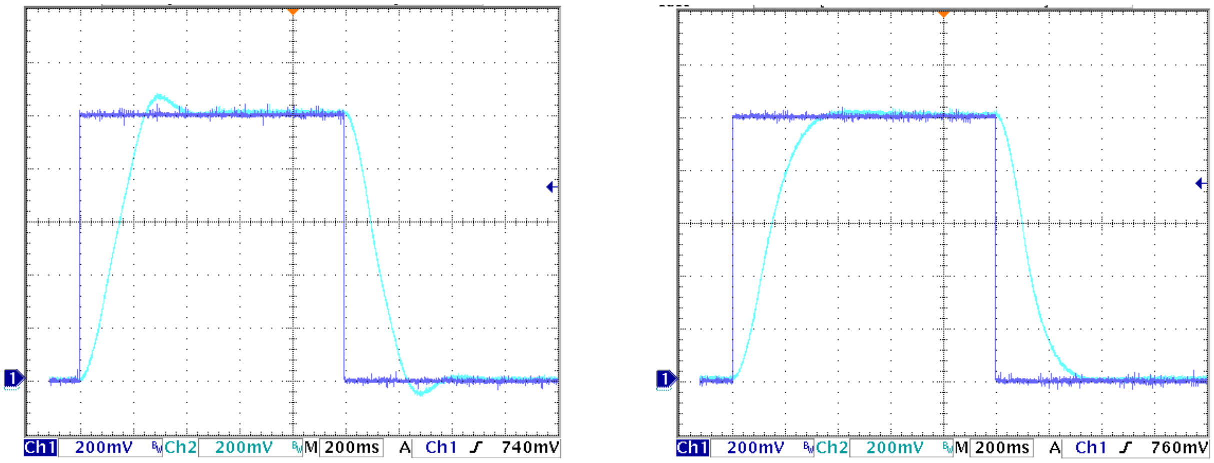
(a) Closed loop response with P controller ($K_P = 60$)

(b) Closed loop response with PD controller ($K_P = 60$, $K_D = 1.2$)



(c) Closed loop response with PI controller ($K_P = 66$, $K_I = 13$)

(d) Closed loop response with PID controller ($K_P = 60$, $K_I = 17$, $K_D = 1.2$)



(e) Closed loop response with PID controller ($K_P = 66$, $K_I = 13$, $K_D = 1.76$)

(f) Closed loop response with PID controller ($K_P = 25$, $K_I = 12$, $K_D = 2.0$)

Figure 9: Closed-loop system response for P, PD, PI, and PID controller configurations.

Summarizing the effects of P, I, and D parameters on closed-loop system response

Based on the above theoretical, simulation, and experimental observations, the effects of K_P, K_I, and K_D values on closed-loop system response is summarized in Table-II. This results summary matches well to the similar information available in the literature[10], however the experiment proposed herein lets students develop this summary table on their own.

Table II: Effects of independent P, I, and D parameters on closed-loop system response.

	Rise Time	**Overshoot**	**Settling Time**	**SteadyState Error**	**Stability**
Increasing K_P	Decrease	Increase	Small change	Decrease	Degrade
Increasing K_I	Small decrease	Increase	Increase	Large decrease	Degrade
Increasing K_D	Small decease	Decrease	Decrease	Minor change	Improve

Student outcomes assessment for the proposed laboratory experiment

As part of a senior-level controls course, a three-hour laboratory experiment based on the material presented herein was conducted by electronics engineering technology students. The associated pre-lab assignment included modeling and simulation of the mass-spring-damper system under open-loop. The assigned pre-lab also asked for an op-amp based circuit schematic to model the open-loop system. The three-hour lab started with the circuit implementation and testing of the plant. This was followed by closed-loop system implementation and testing focusing on time-domain system characteristics. Various controller configurations such as P, PD, PI, and PID were studied to assess system's steady state and dynamic performance parameters such as steady state error, overshoot, damping ratio, frequency of oscillation, rise time, settling time, and system stability. Finally, students were to experimentally come up with an optimal PID controller for the given steady state and dynamic performance specifications.

Indirect student assessment data (Number of students = 20)

Question	Definitely	Possibly	Not really
Did this lab help improve your understanding of various controller configurations (P/PD/PI/PID)?	85%	10%	5%
After completing this experiment, do you feel confident you could design a controller for the end-of-semester course project?	80%	15%	5%
Looking at the output response of a system, would you be able to identify which gains (K_P, K_I, and K_D) to be increased and which gains to be decreased to improve the output response?	75%	20%	5%
Given the transfer function of a system, would you be able to prototype and test an op-amp based model of the system?	70%	20%	10%

Student comments

- A simple yet very effective lab
- The lab was a bit too long
- I now understand what a PID controller is and what it does
- Make this a two-week lab effort. Convert the pre-lab to a full lab by adding the circuit implementation of the open-loop system, and the rest of the lab can become the second full lab.
- This lab will help out with controller design for our end-of-semester course project

Planned update to the laboratory experiment

The experiment presented herein was offered for the first time in spring-2015 as a one-week effort. Based on student feedback, this experiment is currently being expanded to be a two-week effort, and will be implemented in spring-2016. The expanded effort includes assisted prelab work and new material on frequency domain analysis of PID-controllers, including design-oriented relationships among PID parameters, frequency domain parameters (system bandwidth, phase margin, and gain margin), and time-domain parameters (steady state error, damping ratio, overshoot, frequency of oscillation, settling time). Inclusion of a formal approach to PID controller tuning is also under consideration.

Summary

A laboratory experiment is presented encompassing all major aspects of designing a complete feedback control system including system modeling, controller design, simulation, and laboratory implementation and testing. The mass-spring-damper system represents a typical second order system and students can use op-amp based circuits to prototype the plant, and then design various PID controller configurations to study the effects of controller gain(s) on system's steady state and dynamic performance. This simple yet very inexpensive experiment can be used to teach fundamental concepts of PID controller design, leading to an intuitive understanding based in theory and design. The experiment presented herein is currently being updated to include frequency domain analysis and design to complement the time domain analysis and design.

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