AC 2010-1174: AN INQUIRY-BASED EXERCISE INVOLVING A TANK OF WATER WITH A HOLE IN ITS SIDE

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Introduction

The tank draining exercise is part of a larger study on inquiry-based laboratory exercises for undergraduate engineering courses in the fluid and thermal sciences. Our research involves the development of the curricular material, measurement of learning gains, and measurement of changes in student attitude toward laboratory work. In this paper we discuss the laboratory hardware, the laboratory procedure, and typical results of using the tank draining hardware.

Broad Goals

The tank draining exercise provides a laboratory experience to teach students about transient, incompressible flow. Draining of a tank is one of the few practical applications of transient flow that can be analyzed at the level of fluid mechanics knowledge typical of undergraduate engineering students. Mass conservation is applied to the tank to relate the change in height of the free surface to the exit velocity from the hole in the side of the tank. The tank draining experiment also affords a discussion of the whether hydrostatic pressure equation can be applied when the fluid is moving. This issue is explored in the pre-lab reading assignment.

In addition to addressing core concepts of fluid mechanics, the tank draining exercise is designed to develop qualitative reasoning skills. We define qualitative reasoning as the ability to predict trends in the behavior of a system from direct observations (experimental evidence) and qualitative manipulation of mathematical models. This skill is especially useful in a laboratory setting or on a factory floor where there may not be time to perform a detailed engineering analysis. Qualitative reasoning is described by the common expressions “thinking on one’s feet”, “using engineering experience”, and “back of the envelop calculation”.

Qualitative reasoning is a kind of higher order thinking practiced by experienced engineers. A laboratory exercise provides opportunities to develop and demonstrate qualitative reasoning skills because the system response can be predicted and then observed with minimal formal analysis. In the tank draining exercise, students make measurements on one tank. They are then asked to predict the results of repeating the measurements on a tank with a different shape. After making their predictions, they perform the measurements that provide immediate feedback on the accuracy of their predictions.

Previous Work

Similar tank draining experiments have been used in class demonstrations, in science museums, and by other authors[1-4]. For example, the supplemental material to the textbook by Munson, Young and Okiishi[4] includes a video of water draining from three holes in a two liter soda bottle. Libii[1], and Libii and Faseyitan[2] describe a tank-draining experiment where the drain orifice is at the bottom of the tank. Saleta et al. use a configuration similar to that in Figure 1, below[3]. The experiment we have developed uses a digital camera to measure the jet trajectory (like Saleta et al.) and a pressure transducer to measure the fluid height (like Libii and Faseyitan).

Our version of the tank draining exercise is unique in that the arc of the water jet is recorded with a digital camera at the same time a data acquisition system is recording output of the pressure transducer. We also use two tanks with different shapes to show that only the depth of the water, not the shape of the tank, determines length of the jet issuing from the side of the tank. The most significant difference between our work and the preceding work is that our exercise uses guided inquiry to actively engage the students in the measurements as they are conducted.

Building on Tank-Filling Exercise

The tank draining exercise is the second of two exercises that use the same equipment. In the first exercise called "tank-filling", students record pressure as water is added to the tank[5]. The goal of the tank filling exercise is to firmly establish that the relationship between pressure and water depth is independent of the tank shape when the water is stationary. The tank filling exercise is performed in the first week of class, which is appropriate because the hydrostatic equation is introduced early in the academic term. Even if students have not seen the hydrostatic equation in the lecture, the concept of pressure being solely a function of depth is relatively simple to grasp. Despite the simplicity of the concept, a significant fraction of the students hold the misconception that pressure is due to the total weight of water above the point of pressure measurement.

The tank-draining exercise is more complex than the tank-filling exercise. Figure 1 shows the basic features of the apparatus. A jet of water issues from a hole in the side of a tank of water that is a depth h below the free surface in the tank and a distance H above the horizontal base of the tank. The water jet travels a distance L before intersecting the horizontal plane. Because the tank is large relative to the hole in the side of the tank, the tank drains slowly. Furthermore, the pressure on the tank wall opposite the drain hole is still determined only by the depth h. As with the tank filling exercise, the shape of the tank does not determine the hydrostatic pressure. Only near the exit hole does motion of the water have an effect on the pressure distribution. Neglecting flow losses, a simple model of tank draining gives the relationship $L = \sqrt{2hH}$.

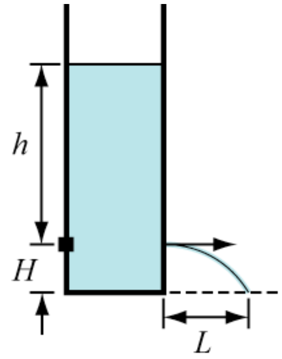


Figure 1: Basic features of the tank draining exercise.

The shape of the tank is important only in that it determines the capacity, and hence the total time to drain the tank. At any instant during the tank draining process, L does not depend on the tank shape. These concepts are exposed in a collaborative, guided-inquiry experience. Students work in small groups, and they analyze results and reach conclusions as they are working in the lab. There is no lab report. Instead, while they are in the lab, students complete a worksheet that requires them to sketch plots, do simple calculations, and answer short essay questions. The students work together to confront their misunderstandings and to resolve any differences between their predictions and the direct observations of the system behavior.

Apparatus

Figure 2 is a schematic of the two simple tanks of water that are used in the tank draining exercise. The first tank is made from a single cylinder of acrylic 18 cm in diameter and 75 cm long. The second tank is made of two cylinders of acrylic with different diameters joined end-to-end. The bottom cylinder of this so-called step-walled tank is 18 cm in diameter and 34 cm tall. The top cylinder is 7.5 cm in diameter and 41 cm tall. Figure 3 on the next page is a photograph of the step-walled tank during a draining experiment.

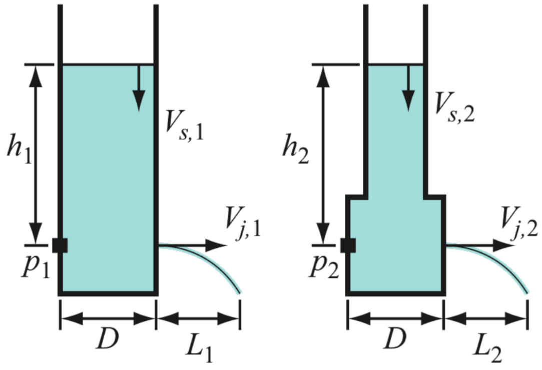


Figure 2: Geometry features of the straight-walled and step-walled tanks.

In the tank filling exercise, the students verify the hydrostatic principle. They record the output of the pressure transducer as water is added to the tank. The pressure transducer is mounted on the side wall of the tank at a distance of 18 cm from the base. The water depth is measured with a flexible ruler taped to the side of the tank.

For the tank-draining exercise, two measurements are made as the tank drains: the instantaneous depth $h(t)$ of the water in the tank, and the horizontal distance $L(t)$ travelled by the jet before it reaches the horizontal plane at the base of the tank. The depth measurement is made with the pressure transducer attached to the side wall. A simple analysis shows that since the hole in the opposite wall is small, and since the tank drains relatively slowly, the pressure transducer is a reliable measure of water depth.

Opposite from the pressure transducer are three small holes of diameter 3.2 mm, 6.4 mm and 7.9 mm,. The holes are at the same depth H as the pressure transducer. The holes are spaced approximately 3 cm apart around the circumference of the bottom section of the tank. Only one hole is active at a time during the experiment.

The jet distance $L(t)$ is obtained from visual inspection of images captured with a digital camera mounted on a tripod and focused on the plane of the jet. Figure 3 on the next page is a photograph of the water jet. A modestly priced digital camera has sufficient resolution to allow L values to be read from a meter stick oriented horizontally and slightly behind the water jet. The images are captured by manually depressing the shutter switch during the experiment. This leaves the problem of synchronizing the L measurements obtained by the camera with the h measurements obtained by the pressure transducer.

Two different schemes have been used to synchronize the jet images with the pressure measurements. At first the images were captured and transferred to the lab computer as the experiment was progressing. Each image was stored with a file name that indicated the time in seconds from the start of the experiment. Capturing and storing the images on the fly was not robust. The operating system (Windows XP) and LabVIEW would occasionally hang up, requiring the experiment to be rerun, which caused the students to be frustrated.

A simpler and more robust solution is to asynchronously capture the images and then transfer the images to the computer after the tank draining is complete. This approach requires a mechanism for adding a time stamp to the images so that the $L(t)$ measurement with the camera can be associated with the $h(t)$ measurement with the pressure transducer. A time stamp is added to each image by positioning a six segment LED display kit (USB7 from Fundamental Logic http://store.fundamentallogic.com) in the field of view of the camera. Figure 3 is a photograph of a tank draining measurement in progress. An enlarged image of the LED is also inset in the image. The digits on the LED display are set from the internal time of the LabVIEW program recording the pressure transducer output. Therefore, each image that records $L(t)$, also indicates the time the image was captured according to the time base of the pressure measurements.

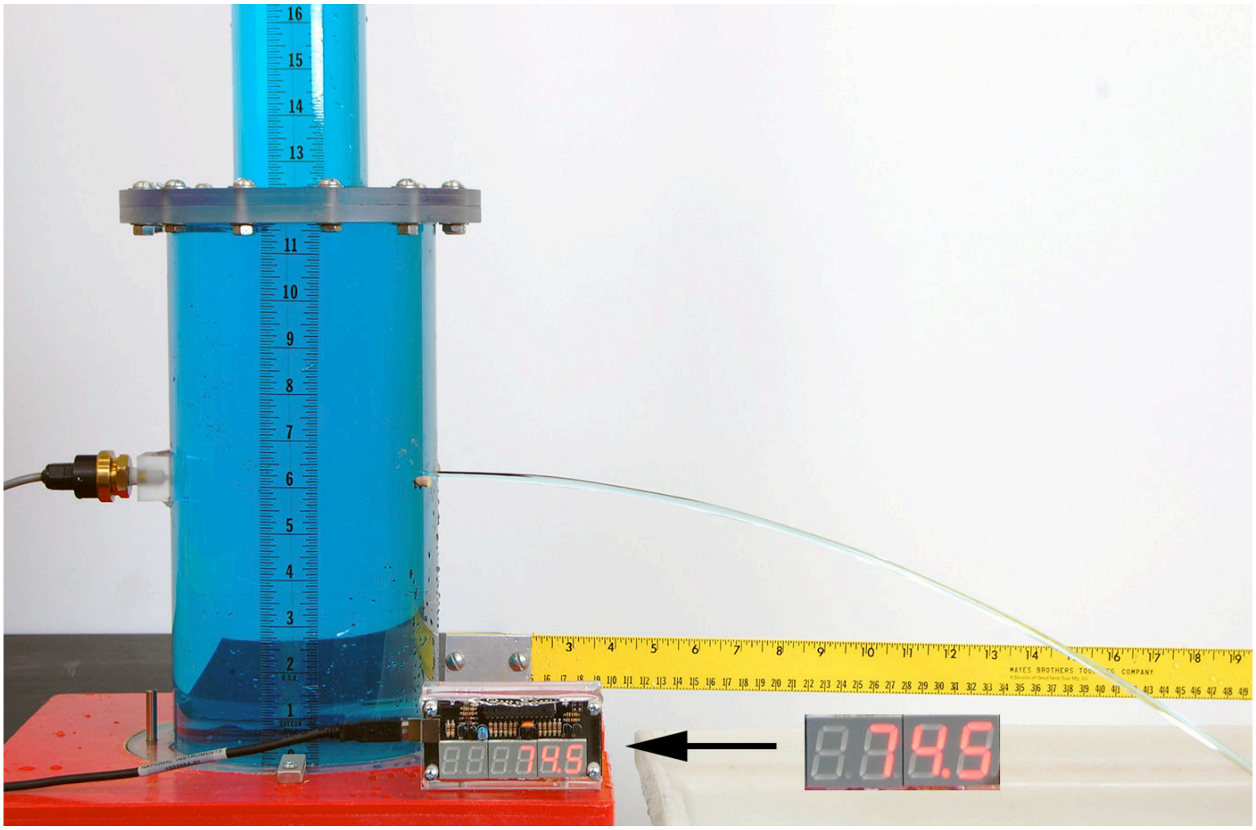


Figure 3. Stepped-tank during draining. The seven-segment LED display at the base of the tank indicates the time in sections from the start of the pressure transducer measurements. Blue food coloring was added to the water to increase the contrast.

Data Collection and Analysis

In a conventional laboratory exercise, students record data in the lab and then go home to analyze the data and write a report. In our guided inquiry exercise, the data collection and analysis both happen in the laboratory. The objective is to engage students in reasoning while they can change the sequence and configuration of the measurements. This requires students to be more active participants in the laboratory. It also creates opportunities to practice qualitative reasoning when students are asked to predict the outcome of a measurement and then confirm that prediction with a measurement.

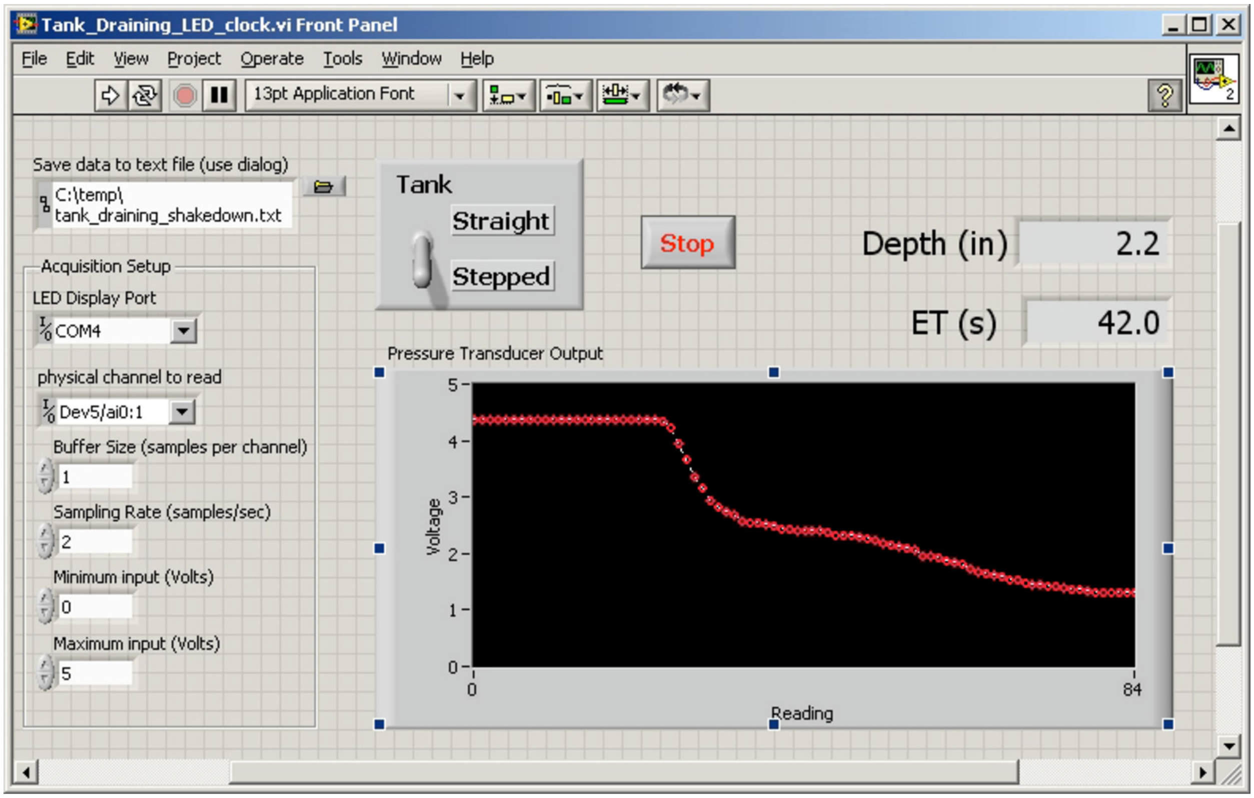
If analysis of measurement data is to occur in the laboratory, the analysis must either be simple enough for students to do by hand in a short time, or the analysis must be supported by software provided to the students. The latter approach is used in the tank draining exercise. The pressure transducer data is recorded with a LabVIEW VI. Combination and analysis of the pressure transducer and image data is performed with a MATLAB program provided to the students.

Figure 3 is a screen capture of the LabVIEW VI that records the pressure transducer output. The center of the VI is a plot of the $p(t)$ signal which is proportional to $h(t)$. The "Depth" and "ET" displayed in the upper right corner are the instantaneous water depth and the elapsed time. The ET value is also sent to the LED display to provide the time stamp for the $L(t)$ images. A toggle switch on the VI determines which channel of the DAQ is stored in the data file. Two pressure transducers – one for each of the two tanks – are connected to the DAQ. Therefore, the toggle positions are labeled with "straight" or "stepped" to indicate the tank shape.

Each experiment with the tank draining apparatus consists of three main phases

1. Record output of the pressure transducer and capture images as water drains from the tank.
2. Use an image viewer to assist in manually extracting the $L(t)$ distance traveled by the jet.
3. Combine the $h(t)$ and $L(t)$ data to obtain $L(h)$.

Figure 4 provides a more detailed conceptual map for the tasks involved in the data capture and analysis. As the tank is draining, images are captured (step a.) and pressure versus time data is recorded (step b). After the draining is complete and the data logging is stopped, the jet length values, L, are extracted from the images captured by the digital camera (step c). The pressure transducer values are converted to depth (step d). The time t_i at which each image was captured is used to interpolate in the depth versus time data, and to extract a set of $h(t_i)$ values corresponding to each of the images used to measure $L(t_i)$, (step e). Finally, the $L(t_i)$ and $h(t_i)$ data are combined to yield a plot of $L(h)$, (step f). Figure 5 is an annotated screen shot of the MATLAB program used to perform the analysis in steps d, e, and f.



Save data to text file (use dialog)
C:\temp\tank_draining_shakedown.txt

Acquisition Setup

LED Display Port: COM4

physical channel to read: Dev5/ai0:1

Buffer Size (samples per channel): 1

Sampling Rate (samples/sec): 2

Minimum input (Volts): 0

Maximum input (Volts): 5

Depth (in) 2.2

ET (s) 42.0

Figure 3 LabVIEW VI used to obtain the pressure measurements during the tank draining exercise.

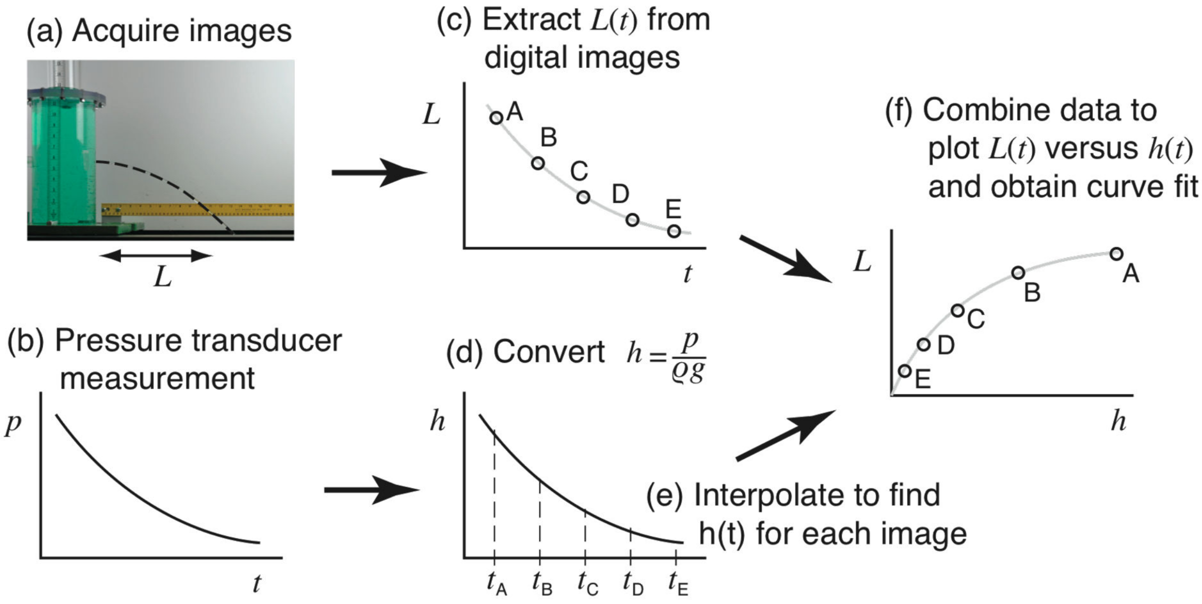
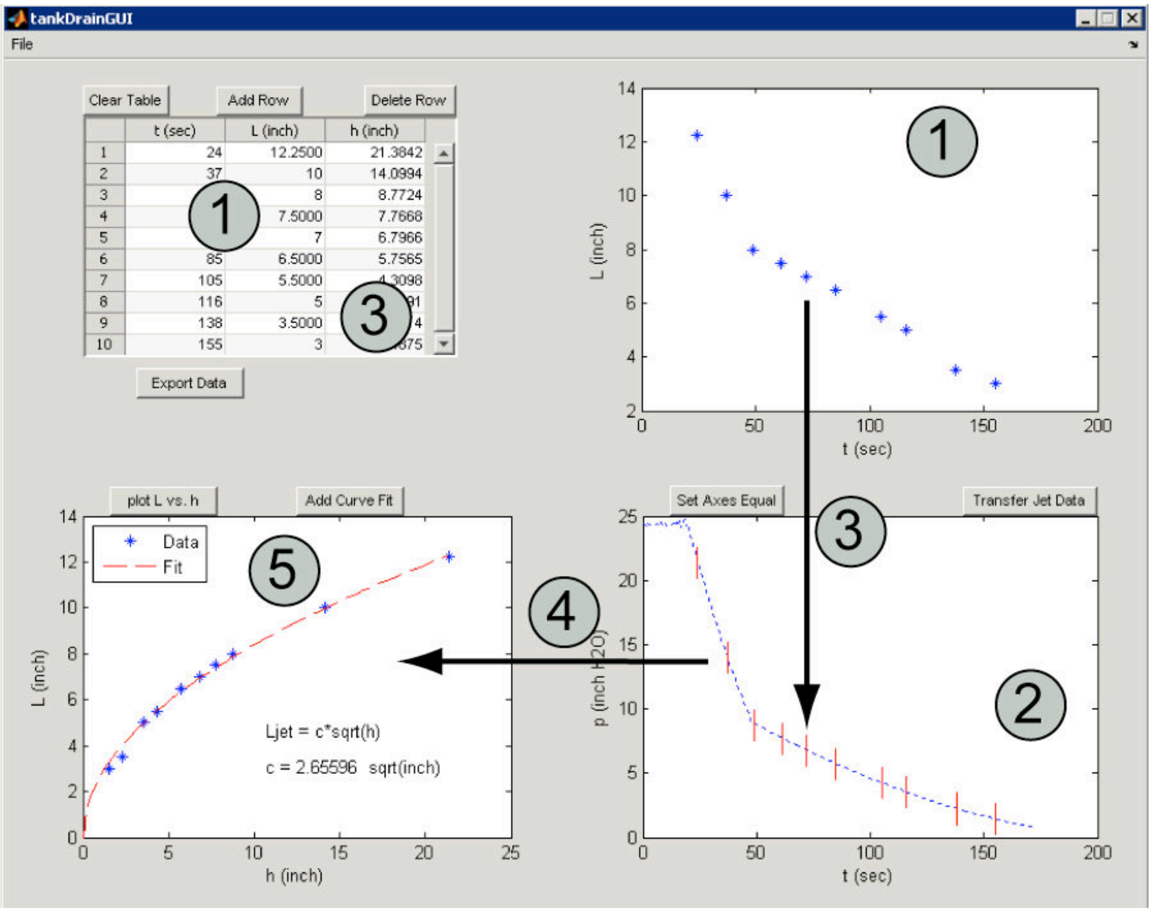


Figure 4 Measurement and data analysis sequence for the tank draining exercise.



	t (sec)	L (inch)	h (inch)
1	24	12.2500	21.3842
2	37	10	14.0994
3	[illegible]	8	8.7724
4	[illegible]	7.5000	7.7668
5	[illegible]	7	6.7966
6	85	6.5000	5.7565
7	105	5.5000	4.3098
8	116	5	[illegible]
9	138	3.5000	[illegible]
10	155	3	[illegible]

Figure 5 Annotated screen shot of the user interface (UI) for the MATLAB program that performs data analysis steps d, e, and f in Figure 4. The data in the plots is from an experiment with the step-walled tank.

The analysis with the MATLAB program is performed in the following steps. The step numbers correspond to the annotations in Figure 5.

1. The $L(t_i)$ data extracted from the images captured during the experiment are entered in a spreadsheet-like data table. The t_i are discrete times at which the images are captured. As the $L(t_i)$ data is entered in the table, it is automatically displayed in the plot in the pane in the upper right quadrant of the User Interface (UI) window.
2. The $p(t)$ data captured by the LabVIEW VI is loaded from a text file saved by the VI. The $p(t)$ data is displayed in the figure pane in the lower right quadrant of the UI.
3. With the $L(t_i)$ data and $p(t)$ stored in program memory, the user clicks a button labeled "Transfer Jet Data", which interpolates the $p(t)$ data to find $p(t_i)$, i.e. the pressure at the discrete times when then images of the water jet were captured. The $p(t_i)$ data is converted to depth, $h(t_i)$, which is added to the data table in the upper left quadrant. To visually represent the interpolation, short red vertical bars are added to the $p(t)$ curve in the lower right quadrant of the UI window.

4. With $L(t_i)$ and $h(t_i)$ data on a common time base t_i, a plot of $L(h)$ is created in the lower left quadrant of the UI window.
5. The user clicks a button labeled "Add Curve Fit" and a least squares fit of $L = c\sqrt{h}$ is performed and added to the $L(h)$ plot. The value of c is displayed.

As a homework assignment or as part of a laboratory report, it would be reasonable to ask students to perform the steps that are automated by the MATLAB program. In our guided inquiry exercise, the computational tasks are automated, and the students are asked to provide reasoning and explanations for the observed behavior.

Guided-Inquiry Exercise

As a result of performing the guided-inquiry exercise, students should achieve the following learning objectives.

- Be able to apply mass conservation to a control volume with a time-varying mass;
- Be able to determine the pressure at depth in a tank with a small hole in its side;
- Use digital photographs to measure geometric features, specifically the trajectory of the water jet emerging from a hole in the side of the tank;
- Apply the Bernoulli and Energy Equations to compute the velocity of a free jet emerging from a hole in a tank;
- Explain the key similarities and key differences between the draining of two tanks with different shapes.

Directions for performing the exercise are written in a worksheet that students complete in the lab. There is no lab report. The calculations on the worksheet are simple enough to be performed with a calculator. The MATLAB program depicted in Figure 5 performs the data-intensive computations required to complete the exercise. Students make sketches of the plots from the data analysis screens, and they are asked to predict how the trends in the plots will change during the next set of experiments.

The exercise begins with both tanks filled with water. A complete cycle of measurement and analysis is performed with the straight-walled tank. Worksheets provide a structured path of inquiry. Questions are posed and the lab group is expected to reach conclusion before moving on to the next phase of the measurements. The lab instructor is required to check the worksheet at key points. The checkpoints are indicated by large "stop signs" in the worksheet. This prevents students from rushing through the exercise in the way that they might in a typical lab exercise that requires them only to record data.

All members of the team progress at the same pace. The goal is to encourage group discussion and resolution of the conceptual questions. A modest amount of instructor involvement is required to make this work. When the team has reached one of the stop signs, the instructor can ask a conceptual question, and whether everyone agrees with the answer provided.

In Fall 2008 the tanks had only one size of drain hole. During a discussion with the instructor, the effect of hole size was debated. In preparation for Fall 2009, the tanks were redesigned to have three drain holes 3.2 mm, 6.4 mm and 7.9 mm in diameter (1/8, 1/4 and 5/16 inch, respectively). The holes are at the same depth as the pressure transducer and are plugged with corks. The tank base was redesigned to allow the tank to be rotated so that the water jet emerging from hole can be aligned parallel with the horizontal meter stick. Only one hole is opened at a time for each experiment. In the guided inquiry worksheet there is no explicit instruction to choose a specific size of drain hole. At the start of the lab session, the instructors position the tank so that the medium size hole is aligned to produce a water jet parallel to the meter stick. The very last question on the guided inquiry worksheet is

> Consider two straight-walled tanks that have holes with different diameters, say d_1 and d_2, with $d_1 > d_2$. At the same h, which tank would have the larger L? At the same h, which tank would have the larger V_j?

The students are not instructed to obtain their answer with any measurements. The overwhelming majority of student groups simply wrote down their opinion of the correct response. Only a small number of student groups took advantage of the opportunity to make measurements to confirm the answers to the last question on the worksheet.

Typical Results

In this section we show results from measurements with the tank draining apparatus. The results are formatted for publication, and are not arranged exactly as the students see the results from the MATLAB data reduction program during the laboratory exercise. However, the data is obtained in the same way that students make measurements, and the data presented here was recorded with the LabVIEW program used by students. Images used to obtain the $L(t)$ data were captured with a more expensive camera than that used by students, and extra care was taken to align the camera to maximize accuracy of the $L(t)$ measurements. Despite these modest differences in procedure and equipment, students can obtain results similar, if not identical, to those presented here.

Figure 6 shows representative data from the straight walled tank. The two plots on the left side of the figure are measurements plotted as a function of time. The top plot is the output of the pressure transducer, converted to depth. The bottom plot is the jet length data obtained from inspection of the digital images. During draining, the changes in depth and the variation in jet length are smoothly varying functions of time. The plot on the right has eliminated time to plot L as a function of h. A curve fit of $L = c\sqrt{h}$, where c is a constant, is a good match to the data.

During the laboratory exercise, students see the data displayed as in Figure 5 (without the numbered circles). They are asked to sketch the $h(t)$, $L(t)$ and $L(h)$ plots on their worksheets. Before making measurements on the step-walled, tank, students are asked to overlay sketches of their predictions for the $h(t)$, $L(t)$ and $L(h)$ data from the step-walled tank. Students then perform the experiment on the step-walled tank to confirm or reject their prediction. Figure 7 shows representative data from the step walled tank. The plots on the left of the time-varying data show a distinct kink at the time when the water depth crosses the change in tank diameter. Although the time varying data shows clear evidence of the influence of the step change in area, the $L(h)$ data is smooth.

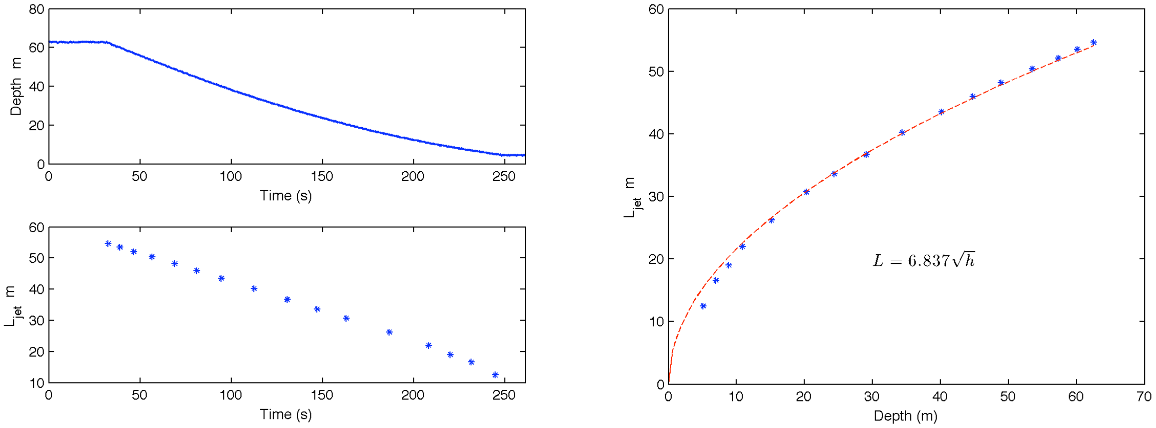


Figure 6 Data from draining of the straight-walled tank. On the left are the $h(t)$ and $L(t)$ curves (top and bottom, respectively). On the right is the combined $L(h)$ data and the curve fit of $L = c\sqrt{h}$ to that data.

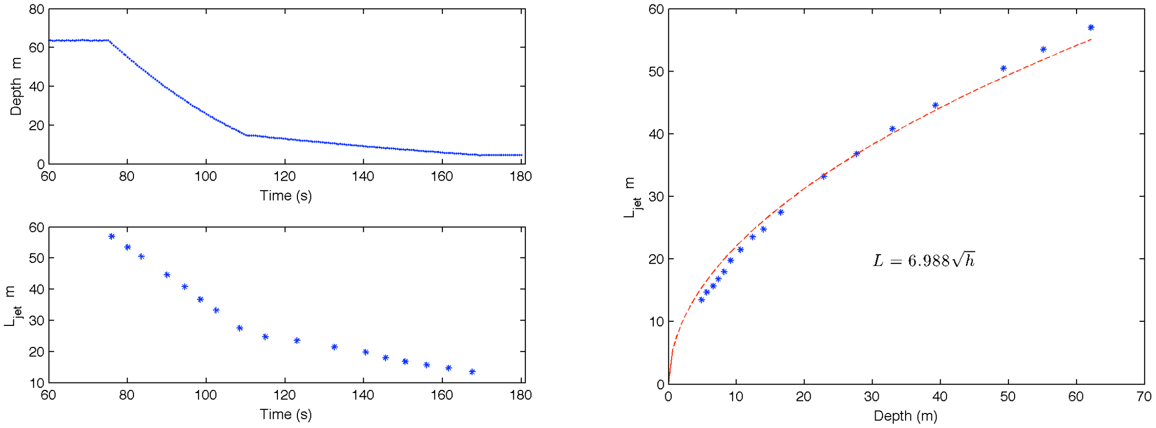


Figure 7 Data from draining of the step-walled tank. On the left are the $h(t)$ and $L(t)$ curves (top and bottom, respectively). On the right is the combined $L(h)$ data and the curve fit of $L = c\sqrt{h}$ to that data.

Having performed the experiment on the straight-walled tank and the step-walled tank, the natural question is how do these tanks compare. Figure 8 is a plot of the $L(h)$ data for the straight-walled tank and the step-walled tank when water is draining from the medium sized hole. The two $L(h)$ data sets are very close. The difference in the $L(h)$ data is due to manufacturing tolerances in the holes, which causes slight differences in the loss coefficients.

The most important information from Figures 6, 7 and 8 is that the $L(h)$ function is independent of the tank geometry. The guided inquiry worksheet begins with a theoretical analysis of the tank draining problem and a short derivation shows that $L = \sqrt{2Hh}$, where H is the height of the hole above the horizontal plane that defines L (see Figures 1 and 2). Students are given the worksheets in advance, and they are told to study the first few pages before coming to the lab. The significance of the simple $L = \sqrt{2Hh}$ is reinforced by the measurements.

Another natural question is how does the hole size affect the jet velocity and travel length. Figure 9 shows the measurements $L(h)$ on the step-walled tank for drain holes, 3.2 mm, 6.4 mm and 7.9 mm in diameter. The $L(h)$ curves for the medium and large holes are close to each other. The small hole is distinctively lower. The jet of water from the small hole is qualitatively different from the jet of water from the larger holes. The jet from the small hole is not round and has a wavy instability. We suspect that the dynamics of the jet after it has left the tank is unlikely to affect the distance traveled by the jet. Rather, we believe that the loss coefficient of the smaller hole is larger than the loss coefficient from the larger holes.

Conclusion

The tank draining apparatus is relatively inexpensive and its operation is easily comprehendible to students. The apparatus is used in two exercises: one focused simply on the hydrostatic equation, and one involving the draining experiment described in this paper. The apparatus also provides an opportunity to engage students in guided-inquiry pedagogy. The first exercise is simple and has the objective of introducing the students to inquiry-based laboratory work. The second exercise is more subtle. It requires students to transform measurements made in real time – $p(t)$ and $L(t)$ – into the more fundamental relationship $L = f(h)$.

Within a two-hour lab session, students can observe and characterize the behavior of the straight-walled tank. From those observations they can predict the behavior of the tank with a cross section that varies with depth. This exercise confirms the hydrostatic equation and extends it to the situation of measuring the depth in a slowly draining tank.

As with any teaching experience, many factors influence on the learning outcomes from the tank draining exercise. We have refined the apparatus and the worksheets over two years. The procedures are robust and can be used at other universities. We encourage instructors to visit eet.cecs.pdx.edu. There they will find the latest copy of the guided-inquiry worksheet, and more details on the experimental apparatus.

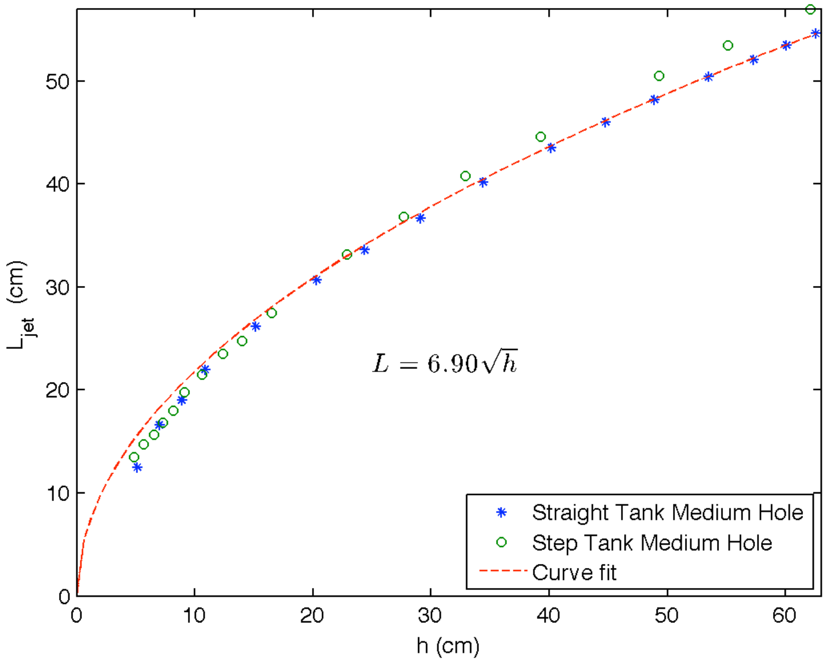


Figure 8 Combined $L(h)$ data for the straight-walled and step-walled tanks with medium-size drain holes.

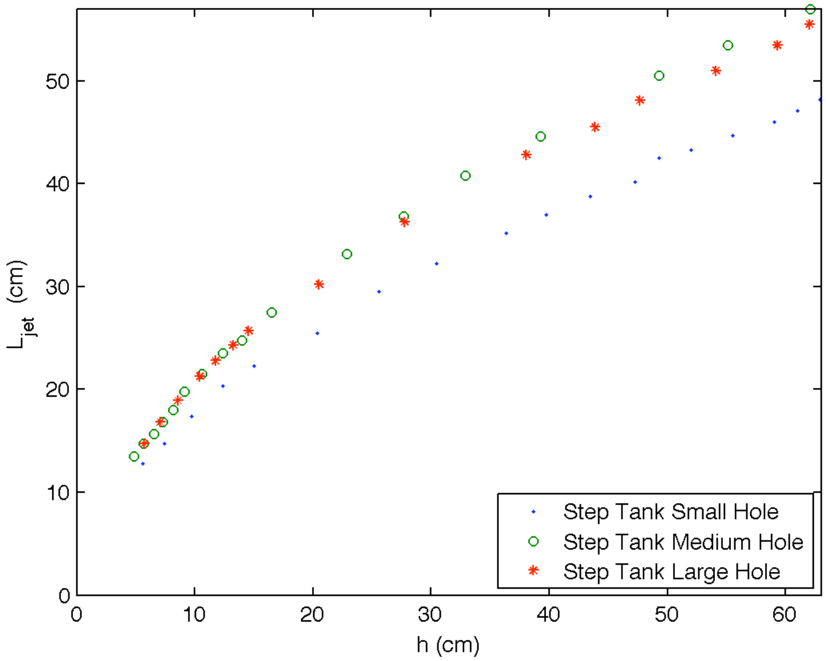


Figure 9 Effect of hole size on the $L(h)$ data for the step-walled tank.

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