AC 2012-5236: AN INTEGRATED MODELING APPROACH TO A SUMMER BRIDGE COURSE

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Current data on the participation of women and minorities in the STEM disciplines continues to show that women and minorities are underrepresented in nearly all fields of engineering at the undergraduate level.[1] Two decades of research on the experiences of undergraduate women and underrepresented minorities in their engineering programs has pointed to a number of factors that contribute to the difficulties in recruiting and retaining talented students.[2, 3, 4, 5] No single factor or simple explanation accounts for the continued underrepresentation of women and minorities in engineering. Rather, this underrepresentation results from the structure of the educational experience and a complex set of interacting factors within that system. A number of studies point to two key factors that are particularly important to retaining students: intellectual engagement with the discipline and social support leading to connections with peers, faculty and the engineering profession.[6, 7] These findings have led a number of institutions to design summer programs that engage students with academic content and build connections to the social setting of the university.

Research on the design and efficacy of summer programs is sparse, and many such programs are focused on improving students' first mathematics course placement.[8, 9] In this paper, we describe findings from our research on the effectiveness of a course (offered as part of a summer bridge program) that was designed to improve engineering students' success rate in their first semester mathematics course. Unlike other summer mathematics course offerings, we did not seek primarily to remediate the weaknesses of students' mathematical preparation that have accumulated over their K-12 schooling. Rather, our goal was to prepare students for success in their first mathematics course, whether that placement was pre-calculus or calculus. To accomplish this goal, we designed a summer course to engage students in challenging mathematics through hands-on modeling projects using collaborative group learning. The following section of this paper describes the design of the course and its implementation. The next section describes results of the students' performance both within the course and in their subsequent courses in mathematics. We then conclude with comments on lessons learned from the first implementation of the course, the continuing re-design of the course and recommendations for future efforts.

Course Design

The overarching objective of this course was to prepare students for subsequent success in pre-calculus and calculus. To accomplish this, we designed the course around four inter-related goals. The first goal was to develop the students' understanding of a topic of study that brings together engineering and mathematics, namely, quantifying and interpreting change. The second goal of this course was for students to develop their problem solving skills. We wanted students to improve their abilities to interpret problem situations and to persist in solving such problems and to do so more independently than in high school. Along with this, our third goal was to develop students' communication skills and abilities to work in a collaborative group. We wanted students to gain experiences and improve in their abilities to read and write about mathematical problems and their solutions, while collaborating with their peers. Finally, a fourth aim of this course was for students to develop and enhance the algebra skills necessary to succeed in this course and in their next math course.

To accomplish this ambitious set of goals, we designed the course around a sequence of modeling activities that would engage students in solving problems, working in small groups, and communicating their thinking throughout the modeling sequence. The central mathematical idea around which this course was organized is a deep understanding of average rate of change through analyzing and interpreting the behavior of linear and non-linear phenomena and how these phenomena change with respect to time. We implemented tasks that were designed to help students understand and apply the concept of average rate of change by engaging them in creating and interpreting models of physical phenomena that change. These tasks included (1) working with motion detectors to analyze linear and quadratic motion and related rates of change; (2) working with computer simulations to interpret velocity and position graphs; (3) using light sensors to model the intensity of light with respect to the distance from the light source and to analyze the rate at which the intensity changes at varying distances from the light source; and (4) building a simple RC circuit to charge a capacitor and then creating a mathematical model that can be used to analyze the change in voltage across the capacitor as it discharges.

Theoretical Background

Modeling approaches to the teaching and learning of science, mathematics and engineering encompass a wide range of theoretical and pragmatic perspectives.[10, 11, 12] Modeling approaches based in the "contextual modelling" perspective draws on the design of activities that motivate students to develop the mathematics needed to make sense of meaningful situations.[10] Much work done within this perspective draws on model eliciting activities (MEAs) developed by Lesh and colleagues and recently applied to engineering education.[11, 13, 14, 15] Model eliciting activities confront the student with the need to develop a model that can be used to describe, explain or predict the behavior of a realistic problem situation. Such MEAs encourage teams of students to engage in an iterative process where they express, test, and refine their own ways of thinking about meaningful problem situations. Solutions to MEAs go beyond what is required of ordinary textbook problems in that the solutions generally involve creating a process or procedure that can be sharable with others and re-used in similar situations. Furthermore, MEAs are designed to elicit a generalizable model that reveals the underlying mathematical structure of the problem situation so that the model can then be applied in a range of contexts.

To date, much of the research on MEAs in engineering education has focused on the effectiveness of a single such task in improving student outcomes in mechanics, dynamics, and thermal science, and in meeting ABET professional standards.[16, 17, 18, 14] However, a single MEA in isolation is seldom enough for a student to develop a generalized model that can be used and re-used in a range of contexts.[13] To achieve this goal, students need multiple opportunities to explore the relevant mathematical constructs and to apply their model in new settings. Sequences of structurally related model exploration activities and model application activities are needed, accompanied by discussions and presentations that focus on the underlying structure of the model and on the strengths of various representations and ways of using them productively. Each stage of this sequence engages students in multiple cycles of descriptions, interpretations, conjectures and explanations that are iteratively refined while interacting with other students. Thus, in this research study, we have designed a model development sequence (described in more detail in the next section) that begins by engaging students with meaningful problem situations that elicit the development of significant mathematical constructs. Students then explore and apply those constructs in other situations leading to the development of a model that is usable in a wide range of contexts. The central mathematical construct around which the instructional sequence was organized is the understanding of average rate of change through interpreting, analyzing and predicting the behavior of linear and non-linear phenomena as they change over time.

The Model Development Sequence

Model development sequences are structurally related tasks, beginning with a model eliciting activity (MEA) and followed by model *exploration* activities and model *application* activities. These tasks are not step-by-step procedures (as often found in laboratory projects), but rather are open-ended tasks that encourage students to express their own ideas about a situation and then explore and apply those ideas in other contexts. These tasks are accompanied by instructor-led discussions, student presentations, and summaries so as to focus attention on the structural similarities among the tasks and on the use of representations across the tasks. In this course, these tasks were investigations of:

- linear and non-linear motion with a motion detector
- linear and non-linear motion with a simulation environment and web applets
- non-linear change with a light sensor
- non-linear charging and discharging of a capacitor in an RC circuit

We began the sequence with a model eliciting activity, using constant and non-constant velocity in moving along a straight path. Students were asked to create graphs using a motion detector and their own bodily motion and to generate descriptions of that motion. The graphs included comparative situations of faster and slower constant speed, changing speed and changing direction, and graphs where the motion was not physically possible.

Following the model eliciting activity, the students engaged in several model exploration activities. An important goal of these activities was to engage students in using everyday language to make sense of the average rate of change in two different contexts and to develop their understanding of the representations for describing change. These activities were designed to help students to think about the underlying structure of the system (or model) and the representation of that structure. The work of Hestenes and colleagues has emphasized the need for instruction to focus on the structure of a system (that is, the set of relationships among the objects in a system) and the representation of that structure.[12] The first model exploration activity used a computer simulation environment called SimCalc Mathworlds.[19] This environment reversed the representational space of the model eliciting activity where bodily motion produced a position graph and extended that space as the students created velocity graphs that produced the motion of a simulated character. From this motion, the students created position graphs, thus developing an understanding of how the position graph could be constructed by calculating the area between the velocity graph and the *x*-axis. In exploring this linked relationship between the velocity and position graphs, students began to reason about the position of characters solely from information about the velocity of the characters. For example, in one task, the students were given written descriptions of the motion of two characters and asked to create appropriate velocity graphs, such as that shown in Figure 1.

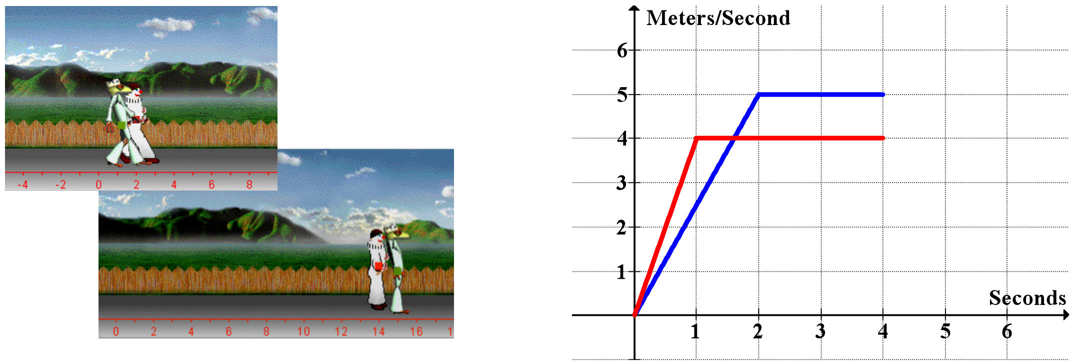

Figure 1: Interpreting Position from a Description of Velocity.

The students had to determine which character had walked farther and who was walking faster at various points in time. The students had to justify whether or not the two characters would ever be at the same position at the same time. This model exploration activity provided an opportunity for students to develop their abilities to interpret position information from a velocity graph and velocity information from a position graph.

The second model exploration activity used the Gym applet from the interactive mathematics textbook by Yerushalmy.[20] This applet (http://www.cet.ac.il/math/function/english/line/rate /rate10.htm) was designed to help students understand how the rate of change is expressed in table values, graphs, and equations. Using the context of training plans on a weight-lifting machine in a gym, the students explored the difference between constant and non-constant rates of change. Specifically, they investigated the graphical and numerical representations of weight training plans where the weight lifted increased or decreased at a constant rate, at an increasing rate or at a decreasing rate. The students were asked to explore the applet in order to make generalized observations about how setting the initial weight, the first change in weight and the change of the change affected the shape of the corresponding graph and the table of values.

The third component of the model development sequence consisted of two model application activities that focused on applying their model to new problem situations. This was intended to lead to a generalized understanding of average rate of change. In the first model application task, students were asked to investigate the relationship between the intensity of light and the distance from a light source. They used a point source of light, a light intensity probe and their graphing calculators to collect data. They used this data to analyze the average rates of change of the intensity at varying distances from the light source and to describe the change in the average rates of change as the distance from the light source increased. The second model application task was an investigation of the rate at which a fully charged capacitor in a simple RC circuit discharged with respect to time. The students built the circuits, charged the capacitor, and then, using their graphing calculator and a voltage probe, measured the voltage drop across the capacitor as it discharged. Students were given a set of resistors and capacitors and were asked to develop a model they could use to answer these three questions:

(1) How does increasing the resistance affect the rate at which a capacitor discharges?

(2) Compare the rates at which the capacitor is discharging at the beginning, middle and end of the total time interval. How does the average rate of change of the function change as time increases?

(3) How does increasing the capacitance affect the rate at which a capacitor discharges?

Taken together, these two model application tasks focused the students' attention simultaneously on the quantity that was measured and on how that quantity was changing with respect to some other quantity (i.e., distance or time). A coordinated understanding of these two measurements is at the crux of representing and reasoning about changing phenomena.[21]

Since a primary goal of this course was to prepare students for subsequent success in their first year mathematics courses, we also wanted to develop students' algebra skills. To do this, we explicitly focused on three topics that we have found to be common sources of student difficulties in our pre-calculus and calculus courses:

- rational expressions and complex fractions
- exponential expressions and equations
- logarithmic expressions and equations

These three topics were also directly related to the mathematical content in the model development sequence, thus providing students with the opportunity to use their algebra skills in a meaningful context. In addition, we provided skills practice through the use of an on-line homework system.

Rate of Change Concept Inventory

To measure students' understanding of average rate of change, we designed a "Rate of Change Concept Inventory" consisting of items in four categories: algebraic expressions, graphical interpretation, symbolic interpretation, and purely contextual. The first year the Rate of Change Concept Inventory contained 17 items (five algebraic, eight graphical, three symbolic, and one contextual). Fourteen of these items were drawn from the research literature on students' conception of rate of change. Three items were developed to test the students' mastery of the algebraic representations involved in expressing and computing average rates of change. In the second year, modifications were made to two of the items and four new items were added to the inventory (giving a total of seven algebraic, eight graphical, three symbolic, and two contextual items respectively). As a result of these modifications, there were 14 identical items over the two years we have used the inventory. The continuing development of this concept inventory is part of our on-going work.

Implementation

The six-week course took place over the past two years during a six-week residential program that provided these pre-freshmen with an opportunity to become familiar with the academic, social, and cultural life at the university. Over the last five years, the average enrollment in the program has been 28% women and 77% underrepresented minorities. We report the results here from two years of the program, during which there were a total of n = 50 students, with n = 33 in the first year and n = 17 in the second year. Fourteen of the participants were female and 36 were male. There was a wide range of student backgrounds and experiences in math and physics. One indicator of this range was the SAT math scores, which ranged from 390 to 790. A second indicator was the highest math course taken in high school, which ranged from not having a course in pre-calculus to a rigorous AP calculus course. All but one participant had completed four years of study of high school mathematics; 29 students had studied calculus in high school and 21 had not studied any calculus. Of the n = 29 students who had taken calculus in high school, n = 17 of these students were in an AP level calculus course. We asked the students how much homework they typically did each day in their high school mathematics class during their senior year. We were surprised to find that n = 9 reported doing 15 minutes or less per day! At the other end, n =10 reported doing an hour or more of homework per day. Clearly a significant number of these students are coming to college without the kinds of experiences that would prepare them to study and work outside of the classroom. Nearly all students had a high school course in physics.

The participants worked in groups of three or four to complete the model eliciting tasks and model application tasks. The model exploration tasks were done individually at a computer; however, the participants were encouraged to discuss their work with each other. Following each task in the sequence, there was a whole-class discussion which usually involved students in presenting the results of the work produced during the model eliciting and model application tasks. The class discussion following the model exploration tasks focused on the structural features of the model and on the relationships among different representational systems. The students worked in pairs to complete final reports on their findings for each of the model application tasks. All participants completed all of the tasks in the model development sequence described above. The written work from these collaborative tasks and from individual course examinations was collected and analyzed.

All participants completed the pre- and post-test of "Rate of Change Concept Inventory." The overall pre- and post-test scores for each year were normally distributed; the combined overall pre- and post-test scores (for the 14 identical items given in both years) were also normally distributed. Hence, these scores were analyzed using t-tests. The four sub-scores (algebraic expressions, graphical interpretation, symbolic interpretation, and contextual items) were not normally distributed and hence were analyzed using the non-parametric Wilcoxon Signed-Rank test.

Results

The results of this study showed a significant improvement in the students' understanding of average rate of change, as measured by the gain on the Rate of Change Concept Inventory. More importantly, these changes to the summer course closed the previous full letter grade gap between summer program students and their non-summer program peers in their first semester mathematics courses. We report each of these gains in the following sections.

Understanding of Average Rate of Change

The post-test results show that each year there was a significant improvement in the students' understanding of the concept of average rate of change. In the first year, the students' overall scores (n = 33) changed from 52% correct to 75% correct, as measured by the 17 items on the Rate of Change Concept Inventory. In the second year, the overall scores (n = 17) changed from 40% correct to 62% correct, as measured by the 20 items on the revised inventory. Finally, the overall performance on the 14 identical items both years on the pre- and post- test (n = 50) improved from 45% correct to 75% correct. There was a significant improvement in three of the sub-score areas: algebraic expressions, graphical interpretation, and symbolic interpretation, as shown in Table 1.

	Pre-test mean (sd), percent correct	**Post-test** mean (sd), percent correct	p-value
Overall (year 1, 17 items)*	8.87 (3.52), 52%	12.69 (84), 75%	<0.001
Overall (year 2, 20 items)*	8.06 (3.10), 40%	12.34 (3.16), 62%	<0.001
Overall (combined year 1 and year 2, 14 items)*	6.33 (2.76), 45%	9.82 (2.47), 70%	<0.001
Algebraic**	1.78 (1.18), 45%	2.94 (1.00), 74%	<0.001
Graphical**	3.37 (1.34), 56%	4.78 (1.31), 80%	<0.001
Symbolic**	0.86 (0.83), 29%	1.64 (0.93), 55%	<0.001
Contextual**	0.32 (0.47), 32%	0.46 (0.50), 50%	n.s.

*parametric t-test; **non-parametric Wilcoxon Signed Rank test

Table 1: Overall Pre-test and Post-test Results and Sub-scores.

While the overall combined scores improved by 25%, there were seven items on the concept inventory for which the improvement was greater than 30% for both years. Two of these were algebraic expression items, three were graphical interpretation items, and two were symbolic interpretation items.

The two algebraic expression items asked the student to find the equation of the line joining two points and to calculate the average rate of change between two points on a parabola. As shown in Table 2, there were substantial gains on two items that measured students' proficiency in being able to express algebraically two basic ideas about average rate of change. On the item that asked for the equation of a function with a constant rate of change n=25 (50%) students answered the item correctly on the pre-test and n=40 (80%) answered correctly on the post-test. Similarly, on the item that asked students to compute the average rate of change for a function with a non-constant rate of change the number correct went from n=14 (28%) on the pre-test to n=32 (64%) on the post-test.

Algebraic Items	Pre-Test		Post-Test			
	n	%	n	%	Δn	Δ%
Q1	25	50%	40	80%	15	30%
Q12	14	28%	32	64%	18	36%

Table 2: Improvement on Algebraic Items.

The symbolic interpretation items required the student to create appropriate symbolic expressions when given a problem context or to interpret the meaning of the parameters in symbolic expression. As shown in Table 3, there were substantial gains on two of these items. One of the symbolic items (Q9) asked the students to create an appropriate expression for the area of a circular ripple in terms of time when the outer edge of the ripple is moving at a constant rate. There was a substantial gain on this item of 32 percentage points from the pre-test (n = 8, 16% correct) to the post-test (n = 24, 48% correct). In the other symbolic item (Q18), the students had to interpret the meaning of the parameters in an exponential growth function: “The model that describes the number of bacteria in a culture after t days has just been updated from $P(t)=7(2)^t$ to $P(t)=7(3)^t$. What implications can you draw from this information?” There was a substantial gain on this question of 36 percentage points from the pre-test (n=19, 38% correct) to the post-test (n=37, 74% correct). This likely reflects the emphasis in the model development sequence on making meaningful interpretations of data and on giving descriptions of the average rate of change in various contexts, including the exponential change in the circuit task.

Symbolic Items	Pre-Test		Post-Test			
	n	%	n	%	Δn	Δ%
Q9	8	16%	24	48%	16	32%
Q18	19	38%	37	74%	18	36%

Table 3: Improvement on Symbolic Interpretation Items.

There were substantial gains on three items that measured students’ proficiency at interpreting rate of change when given graphical information. These gains are summarized in Table 4.

Graphical Items	Pre-Test		Post-Test			
	n	%	n	%	Δn	Δ%
Q5 C	28	56%	46	92%	18	36%
Q8 A and B	24	48%	43	86%	19	38%
	26	52%	44	88%	18	36%
Q10	7	14%	22	44%	15	30%

Table 4: Improvement on Graphical Interpretation Items.

Questions Q5 C, and Q8 addressed interpreting information about velocity when given a position graph. Question Q10, on the other hand, involved interpreting position information when given a velocity (or speed) graph. Students often confuse the interpretation of these graphs, experiencing difficulty in inferring velocity information from a position graph and mis-reading a velocity graph as if it were a position graph.[22] This coordination of representational systems (shifting between the velocity or rate graph and its associated position graph) was the main focus in the model exploration tasks described earlier. We will illustrate these results with Q5 C and Q10.

Figure 2: A Graphical Representation of a 20-meter Race between Two Runners.

Item Q5 C asked for a description of the runner whose graph is shown by the solid line in Figure 2 over the time interval from 7 seconds to 8 seconds. On the post-test, 92% of the participants were able to correctly describe the runner as moving back towards the starting position. In other words, the participants were able to correctly reason about the velocity (or average rate of change) over an interval when given a position graph.

Item Q10 represents an important reversal of the above problem and one that is a well-known source of difficulty for calculus students.[23] The item asks for the relationship between the position of Car A and Car B at $t = 1$ hour. To successfully answer this item requires an understanding of how to reason about position when given a velocity graph. We found a 30% improvement in the number of students who were able to correctly interpret the relative position of two cars, starting from the same position and travelling in the same direction, when given the speed graph shown in Figure 3.

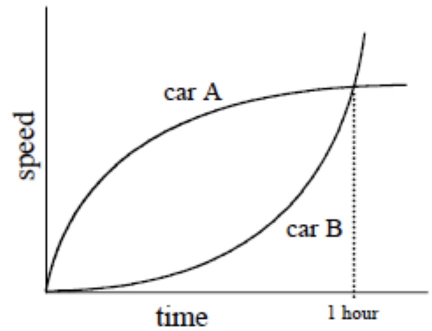

Figure 3: Interpreting the Relative Position of Two Cars Given their Speed.

The item asks for the relationship between the position of Car A and Car B at $t = 1$ hour. The substantial 30% gain on this question suggests that the model exploration tasks within the model development sequence helped the students understand how to reason graphically about rates of change.

First Year Mathematics Course Grades

Upon completion of the summer bridge program, most students were placed into a course in pre-calculus or a first course in calculus taken with other engineering students. Those students who earned a grade of 4 or 5 on the AP Calculus examination were placed into a second course in calculus. In the two years prior to this study, we found that the students who were in our summer bridge program performed about a letter grade below their peers who were not in the summer program. As shown in Table 4, for the three years prior to the implementation of this course, regardless of whether the students were placed into pre-calculus, calculus I or calculus II, their achievement in their first year lagged about a letter below their peers. For comparison purposes, we selected at random a group of students who matched the summer students in terms of gender, ethnicity and, within those categories, as close as possible SAT mathematics score.

Year	First math course	Number of summer students	Mean Fall math grade Summer students	Mean SAT math Summer students	Mean Fall math grade non-summer students	Mean SAT math non-summer students
2007	Pre-calculus	17	1.2 (D)	492	2.3 (C)	531
	Calculus I	26	2.0 (C)	558	2.5 (C+)	595
	Calculus II	7	1.6 (D)	654	2.6 (C+)	652
2008	Pre-calculus	14	1.7 (C-)	495	2.4 (C+)	535
	Calculus I	17	0.8 (F)	571	2.0 (C)	586
	Calculus II	11	2.3 (C)	676	2.7 (B-)	669
2009	Pre-calculus	22	1.8 (C-)	530	2.5 (C+)	532
	Calculus I	12	2.1 (C)	625	2.2 (C)	624
	Calculus II	4	1.6 (D)	575	2.2 (C)	630

2010	Pre-calculus	12	2.1 (C)	539	2.1 (C)	507
	Calculus I	21	2.1 (C)	585	2.1 (C)	610
	Calculus II	5	3.1 (B)	722	3.0 (B)	688

Table 5: Grades in First Mathematics Course.

The integrated modeling course closed achievement gap between those who participated in the summer bridge program and their peers who did not participate. We are cautiously optimistic about these results. In part, these results may suggest that we have succeeded in recruiting into our summer program those students who are most at risk for success in engineering and that the re-designed summer course contributed to closing the gap between those who are at risk and their peers. However, these results also suggest that the summer program does not yet go far enough in supporting those students in achieving at a high level.

Lessons Learned and Continuing Re-design

We begin this section by reporting our observations from the first year implementation related to our course goals for problem solving, collaboration, and communication. These observations influenced the re-design of the course for the second year.

The students were overall very enthusiastic about the modeling tasks, especially the model eliciting task with the motion detectors and the model application task with the circuits. All of the tasks had high levels of student engagement. While nearly all students had some basic exposure to circuits in high school physics, hardly any of them had any actual experience building or measuring real circuits. They were all able to use breadboards to build their circuits and complete some basic analyses of voltage changes across a capacitor in a simple RC circuit. Since all of the students had taken a course in high school physics, they all came with varying degrees of misconceptions about velocity, position, and speed. We note that students' difficulties with these concepts are well-known from research in physics education.[24, 25, 26] We under-estimated the degree to which these misconceptions were present. For some students these difficulties persisted through to the end of the course. We directly addressed this in the course re-design by giving more attention in the model exploration activities to:

- the distinction between speed and velocity;
- an analysis of negative velocity;
- changing velocity and acceleration
- interpreting the relationship between changing velocity and position.

This will potentially have a positive impact when students study physics in their second semester.

We saw good (and in some cases outstanding) improvements in communication. It appeared that the students had limited experiences in being asked to explain their thinking or justify their reasoning or listen to other students' arguments. However, across the board, we saw improvements in students' abilities to (a) express themselves in writing (in lab reports and in homework assignments) and (b) express themselves in making arguments in class and in responding to other students' arguments.

We saw varying degrees of success in students' collaboration with each other. It appeared that many of the students had limited collaborative experiences in high school. Some pairs and groups of students appeared to steadily improve in their abilities to work productively with each other. Unfortunately, some pairs and groups had less success in collaborating. Since the modeling tasks (as described earlier) are designed to engage students in expressing, testing, and revising their ideas, we found that we needed to more directly address the expectation for collaborative work among the students. We attributed some of the difficulty in this area to a "high school mind set" with the kind of tasks where students wait for their instructors to give them the correct answer. Modeling tasks move beyond this approach to problem solving. Thus, in the re-design of the course, we made clearer the expectation that when members of a group have different solutions to a task that they need to make a sustained effort to meaningfully resolve the discrepancies.

We were surprised at the limits of the technology skills of most of the students. We had assumed that the students were skilled in using their graphing calculators. As it turns out, many students had not used their graphing calculator to do anything much more than calculations and simple graphing. Many students had limited experience in using Excel for graphing and data analysis. This turned out to be problematic in the modeling tasks. We were further surprised at the difficulties that they encountered in learning new technologies. As engineering students and students who have grown up with technology, we had erroneously assumed that they would have an enthusiastic "dig into it" approach to technology. For example, we had erroneously assumed that they would download and learn to use the graphing software we had recommended for their reports; many did not. When students were not able to create data graphs or access data on their graphing calculator, we did not have the supports in place for them to learn how to do this. We addressed this in two ways in the re-design of the course. First, from the beginning of the course, we asked students to evaluate their own expertise with the graphing calculator. We used this self-evaluation to group students together and made explicit the expectation that everyone in the group needs the technology expertise. Second, we created a summary sheet of basic technology instructions so that those students who did not have prior experience with technology would have easy access to the information needed for the modeling tasks. In the next implementation of the course, we anticipate having two or three optional work sessions where students can learn basic spreadsheet skills in a just-in-time mode, where these skills will be directly useful in the model application tasks.

Conclusions

The results of this study provide some evidence that the model development sequence had a positive impact on the students' understanding of average rate of change, as measured by the statistically significant overall gain on the "Rate of Change Concept Inventory" and a positive impact on their subsequent performance in their first year mathematics course. The gains on the graphical items on the concept inventory may be due to the model exploration tasks that focused on the coordination between representational systems. This coordination included shifting between position and velocity graphs, thus distinguishing between the function's graph and the graph of its rate of change, and shifting between numerical and graphical data, thus linking the value of the average rate of change with the graph of the function. Building students' foundational understandings of functions and their rates of change, and applying these understandings in meaningful contexts, while at the same time developing their basic algebra skills, appears to have contributed to their subsequent success in their first college mathematics course.

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