ASEE AMERICAN SOCIETY FOR ENGINEERING EDUCATION

Paper ID #35992

Analysis of Beams on Elastic Foundations by NASTRAN/PATRAN Finite Element software

Dr. Cyrus K Hagigat, The University of Toledo

Dr. Hagigat is an associate professor in the Engineering Technology department of the College of Engineering of the University of Toledo. Dr. Hagigat has an extensive industrial background, and his teaching technique is based on practical aspects of engineering.

Analysis of Beams on Elastic Foundations by NASTRAN/PATRAN Finite Element software

I: Abstract

NASTRAN/PATRAN is one of the more commonly used Finite Element technique software packages which can be used to obtain solutions for non-standard loading scenarios.

One of the non-standard loading scenarios is "Beams on Elastic Foundations." The Elastic foundation can be continuous or applied at certain point(s) along the beam. Closed form solutions for Static Stress analysis for such structures are available for "Beams on Elastic Foundations" with various boundary conditions in classical textbook such as "Roark's Formulas for Stress & Stain." However, the formulas are tedious to use and apply to limited loading conditions.

In this article, NASTRAN/PATRAN is used to perform Static analysis of beams on elastic foundations subjected to a specific end of beam loading condition and the potential of expanding the technique to other loading scenarios is discussed. The techniques illustrated in this article were used in an "Advanced Mechanics of Materials" course. The study involved developing models for continuing bases and then adjusting the techniques to a number of elastic supports. The solution of the beams on multiple elastic foundations by Finite Element technique and the unique requirements of the finite element technique are then discussed.

Use of Finite Element technique to expand the types of loading scenarios beyond the elasticity based techniques are discussed.

II: Introduction

Classical solutions on beams on continuous foundations such as clay or sand are well developed using the theory of elasticity. These solutions are modified in Roark's formulas for stress and strain to include much expanded loading and boundary conditions.

Roark's formulas are tedious to use and are limited to the scenarios presented. Finite Element techniques can be used to replace the Roark's formulas and to also expand the loading and boundary conditions. However, unique issues arise in finite element modeling. This article discusses the unique issues of using springs in a finite element model and discusses a unique technique in addressing the issue. The techniques discussed in this article are used in a "Advanced Mechanics of Materials" course by the author.

III: Description of classic formulation of stresses of beams on elastic foundation and adjusting the formulation for spring supports:

When a beam is placed on an elastic foundation such as sand or clay and transverse load(s) are applied to it, the foundation develops continuous reactions that are proportional at each position along the beam to the deflection of the beam [1]. Figure 1 illustrates this concept where an infinitely long beam on a continuous elastic foundation is subjected to a concentrated force P.

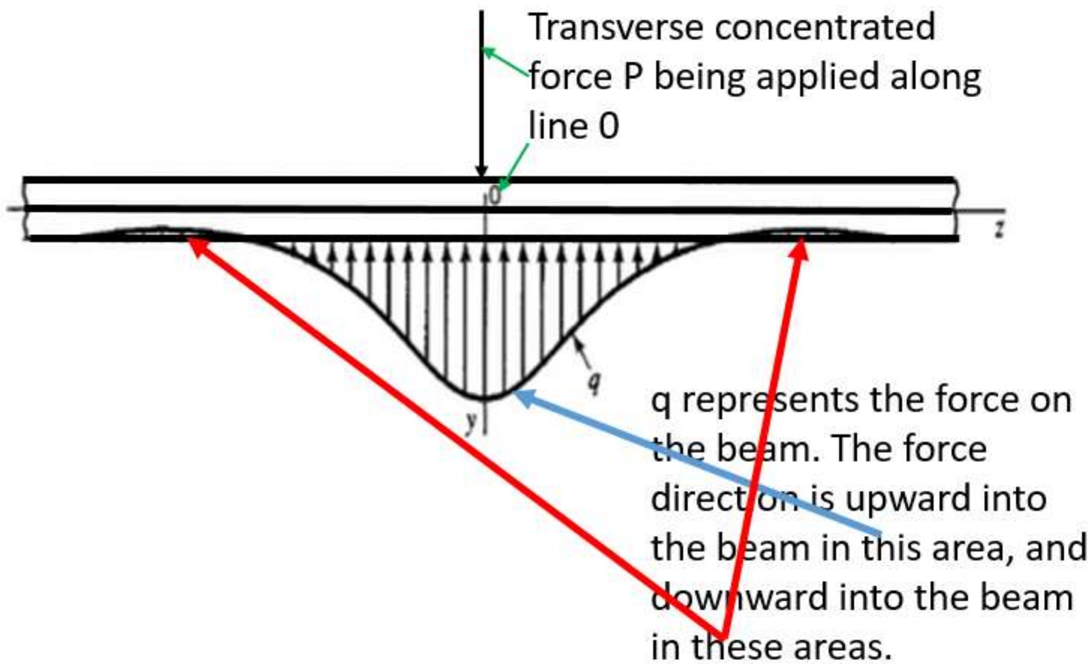


Figure 1: Schematic diagram of reaction forces from a continuous foundation on an infinitely long beam

Figure 2 is the illustration that the reaction force on the beam is a function of beam displacement.

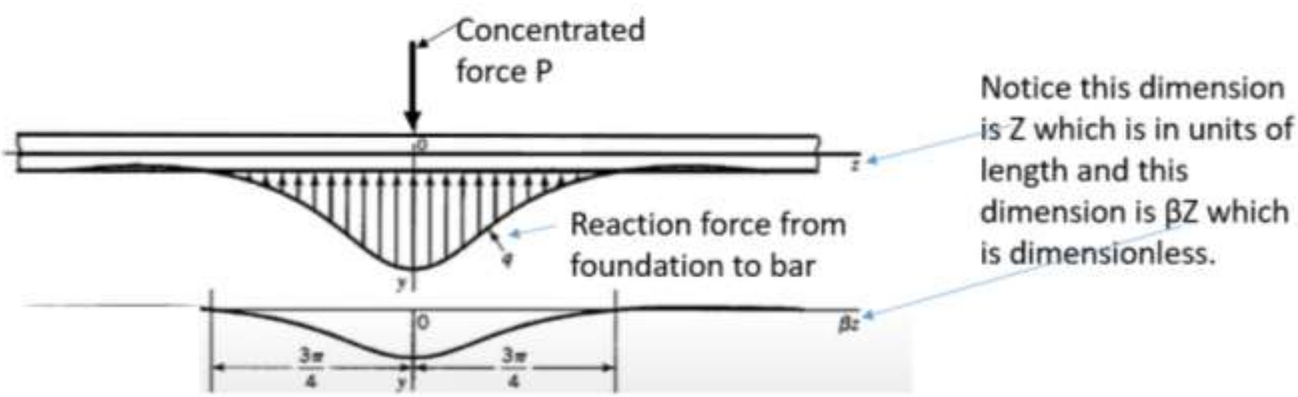


Figure 2: Comparison of reaction forces on the beam as a function of foundation reaction caused by beam displacement
(β will be defined in a later section of this article)

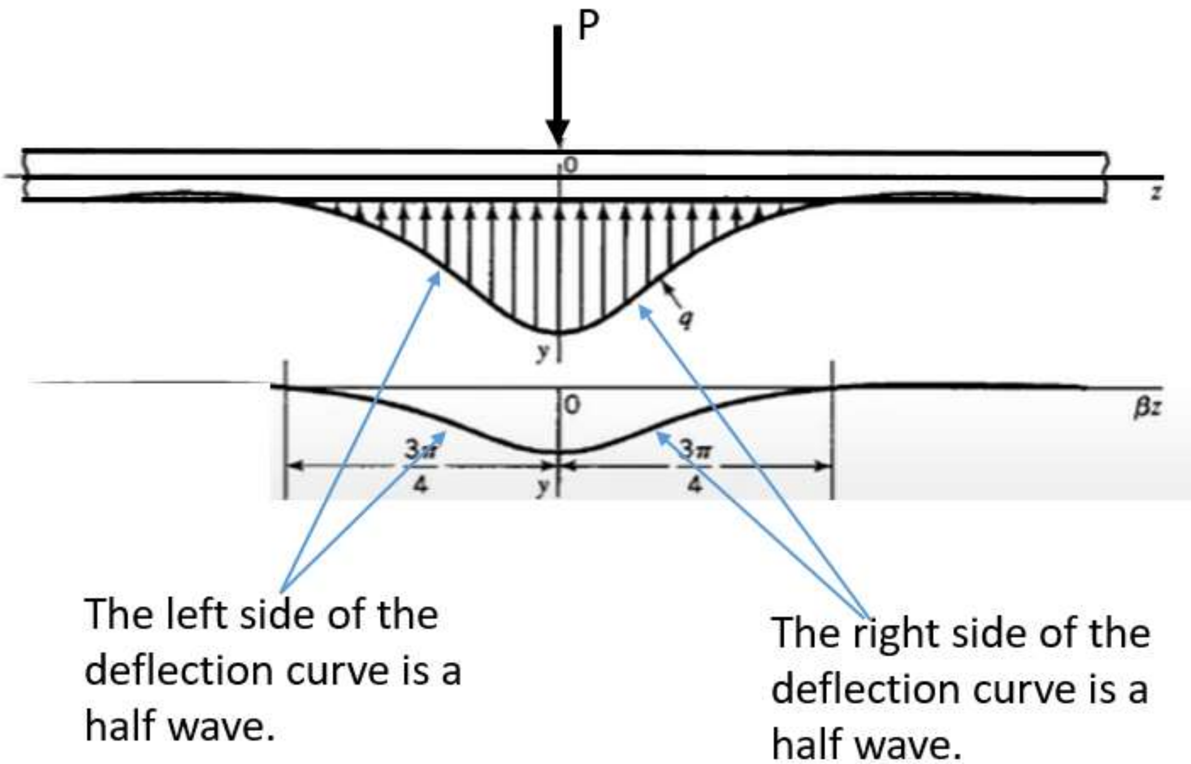


Figure 3: Illustration of the concept of half-wave

In this type of analysis, a variable k_0 is used. k_0 is a function of the foundation material. Units for k_0 are "force/ length^3". The stiffness of the foundation "k" is defined by "b.k_0" where b is the width of the beam and its units are "length". Therefore, the units of k are "force/length^2". β is defined by equation (1).

$$\beta = \sqrt[4]{K/4EI} \qquad \text{equation (1)}$$

In equation (1), E is the modulus of elasticity of the beam material in units of "force/ length^2", and I is the area moment of inertia of the beam and its units are "length^4". Figure 4 shows the axis about which "I" is calculated. An examination of equation (1) and the units for each component shows that the unit for β is "1/length". Since the unit for the beam dimension is "length", "βz" is dimensionless as indicated on figure 3. The I in equation (1) is calculated about the axis shown in figure 4.

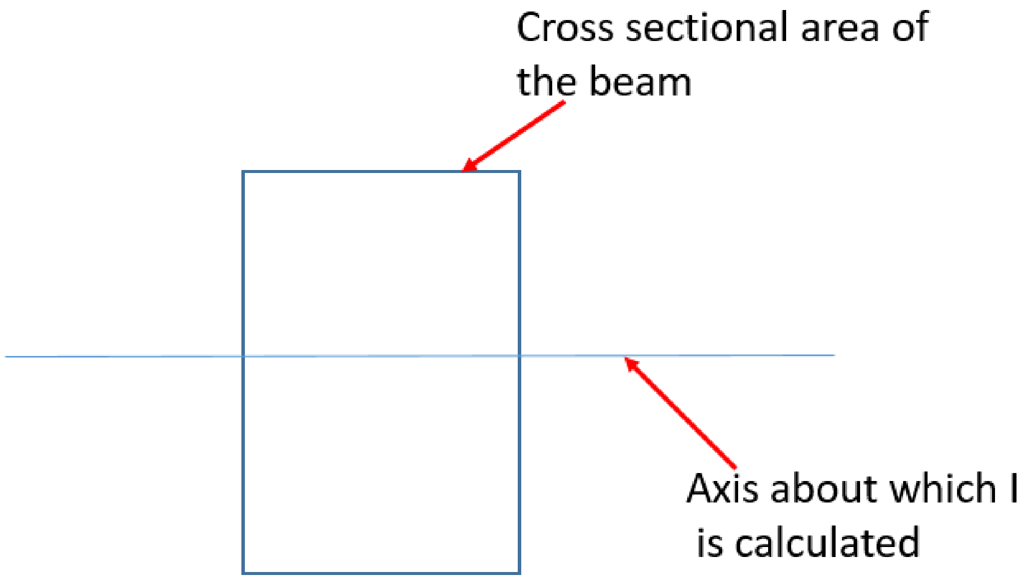


Figure 4: Illustration of axis for calculation of beam
moment of inertia

The analysis of a beam on a continuous elastic foundation can be used to develop formulas for a beam placed on springs as shown in figure 5 [1]. Figure 5 and the subsequent theoretical formulas for analyzing it assume the spring distances from one another are all the same.

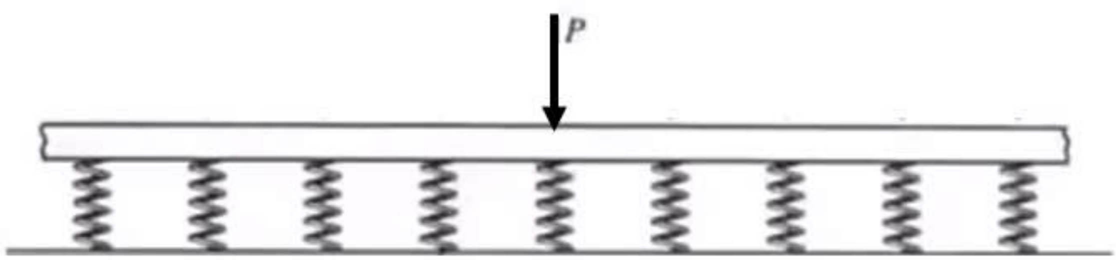


Figure 5: A beam supported by elastic springs

If a discontinuous foundation has at least three concentrated reaction forces in every half-wavelength of the deflected beam, then the solution for a continuous foundation can be modified and provide the solution for a beam with elastic discontinuous supports as shown in figure 5 [2].

In the case of elastic concentrated supports of figure (5), equation (1) is re-written as equation (2) [2]. b in equation (2) is as shown in figure 6. Figure 6 shows that b is the length that one spring acts on.

$\beta = \sqrt[4]{b.K0/4EI}$ equation (2)

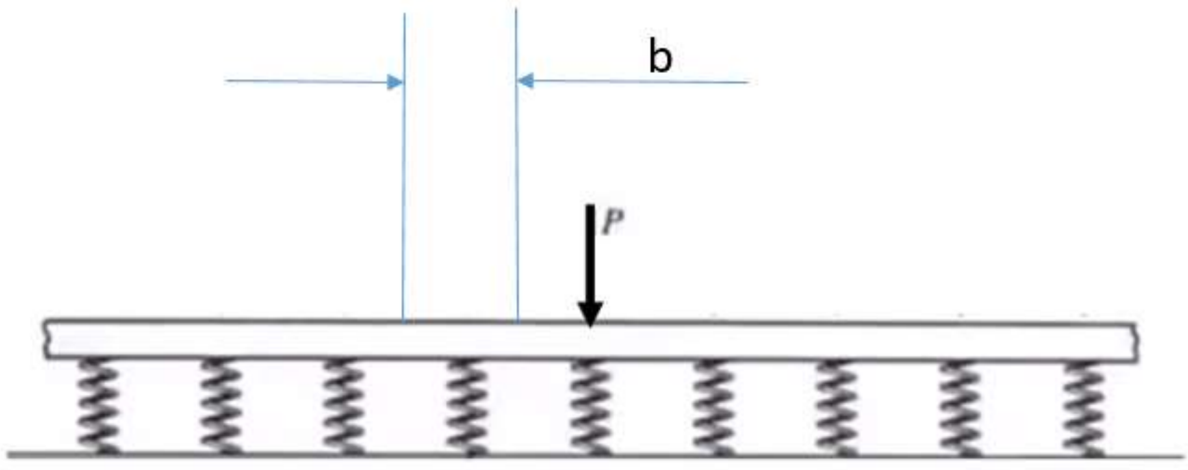
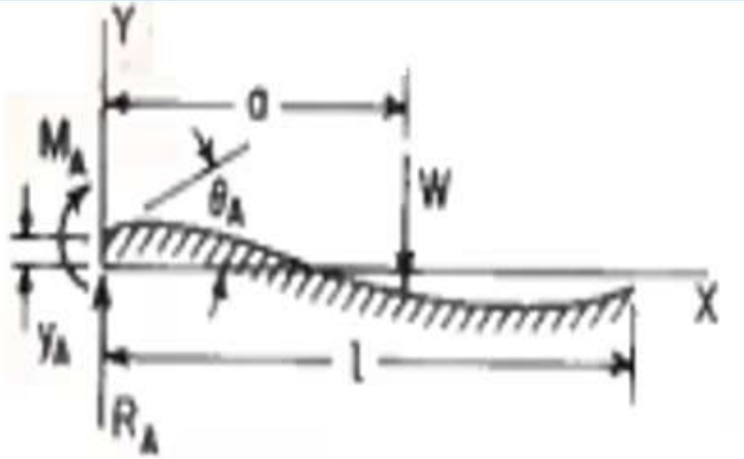


Figure 6: Defining b of equation (2)

Equation (2) applies if the springs are equally spaced, and if in a span "L" of the beam there are at least 3 concentrated supports (springs in figure 5) where "L" is defined by "Π/β", and where β is defined by equation (2).

In reference [2], there are 2 sets of tables related to a beam supported by equally spaced elastic supports. One of the tables is for "finite-length" beams (table 7 on page 140 of reference [2]). The condition for a "finite-length" beam is that "βL" is less than 6. The other is table 8 on page 148 of reference [2]. Table 8 is for "semi-finite" beams. The condition for a "semi-finite" beam is that "βL" is greater than 6. Figures 7 and 8 are partial screen shots of tables 7 and 8 of reference [2].



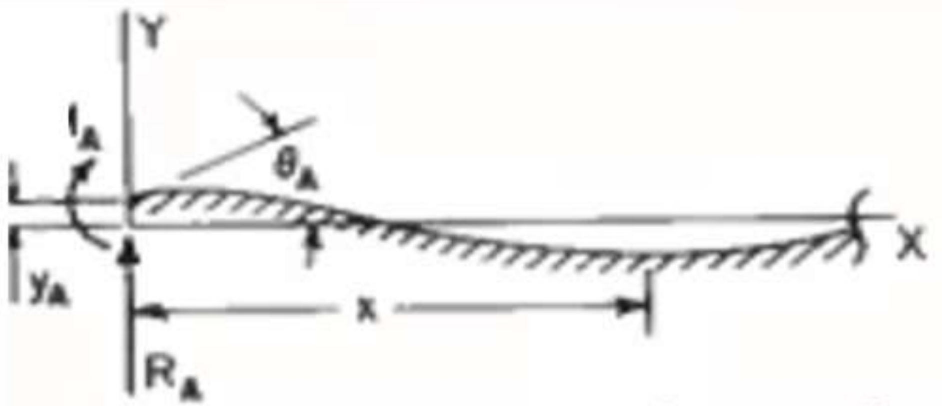
$$\text{Transverse shear} = V = R_A F_1 - y_A 2EI\beta^3 F_2 - \theta_A 2EI\beta^2 F_3 - M_A \beta F_4 - W F_{a1}$$

$$\text{Bending moment} = M = M_A F_1 + \frac{R_A}{2\beta} F_2 - y_A 2EI\beta^2 F_3 - \theta_A EI\beta F_4 - \frac{W}{2\beta} F_{a2}$$

$$\text{Slope} = \theta = \theta_A F_1 + \frac{M_A}{2EI\beta} F_2 + \frac{R_A}{2EI\beta^2} F_3 - y_A \beta F_4 - \frac{W}{2EI\beta^2} F_{a3}$$

$$\text{Deflection} = y = y_A F_1 + \frac{\theta_A}{2\beta} F_2 + \frac{M_A}{2EI\beta^2} F_3 + \frac{R_A}{4EI\beta^3} F_4 - \frac{W}{4EI\beta^3} F_{a4}$$

Figure 7: screen shot of table for analysis of finite-length beams on elastic foundation



Transverse shear $= V = R_A F_1 - y_A 2EI\beta^3 F_2 - \theta_A 2EI\beta^2 F_3 - M_A \beta F_4 + LT_V$

Bending moment $= M = M_A F_1 + \frac{R_A}{2\beta} F_2 - y_A 2EI\beta^2 F_3 - \theta_A EI\beta F_4 + LT_M$

Slope $= \theta = \theta_A F_1 + \frac{M_A}{2EI\beta} F_2 + \frac{R_A}{2EI\beta^2} F_3 - y_A \beta F_4 + LT_\theta$

Deflection $= y = y_A F_1 + \frac{\theta_A}{2\beta} F_2 + \frac{M_A}{2EI\beta^2} F_3 + \frac{R_A}{4EI\beta^3} F_4 + LT_y$

Figure 8: screen shot of table for analysis of
Semi-infinite beams on elastic foundation

As it is shown in figures 7 and 8, the Roark's formulas are long and tedious to implement. EXCEL spread sheets can be used to automate Roark's formulas.

Examples 1 and 2 describe the use of the formulas in figures 7 and 8.

Example 1:

An I-beam that is 20 ft long is used as a rail for an overhead crane. The rail is being supported every 2 ft of its length by being bolted to the bottom of a lateral I-beams at mid-length. The supporting beams are simply supported at their ends. A 2000 lb weight is applied (being supported) to one end of the structure. Figure 9 illustrates the structure and its supports. Figure 10 is a 3 dimensional view of the structure showing the applied load.

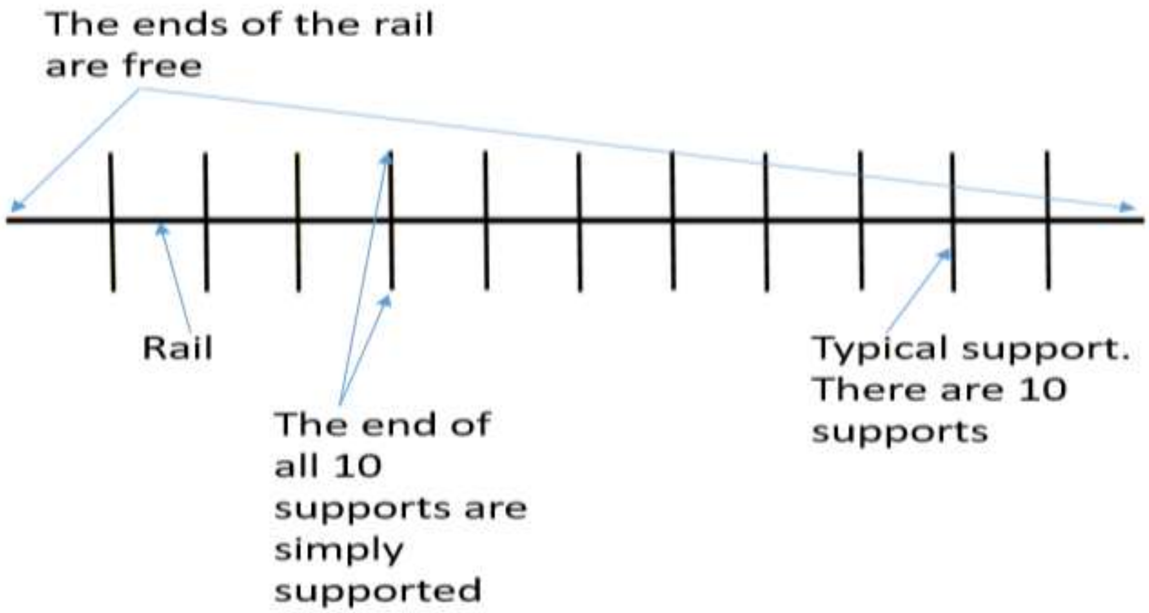


Figure 9: Top view of the rail and its supports
(drawing not to scale)

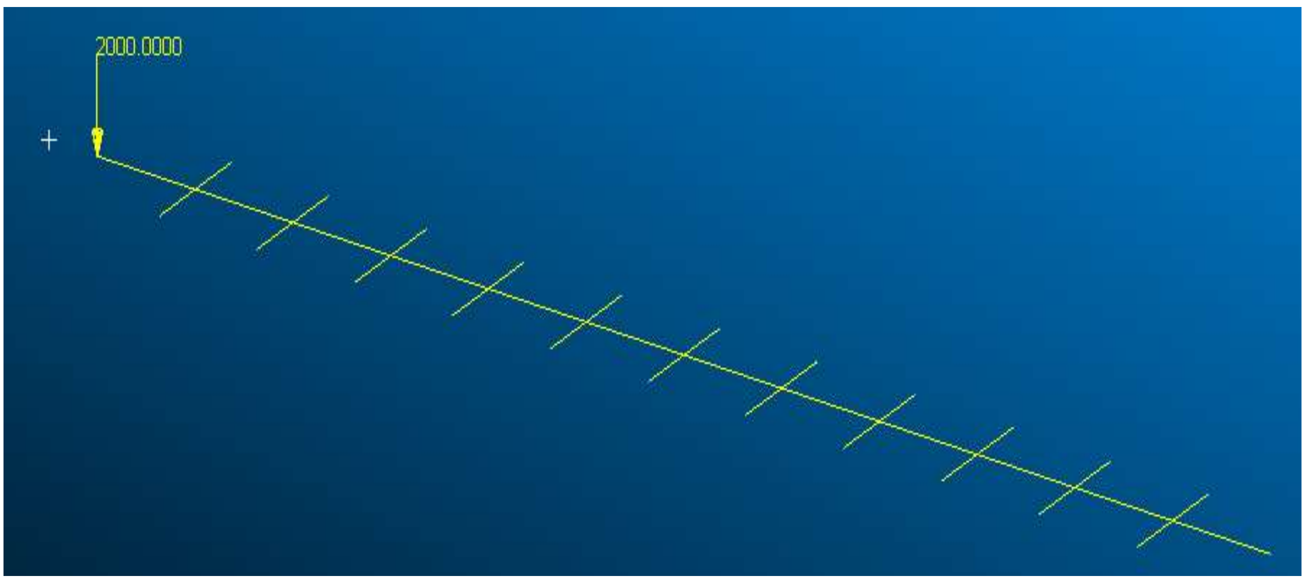


Figure 10: 3 dimensional sketch of the rail and the location of the applied load
(drawing not to scale)

The rail is being analyzed using Roark's table shown in figure 7.

Figure 11 shows the displacement for a beam simply supported at its ends and loaded at its center [3].

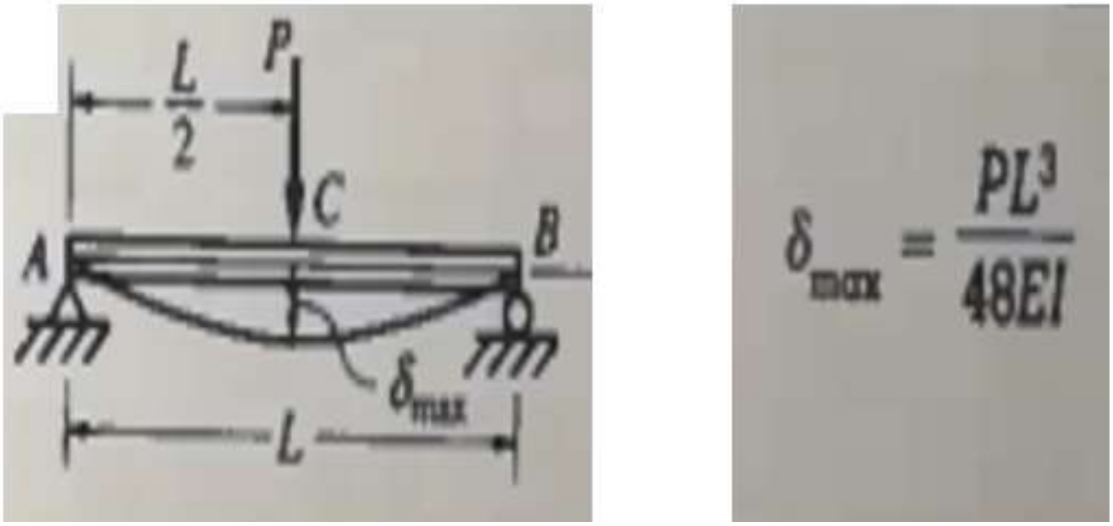


Figure 11: Displacement formula for a simply supported beam
loaded with a concentrated load in the middle of the beam span

Using the formula shown in figure 11, it is determined that the equivalent spring stiffness of a simply supported beam loaded at it center is defined by equation (3). Equation (3) can be used to calculate the equivalent spring stiffness for the supporting rails of figure 10.

Equivalent spring stiffness of the beam of figure 11= $48EI/L^3$ equation (3)

If the supporting beams of figure 10 are replaced by their equivalent springs as defined by equation (3), then the model becomes similar to the model of figure 6.

If spring constant for each supporting beam is 1013 lb/in, and it is assumed that the load is distributed over a 2 ft length (distance between supports), and using equation (2), a value 0.01127 $in^{-1}$for β is calculated. Since the distance between supports is 240 inches (2 ft), βl = (0.01127)(240) = 2.70. This value of βl is lower than 6 and therefore table of figure 7 should be used. Using table 7, a value for bending moment along the entire rail can be calculated. The maximum bending moment can be used to calculate the stress in the rail.

Example 2:

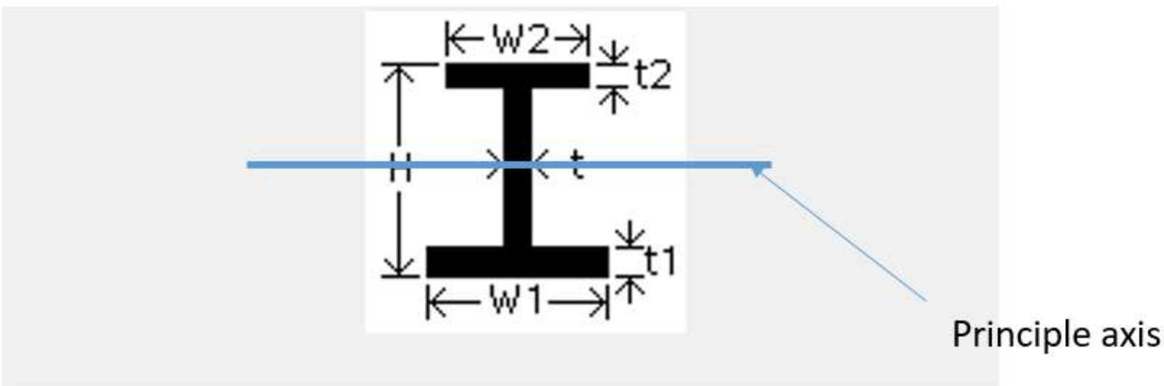
If the I-beam in example 1 had been much longer such that the value of βl becomes greater than 6, then the table of figure 8 should be used.

As it can be observed, using the elasticity based formulas presented in reference [2] is cumbersome. It is also observed that the elasticity based formulas in reference [2] use the span between the springs to calculate the stresses.

IV: Finite Element based approach

Finite Element analysis provides a more efficient technique, and also provides more detailed information. Figure 13 is a finite element model of a beam supported by elastic springs using NASTRAN/PATRAN. In the model of figure 12, the properties reflect the properties used in examples 1 and 2.

The section properties used for the beam of the model of figure 13 is shown in figure 12. The dimensions shown in figure 12 are in inches. The area moment of inertia for the principle axis if the I-beam of figure 12 calculated by PATRAN 21.41 in^4.



H	5
W1	6
W2	6
t	0.29
t1	0.29
t2	0.29

Figure 12: section properties of the I-beam of the model of figure 13

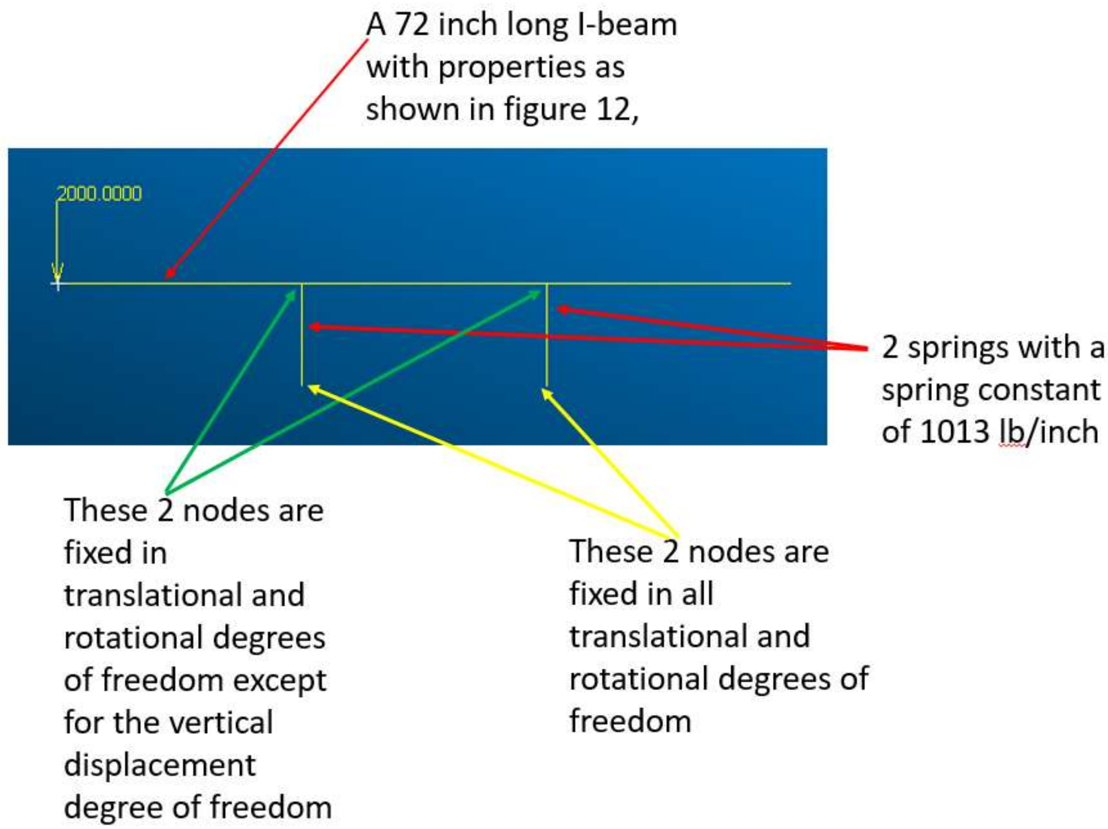


Figure 13: Finite Element model of an I-beam supported by 2 springs.

The model of figure 13 appears as shown in figure 14 once the displacement are shown on the model.

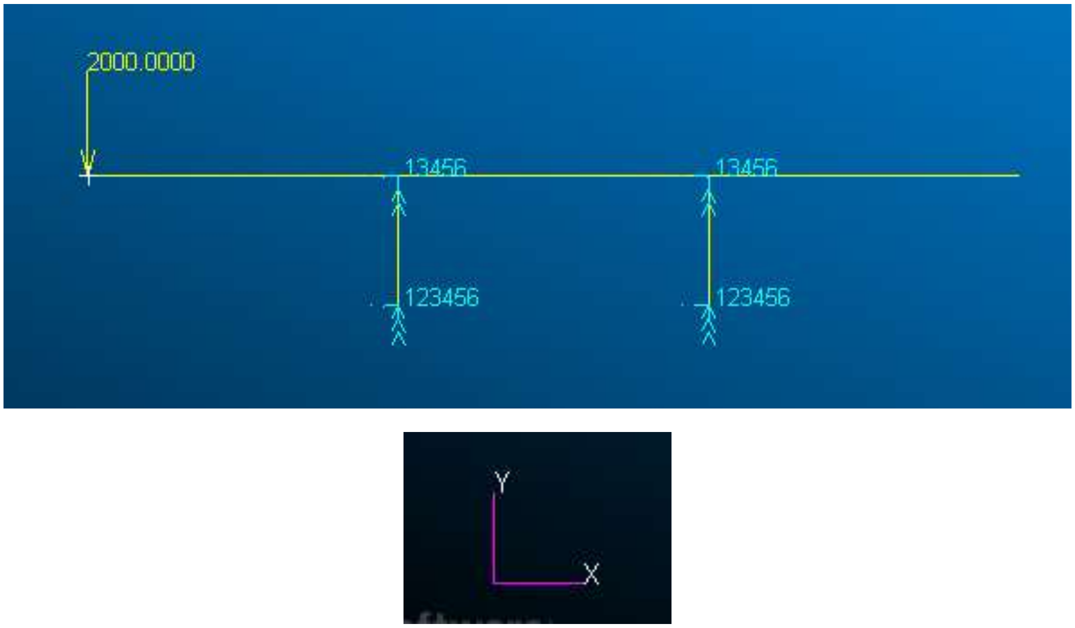


Figure 14: Model of figure 14 with all restricted displacement and rotations shown. 1, 2 and 3 refer to displacement in X, Y and Z direction. 4, 5 and 6 refer to rotation about X, Y and Z axes.

Although as shown in section III of this document, a model similar to the model of figure 14 (figure 6) has a theoretical solution, the Finite Element model of figure 14 does not work due to the internal matrix formation mechanism in NASTRAN/PATRAN [4].

The model of figure 15 is the same as the model of figure 14 except that all springs are replaced by equivalent I-beams that produce the same stiffness as the springs (as shown in figure 11) and are fixed at their ends. By employing cross beams, the matrix formation requirements of the Finite Element model is met and a beam on strings is simulated.

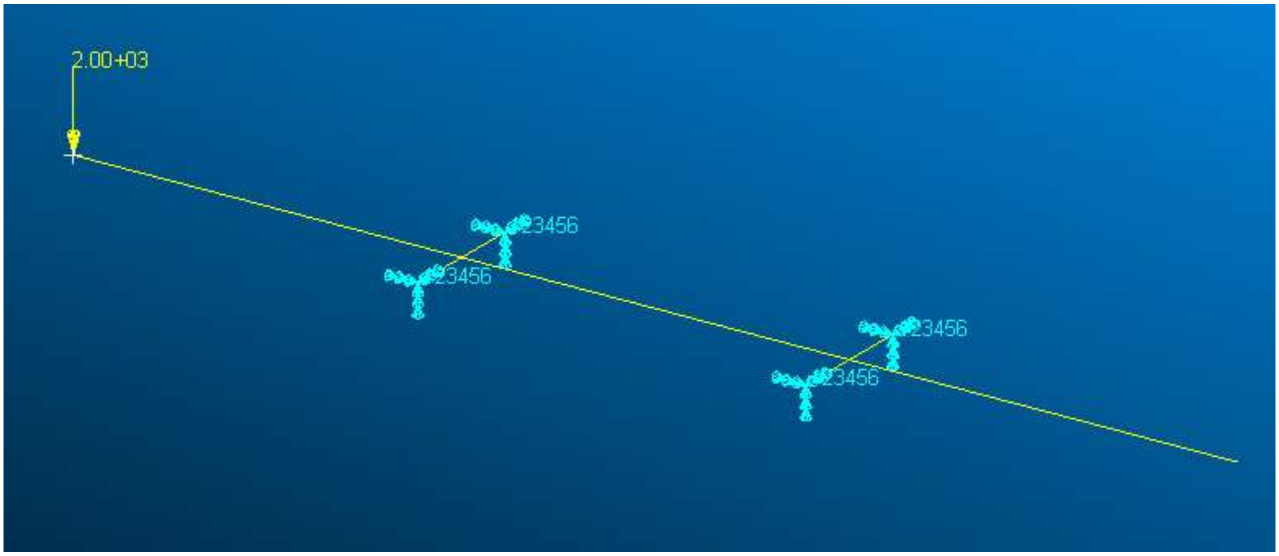


Figure 15: The same model as figure 14 except that the springs are
replaced by cross I-beams fixed at both ends that produce equivalent spring effects

Figures 16 and 17 show the displacement and stresses for the model of figure 15.

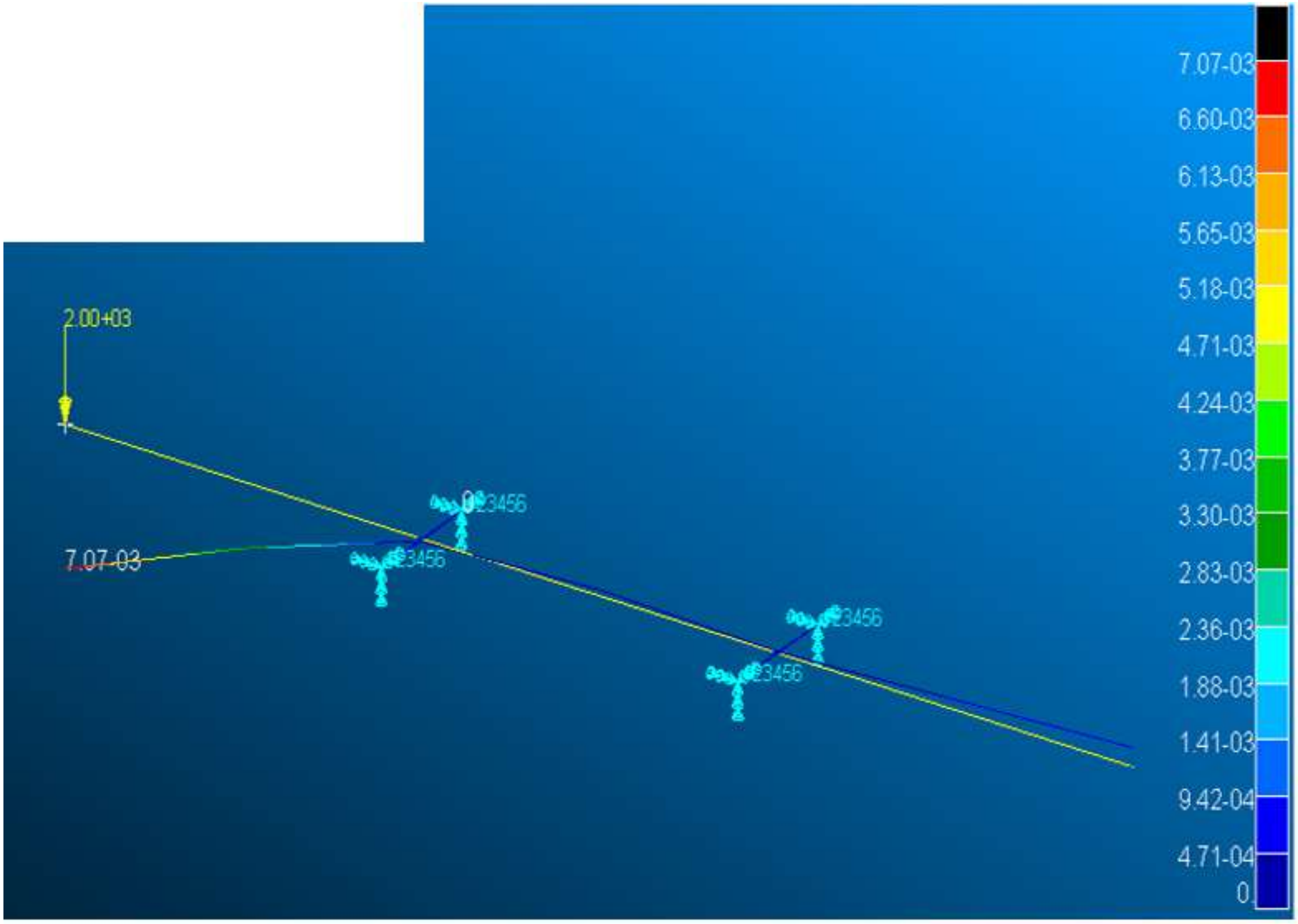


Figure 16: Displacement of the model of figure 15

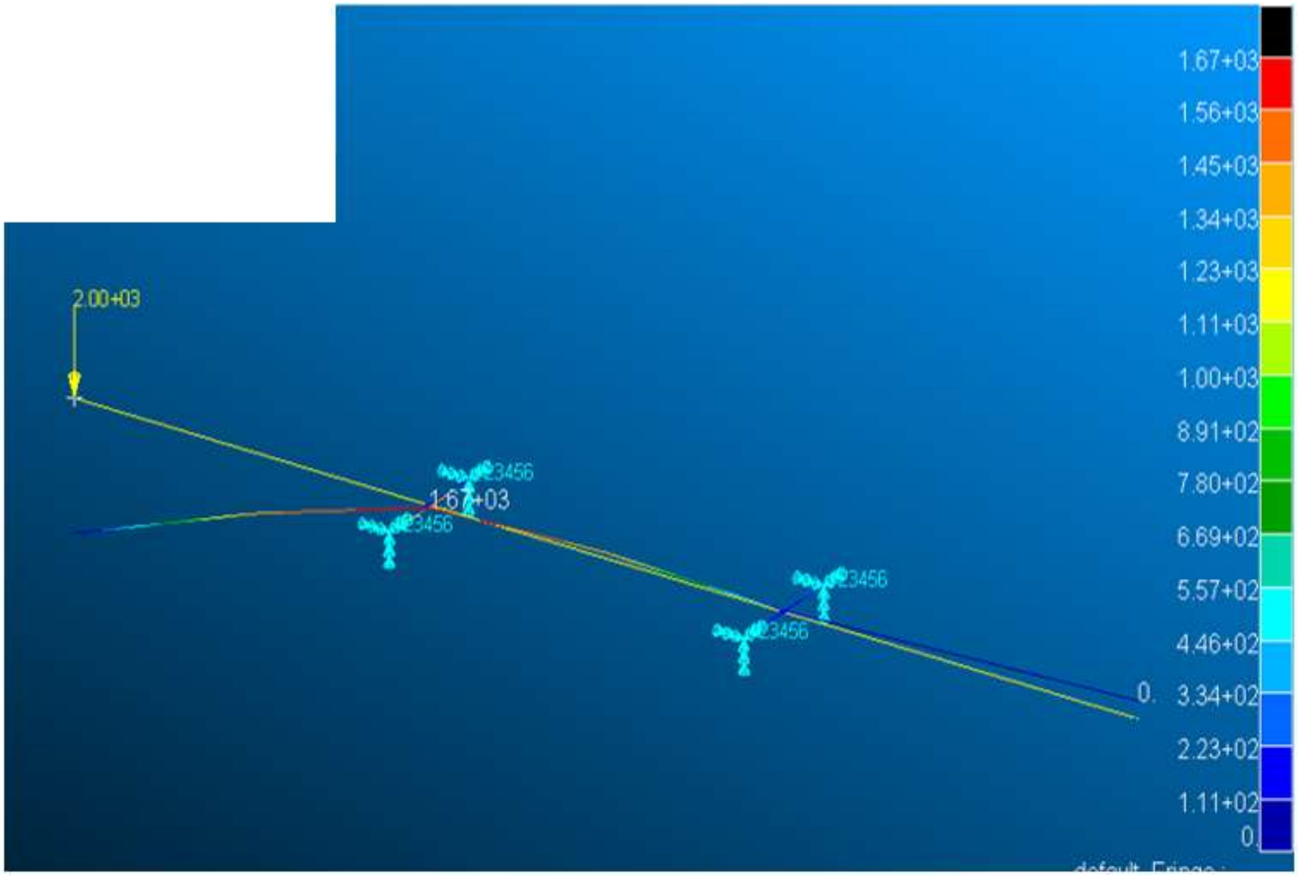


Figure 17: Stresses for the model of figure 15

V: Student response and additional concepts taught in class

Students had difficulty with adjusting their Finite Element modeling techniques such that the model would run. One of these adjustments in described in section IV of this document where inclusion of only springs would result in a runtime error and the springs had to be simulated by using beams. The students also had difficulty interpreting the differences between the formulation of a beam on a continuous base versus a beam on a number of springs as described in section III of this document. However, after the initial glitches were fixed, the majority of students enjoyed the course.

There is significant work that must be mastered by students related to Finite Element modeling techniques that are not covered in this article due to space limitation.

VI: Summary and Conclusion

Elasticity based solution of beam on a continuous elastic foundation is used to develop formulas for beams on multiple discontinuous elastic foundations that are equally spaced. Roark's formulas for stress and strain have formulas for discontinuous elastic foundations for a variety of loading and boundary conditions.

Unique issue arise when attempts are made to model the discontinuous elastic foundations in the form of springs. Unique modeling approaches are demonstrated to address these issues. The finite element modeling techniques can be used to analyze when the discontinuous elastic foundations are not equally spaced and for any combination of loading and boundary conditions.

The techniques demonstrated were used in an “Advanced Mechanics of Materials course”.

VII: References

[1]. Advanced Mechanics of Materials, by Boresi & Schmidt.
[2]. Roark’s formulas for Stress and Strain, 6th edition, section 7.5.
[3]. Statics and Strength of Materials, second edition by Cheng.
[4]. NASTRAN theoretical manual.