Analysis of Directional Beam Patterns from Firefly Optimization

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***Abstract*—An analysis of the positional distributions of the elements of a linear antenna array is conducted. The movement and clustering of fireflies based on their intensity is applied as an optimization technique to determine the element positions that achieve a desired radiation pattern, consisting of a main beam bandwidth and sidelobe level. The expected value and variance of the resulting beam pattern with respect to angular position and probability distribution of inter-element distances as the number of elements are discussed. It is shown that the distribution for inter-element distances that meet the desired radiation pattern yields a bimodal structure, one where arrays consisting of a high number of elements experience clustering, with elements having a higher probability of being spaced closer together.**

I. Motivation

Nonuniformly spaced antenna arrays may achieve desired beamforms using constant current through proper positioning of the individual elements. This has many applications due to the miniaturization of transducers, coupled with their economic viability, that allows a great many of them to be placed in an array. Individual elements may then be activated in accordance with a specific inter-element distance distribution that ensures the array produces a specified beamform.

Applications of this are arrays consisting of wireless nodes where a nonuniform spacing offers robust performance [1] and low power wireless sensor networks [2]. The analysis of nonuniformly spaced wireless elements in a one dimensional array is conducted in this work.

II. Introduction

Consider an antenna array that is composed of $2M$ elements positioned symmetrically about the origin on the x-axis in three-dimensional space. The beam pattern in the $x-y$ plane is described with respect to the angle ϕ as,

$$G(\phi) = \frac{1}{2M} \sum_{n=-M}^{M} cos(k\ x_n\ cos\phi) \tag{1}$$

where k is the wave number and x_n is the position of the n^{th} element. The selection of the positions x_n such that the side-lobe amplitudes are constrained to be below a level of S_{des} decibels is of interest. The desired beam pattern should satisfy the constraints,

$$G_{des}(\phi) = \begin{cases} 0dB, & -\frac{BW}{2} < \phi < \frac{BW}{2} \\ S_{des}, & \text{otherwise} \end{cases} \tag{2}$$

where BW is the bandwidth of the main beam.

Various methods have been presented for generating the beam pattern that include positional error correction of elements using least squares [3], determining element positions using a genetic algorithm with the conjugate gradient method for a specified bandwidth and sidelobe level [4], and node selection from a two-dimensional space using a genetic algorithm for a linear array [5]. [6] et al. utilize a firefly algorithm and a profile of a desired radiation pattern to determine iterations for convergence of the fitness function for a set of fireflies. They also utilized a desired radiation pattern with specific null locations and widths. A comparison of particle swarm optimization and the firefly algorithm for both design problems was also shown, demonstrating that the firefly algorithm converged at a faster rate and achieved a better sidelobe level, making it very suitable for design of nonuniformly spaced arrays.

The performance of the firefly algorithm is examined in this work with respect to the probabilistic metrics of the beam pattern. The characterization of the inter-element spacing that results from this approach is also discussed. The firefly optimization algorithm is referred to as a metaheuristic based approach, in that few assumptions are made about the particular problem being solved unlike heuristic based optimization. The algorithm attempts to emulate some of the clustering dynamics of fireflies that is based on the observable intensities of other fireflies. Bounds and other restrictions may be imposed on their movement but in general the fireflies have little to no knowledge of the problem itself, and can thus be used for a wide variety of problems. Yang [7] first presented the spatio-temporal dynamics of fireflies as candidates for designing optimizaton methods.

The basic dynamic utilized is the movement of fireflies towards each other based on their intensity. A fitness function that defines the characteristic to be optimized for the particular problem being solved represents the intensity. Fireflies will move towards other fireflies which better meet the fitness function. The visibility of the intensity of fireflies decreases with distance and this feature allows the formation of clusters of fireflies. In each iteration of the algorithm the fitness of each firefly is computed and the movement of fireflies is controlled by two factors. One is a random movement, the length of which decreases exponentially in time and the second movement is towards all other fireflies with a greater fitness than its own. This combination allows the incorporation of both individual exploration and directed movement towards fireflies with high intensity.

For the application considered in this paper of determining the positions of the M linear array elements, each firefly denoted by the index i is described by an attribute vector $\underline{x}_i : [x_i[1], ...x_i[M]$ where $x_i[m]$ is the position of the m^{th} element of the i^{th} firefly.

The fitness function that characterizes the difference between the pattern $G(\phi; i, t)$ obtained from the position vector $\underline{x}_i^t$ at time t and the desired profile $G_{des}(\phi)$ is defined as,

$$f_i^t = \sum_{\phi=0}^{\pi} [G(\phi; i, t) - G_{des}(\phi)]^2 \ I(\phi) \tag{3}$$

where the indicator function $I(\phi) = 1$ for $G(\phi; i, t) > G_{des}(\phi)$ and zero otherwise.

The function f_i^t is computed for every firefly i at every iteration t. The objective of the optimization is to minimize f_i^t, where the minimum value of 0 implies that the pattern generated meets the requirements of the desired beam pattern for every angle in ϕ. This minimization leads to the optimal positions of the array elements.

The algorithm proceeds as follows. A set of M fireflies are initialized with random positions of array elements such that the inter-element distances are uniformly distributed in the range $(x_{min} = 0.35\lambda, x_{max} = 0.9\lambda)$. We consider the parameters used in the work by [6], specifically the choice of initial distribution range and bandwidth requirement. Here the positions are non-dimensionalized with respect to λ, the wavelength of radiation. Note that, these bounds are enforced only at the initial time step.

An individual firefly with index $\hat{i}$ will then move towards each firefly in the set of i which possess a fitness strictly less than that of $\hat{i}$. The updates to the element positions takes place as:

$$x_{\hat{i}}[m] = \begin{cases} x_{\hat{i}}[m] + \beta_{\hat{i},i}(x_i[m] - x_{\hat{i}}[m]) + \alpha_t, & * \\ x_{\hat{i}}[m] + \alpha_t, & \text{else} \end{cases} \tag{4}$$

where $f_i^t < f_{\hat{i}}^t$

The term $\alpha_t = \alpha_0 \gamma (1 - \frac{t}{T})(rand - 0.5)$ is linearly decreased over the total number of iterations T and $rand$ is a uniform random number from [0,1], where γ is related to the range of the space $[(x_{max} - x_{min})/M]$, which is a function of the initial inter-element distribution and the total elements. A choice of $\gamma = 0$ results in the standard particle swarm optimization. This relation of γ to the search space is what enables fireflies to converge upon neighbours without being mitigated by the effects of a large M, which effectively increases their distance.

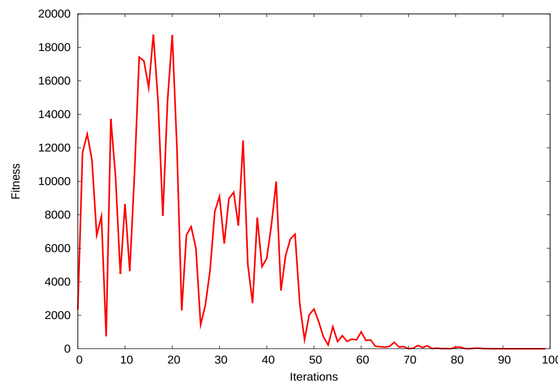

Fig. 1. Average Fitness for All Fireflies

The function $\beta_{\hat{i},i}$ is defined as:

$$\beta_{\hat{i},i} = \beta_0 e^{(-\gamma r_{\hat{i},i}^2)} \tag{5}$$

and provides a distance based weighting of the firefly intensity such that pairs of fireflies separated by large distances are not impacted to move. The value of $\beta_0 = 1$ is fixed in this analysis and $r_{\hat{i},i}$ is the distance between the $\hat{i}^{th}$ and i^{th} fireflies, defined as:

$$r_{\hat{i},i} = \sqrt{\sum_{m=1}^{M} (x_{\hat{i}}[m] - x_i[m])^2} \tag{6}$$

To summarize, with each iteration of the algorithm the fitness of each firefly is calculated and each firefly moves both randomly, with a distance decreasing with each iteration and towards other fireflies, taking larger steps towards those fireflies that are closer.

III. Firefly Algorithm Results

The firefly algorithm is capable of finding multiple equally valid solutions. As the fitness function used here is a very simple function containing only a main beam bandwidth and a total sidelobe level, many fireflies will meet this criteria but may differ from each other significantly, having different absolute element positions or inter-element distances. This features results in radiation patterns with different side lobe levels and positions.

An initial configuration is used where:

- M=10
- firefly count=20
- iterations $T = 100$
- α_0=0.5
- BW_{des}=0.234 radians
- S_{des}=-35 dB

Figure 1 shows how the fitness of the fireflies progresses as the algorithm iterates. Plotted is the average fitness for all fireflies for each iteration of the algorithm. There are large variations in the fitness near the start of the algorithm due to the choice in α, which allows the fireflies to take larger random

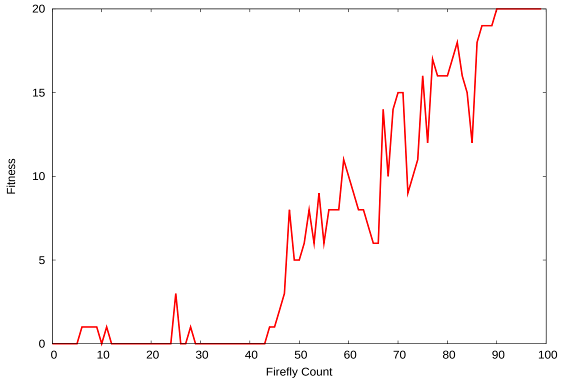

Fig. 2. Number of Fireflies with a Fitness of Zero

steps. A very large α enables the fireflies to explore to a greater degree, but may also potentially prevent convergence as their random movements are much larger than their movements towards fireflies with a better fitness. An average fitness level of zero implies that all fireflies have converged to a solution in the position of the array elements that meet the specified criteria. Note that at each iteration, a certain number of fireflies would have a fitness function of zero. Although these fireflies now meet the desired radiation pattern, in successive iterations they will still experience some random movement, leading their fitness to potentially rise. However, this random movement is decreased with each iteration, which leads to convergence towards a good fitness value.

Figure 2 is a plot of the number of fireflies whose fitness is zero as a function of the number of iterations. The plot shows that there is a rapid rise in fireflies roughly halfway through the total number of iterations, which occurs due to the linear decrease in α as a function of iterations. At the start of the algorithm the random movement values will be high, allowing the fireflies to explore, and as the iterations increase the fireflies will converge upon a good fitness value, which results in the rapid increase.

Since position elements of individual fireflies that have converged are not identical, an average radiation pattern and corresponding variance can be derived from those that satisfy the beam pattern criteria. The distribution of inter-element distances of the corresponding fireflies will also be investigated.

All three statistics were calculated using all fireflies that had a fitness cost of zero for each iteration of the algorithm, which is in the range of 0:20 fireflies for each iteration. This allows the variance of exploring fireflies at the start of the algorithm who meet the desired radiation pattern to be included alongside that of those converged fireflies at the end of the algorithm. Figure 3 demonstrates the radiation pattern of the mean of those fireflies who meet the fitness function perfectly. Figure 4 plots the normalized variance in decibels. The variance rapidly increases away from the main beam and is minimal at the peak of each sidelobe.

Figure 5 is a probability mass function (PMF plot) of the inter-element distances. The inter-element distances have a

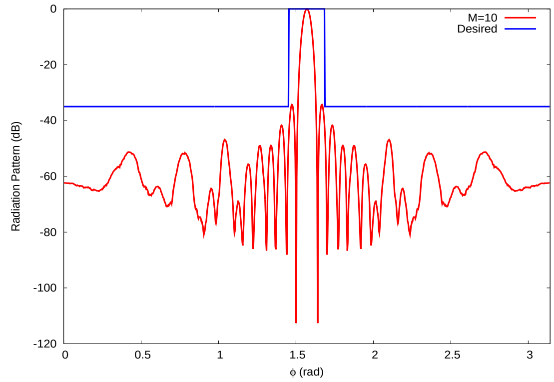

Fig. 3. Mean Radiation Pattern for All Optimal Fireflies

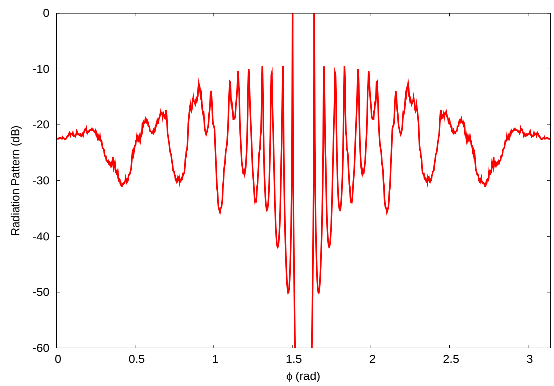

Fig. 4. Variance of Radiation Pattern for All Optimal Fireflies

resolution of 0.008λ in the results shown. This plot shows that most elements are spaced within the x_{min} and x_{max} bounds specified as initial conditions. The algorithm allows fireflies to acquire inter-element positions that are outside this bounded range. The radiation pattern resulting from arrays consisting of these unbounded elements can still meet the desired radiation pattern, allowing for an extended distribution of element distances. However, further constraints may need to be considered as unbounded inter-element distances may result in infeasibly long arrays or arrays where elements are placed in

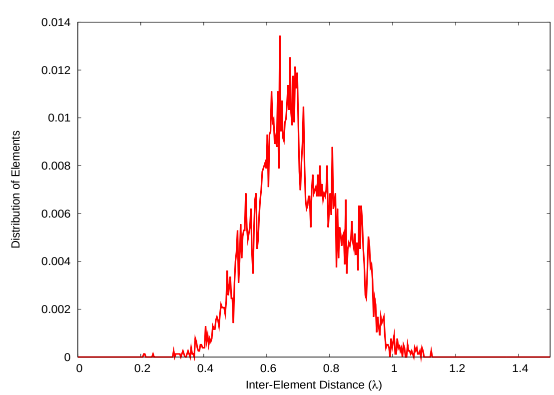

Fig. 5. Distribution of Inter-Element Distances for All Optimal Fireflies

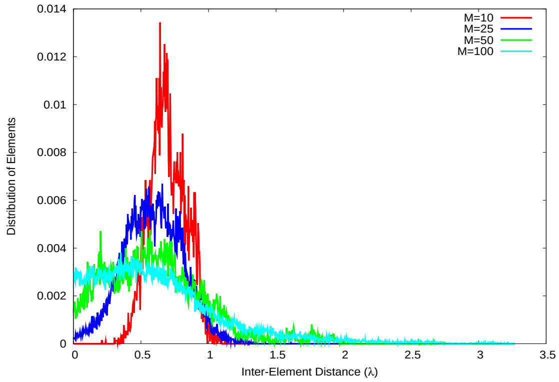

Fig. 6. Inter-Element Distance Distribtion for $M = 10, 25, 50, 100$

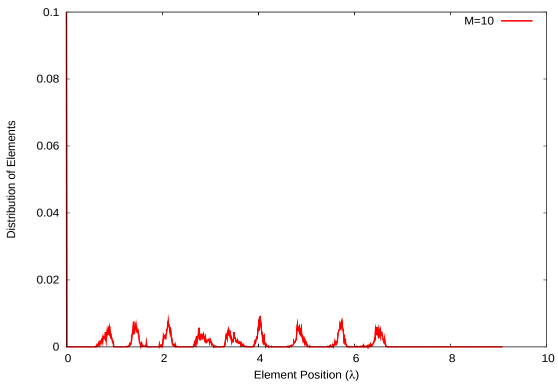

Fig. 7. Positional Distribution for Elements for $M = 10$

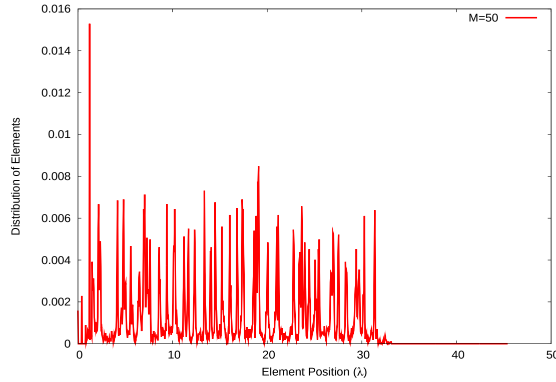

Fig. 8. Positional Distribution for Elements for $M = 50$

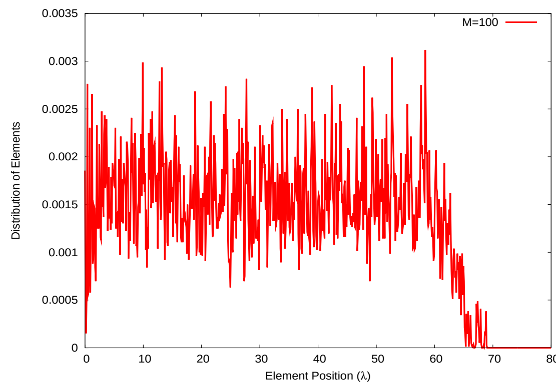

Fig. 9. Positional Distribution for Elements for $M = 100$

such short distances that achieving this with a physical element of a certain size would be impossible.

The number of array elements and the initial conditions on inter-element bounds can influence the performance of the algorithm and the results. The effects on the variance, inter-element distance distribution, and the overall convergence effects of the firefly algorithm as these parameters are varied is investigated next.

IV. Parameter Exploration

One of the key parameters that determines the radiation pattern is the number of elements within the array. As M increases the number of configurations that produce viable radiation patterns in accordance with the desired pattern also increases.

Figure 6 shows the effects of increasing number of elements on the inter-element distance distribution. As M increases, it becomes much more possible for arrays to consist of elements having a spacing that lies outside the bounded region and still meet the overall desired pattern. Arrays consisting of larger M result in many more inter-element distances that are less than x_{min}, but there is no significant increase in those that are greater than x_{max}. This implies that there may be clustering of elements, so the absolute element position distribution will be shown next.

The probabilities for the spatial position of array elements is constructed similar to the inter-element distribution with a spatial resolution of $Mx_{high}/1000$. Figure 7 is the distribution of the element positions for $M = 10$. There are ten distinct clusters, separated roughly equidistant from each other, with one cluster around zero. Figure 8 is the result for $M = 50$ and Figure 9 shows the distribution for $M = 100$. As the number of elements increase the distribution approaches a uniform probability structure across the region.

To analyze the dependence on the initial random distribution, the simulation is run for $NENS = 10$ ensembles with different initial distributions for the inter-element positions. Therefore, the mean and variance considered here consists of all fireflies that meet the fitness criteria at any iteration, for any run.

Figure 10 shows the mean radiation patterns for $M = 10$ and $M = 100$ for $NENS = 10$. This mean beam pattern is thus the average over ten runs of the algorithm using new initialized positions each time. This result can better represent the generation of a beam that is captured from the activation of multiple antenna arrays, each with random positions for the elements. This will reduce the variation that occurs from consideration of only one set of initial conditions. The firefly algorithm may become easily stuck in local minima and multiple ensembles allows a reduction in variance. These

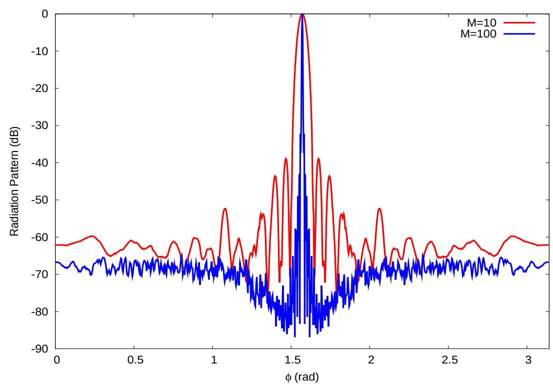

Fig. 10. Mean Radiation Pattern for $M = 10, 100$ and $NENS = 10$

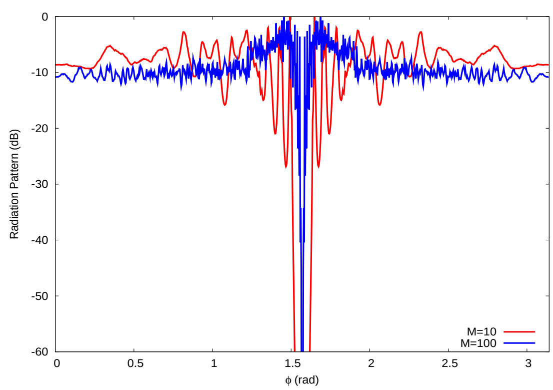

Fig. 11. Variance of Radiation Pattern for $M = 10, 100$ and $NENS = 10$

results may be compared to that of only a single ensemble, where the variation in the inter-element distribution is much higher.

Increasing M from 10 to 100 results in a much better radiation pattern, one that is consistently lower. Figure 11 shows the variance for this increase in M and it displays the opposite, have a consistent variance that is greater than that of $M = 10$, but not significantly so.

Figures 12 and 13 show the effects on the element positions for different numbers of elements. Both distributions show a

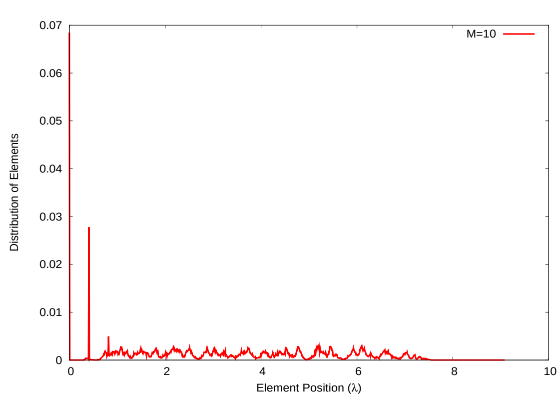

Fig. 12. Absolute Element Position for $M = 10$ and $NENS = 10$

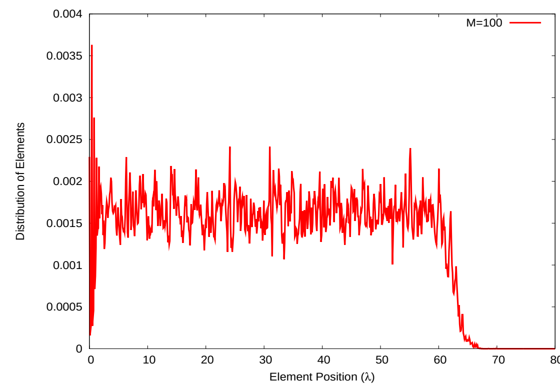

Fig. 13. Absolute Element Position for $M = 100$ and $NENS = 10$

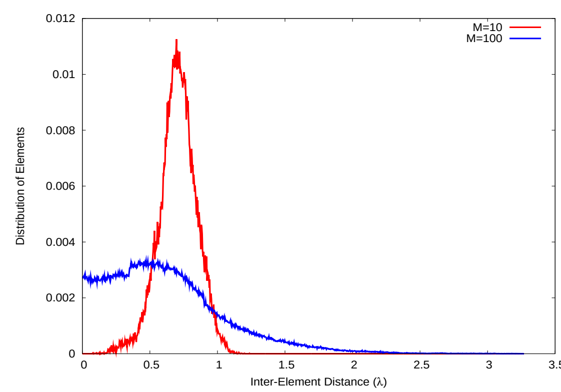

Fig. 14. Inter-Element Positions for $M = 10, 100$ and $NENS = 10$

very high likelylihood of having an element near or at the origin (Or both), but after that their distributions differ. For $M = 100$ the distribution is fairly uniform across the range, though the likelihood of having a position greater than $\frac{2}{3}$ of Mx_{max} decreases to zero. There is also a very large number elements that occur near zero, which explains the increase in inter-element distances that are less than x_{min} as was seen previously. $M = 10$ results in various peaks and valleys whose positions may depend largely on the initial positions and so a smooth distribution is unlikely to be seen without a high value of $NENS$.

Finally, the effects on the inter-element distances will be seen. Although this was investigated earlier, a smooth distribution was not obtained.

Figure 14 shows a clear distribution in the inter-element positions for both $M = 10$ and $M = 100$. As the number of elements increases the distribution is affected such that many more elements have an inter-element distance less than x_{low}, but few are spaced further apart than x_{max}. These distributions exhibit a bimodal structure. As such, this unbounded algorithm has demonstrated the possibility of not only unbounded inter-element distances that result in desired radiation patterns, but also the ability for arrays with large M to have a higher likelihood of this occuring. Arrays with a larger M result in an increasing probability of elements being placed closer together than that of x_{min}, which implies that clustering may occur, reinforced by the results shown in Figure 13.

V. Conclusion

The application of firefly dynamics to determine the random positions of linear array elements was investigated in this work. The convergence to the desired beam pattern was achieved within a hundred iterations. An inter-element distance distribution was obtained that may be utilized for identifying elements to be activated for meeting a specific radiation pattern. The inter-element distributions are affected by the total number of elements. An increase in element count results in an increase in the number of elements spaced closer together than x_{min}. This implies that clustering of antennas may result in desirable radiation patterns, for those antennas which possess a high element count. High variance occurs in the positions of the nulls between sidelobes and this may be further investigated to refine the inter-element distribution such that the resulting variance is minimized.

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