Analysis of reasoning paths of engineering students

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Abstract

There are several factors that have an effect on physics learning for engineering students: from strong alternate conceptions, to attitudes toward their learning or expectations of the use of physics in their other courses during their undergraduate education, or, even further, to their professional practice once they graduate. One of the factors proven to have a strong effect when using active learning strategies has been students' reasoning. Studies show that physics learning does not correlate to students' reasoning level for traditional teaching. On the other hand, physics learning has a positive correlation to students' reasoning level for those taking active learning physics classes. There are several instruments to assess reasoning, such as the Lawson test. This test is well known among the physics education research community and is often used to measure fundamental reasoning elements through simple context situations. The test consists of 12 pairs of multiple-choice items in which, for each pair, the second item consists of the different reasoning students might have to answer the first item. Although Lawson test's results have been documented in several different studies, often the use is limited to analysis using the general score or, at most, using the different Lawson test levels of reasoning. The objective of this study was to conduct an in-depth examination of the tests results by analyzing students' paths for each pair of items. To that end, 500 undergraduate engineering students took the Lawson Test. For each pair of items, the analysis looked at the most common students' paths when answering the question correctly or incorrectly. By combining the results for each dimension of the test, an in-depth analysis on students' reasoning was performed for two dimensions. Moreover, by characterizing students' paths, we believe that physics education researchers will have another tool to design activities to develop students' reasoning skills and therefore, increase engineering students' physics learning.

Introduction

Scientific reasoning refers to "cognitive abilities such as critical thinking and reasoning" (Bao et al, 2009, p. 586) or "skills involved in inquiry, experimentation, evidence evaluation, and inference that are done in the service of conceptual change or scientific understanding" (Zimmerman, 2007). It is needed in problem solving situations and requires methods of scientific inquiry such as the cycle of analysis, testing, reflection and revision, in order to construct a deeper understanding of the situation. Scientific thinking is "purposeful thinking that has the objective of enhancing the seeker's knowledge" (Kuhn, 2010, p. 2).

To measure scientific reasoning, Lawson's Classroom Test of Scientific Reasoning (Lawson, 1978, 2004) is an instrument that has been extensively used. This multiple choice test includes probabilistic reasoning, combinatorial reasoning, proportional reasoning, and controlling of variables in the context of the broader science domain (Lawson, 1978). The test focuses on general scientific reasoning rather than specific concepts; that is, expertise in certain content domain is not required (Tiruneh et al., 2014).

The Classroom Test of Scientific Reasoning (CTSR) has not been characterized in terms of its use to understand the level of results that can be obtained from it. This work has the objective to analyze the test and understand the different answers from students and what we can learn from that.

Background

Since scientific reasoning could be related to other factors that affect learning, we have developed some background ideas in this work. In the first part we see the connection of teachers' scientific reasoning and their effective use of inquiry-based learning. The second part is about the relationship between students' scientific reasoning and their problem solving abilities. Then, we analyzed the relationship of students' Force Concept Inventory (FCI) performance and their scientific reasoning abilities. At the end, we present a study which examines the relationship between students' content knowledge and their scientific reasoning ability.

Scientific reasoning and effective inquiry-based learning

Benford and Lawson (2001) conducted a quantitative study to test the hypotheses: a) teacher' scientific reasoning positively relates to an effective use of inquiry-based learning, and b) an effective implementation of inquiry-based learning fosters the development of scientific reasoning in students. Both hypotheses were accepted, meaning scientific reasoning works as a predictor of an effective implementation of inquiry-based learning and students improve their scientific reasoning when effectively using inquiry-based learning.

Scientific reasoning and problem solving

Aguilar et al. (2002) conducted a quantitative study to determine the relationship between problem solving skills and different levels of scientific reasoning. In this study, 78 high school students were administered the Test of Logical Thinking. This test measures formal reasoning: controlling variables, proportional reasoning, combinatorial reasoning, probabilistic reasoning, and correlational reasoning (Tobin and Capie, 1981). Aguilar confirmed that there is a relationship between scientific reasoning and mathematical problem solving skills, that is, the higher the formal reasoning the better the problem solving skills. However, Aguilar argued that, in the case of proportionality problems, students with high formal thinking had difficulties solving those problems, as only 36% percent of those students solved proportionality problems correctly. Aguilar concluded that a high formal reasoning level is not a sufficient predictor to determine problem solving skills for certain topics.

Relating FCI scores and scientific reasoning ability

The Force Concept Inventory (FCI) is a widely used test that evaluates basic, yet fundamental, Newtonian concepts. It is often administered in a Pretest-Instruction-Posttest sequence to evaluate students' understanding of those concepts. Coletta and Phillips (2005) report a positive correlation between class average normalized FCI gains and class average pretest scores. Coletta and Phillips used the Lawson Classroom Test of Scientific Reasoning to prove that students' background plays an important role in their learning outcomes. Hake's study (1998) argues that interactive engagement classes obtained a higher relative gain, and Coletta and Phillips argue that the background of the students may be a variable to consider. Precisely, they concluded that the normalized gain and the FCI pretest scores is far less significant than the correlation between the normalized gain and the Lawson test scores (p. 1176).

Content knowledge and scientific reasoning ability

Bao et al. (2009) designed a comparative study between freshman college students in China and the US. The authors focused on students' conceptual understanding in physics and their scientific reasoning ability. To that end, students from four universities in the US and from three universities in China took either the Force Concept Inventory (523 students from China and 2681 students from the US), the Brief Electricity and Magnetism Assessment (331 students from China and 650 students from the US) or the Lawson Test (370 students from Chinese universities and 1061 students from the US' universities). Results show that Chinese students had a higher average in both conceptual tests (FCI and BEMA). However, the reasoning ability results have a very similar pattern for both groups. This seems to indicate that educational differences between China and USA, as depicted by the conceptual knowledge tests, do not have an impact on the development of the scientific reasoning ability.

Several studies have focused on trying to find a correlation between scientific reasoning and content knowledge in STEM; particularly in biology, mathematics and physics (Aguilar et al., 2002; Bao et al., 2009; Coletta and Phillips, 2009; Lawson, 2007); Lawson's Classroom Test of Scientific Reasoning has been documented for different studies. However, those studies limited the analysis to the general score or different Lawson test levels of reasoning. The objective of this study is to conduct an in-depth examination of the tests' results by analyzing students' paths for each pair of items.

Methodology

Engineering students in four different courses took the Lawson test in the fourth week of classes. These physics courses go from the introductory physics course up to the electricity and magnetism course; these are the four basic physics' courses that most engineering students take as part of their program.

The semester in which the study was implemented had 2321 students enrolled in any of the several sections offered for those four physics courses. The test was administered as a diagnostic test. Each student was given a pair of sheets, the questions sheet and the answer sheet (bubble sheet). The answer sheet was later scanned to collect the results.

The Lawson test consists of 24 multiple-choice questions that are paired-up, that is, for each question, a following question asks for the reasoning. The questions include topics such as conservation, proportional thinking, probabilistic thinking, deductive and inductive reasoning, correlational thinking, and hypothesis evaluation (Lawson, 2004).

Results

We present the tests results in general in the first section and for each dimension in the following section. The discussion will take place after the results focusing only on two dimensions in this study.

Test results

There are a number ways of presenting results, depending on the objective. Since the Lawson test has twelve pairs of items, one way to present results is by using a histogram with the percentage of students having a number of correct pairs. Figure 1 presents that histogram for the 2321 students in this study.

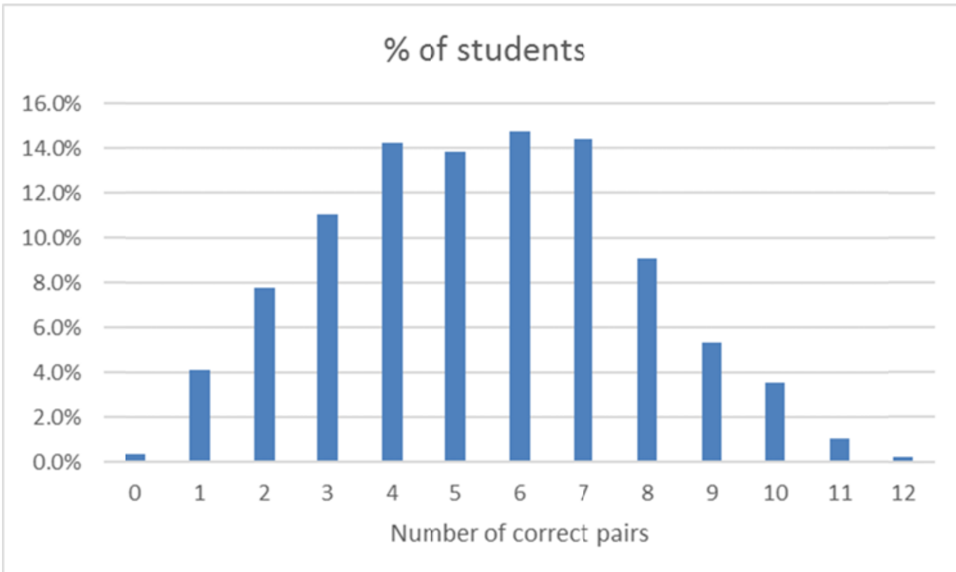


Figure 1. Histogram of percentage of students having a number of pairs of items correct (N = 2321).

The Lawson test is divided into a classification of students' level of reasoning according to the number of correct pairs. Students having a score between zero and three correct pairs, fall in the classification of concrete reasoning (CR). Students having a score between eight and twelve correct pairs fall in the classification of formal reasoning (FR). Students having a score between four and seven fall in the classification of transitional reasoning (TR). Figure 2 presents a histogram taking into account the reasoning level as the bins.

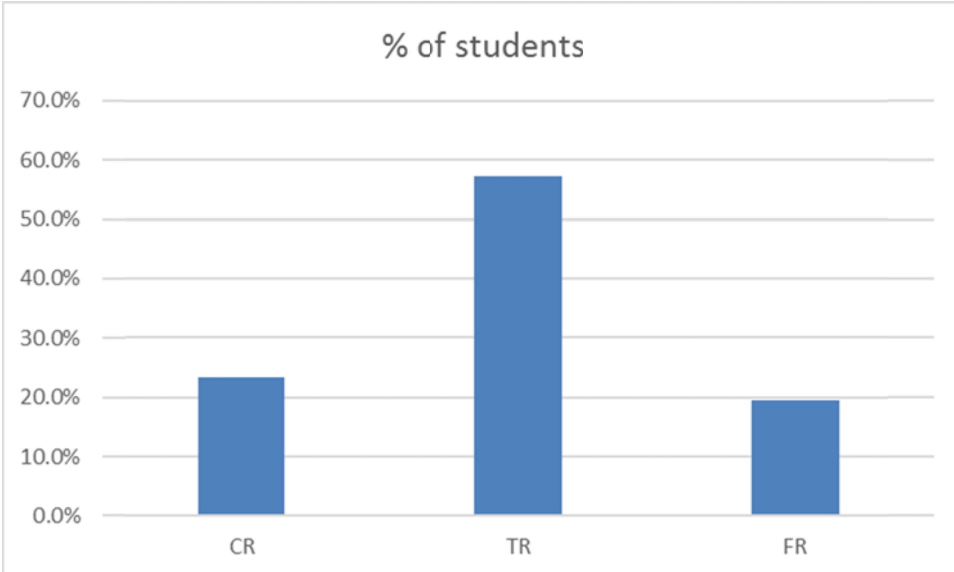


Figure 2. Percentage of students in each of the categories of reasoning levels (N=2321). CR: Concrete Reasoning, TR: Transitional reasoning and FR: Formal reasoning.

It is common to see that university students are in a transitional level of reasoning; 54% of students in this study are in that level. Only 19% of students are in a formal reasoning level and 23% of students are in a concrete level of reasoning.

Table I shows the results of each item in the Lawson test. The highlighted numbers show the number of students who answered that item correctly.

Table I
Results of the 24 items of the test

	Item 1	Item 2	Item 3	Item 4	Item 5	Item 6	Item 7	Item 8	Item 9	Item 10	Item 11	Item 12
A	21	45	1770	24	590	112	362	1032	59	72	718	486
B	2213	52	491	65	1460	528	543	51	576	259	796	588
C	86	18	54	381	67	1376	94	106	90	1351	601	264
D	N/A	2200	N/A	83	86	261	1065	332	219	497	190	833
E	N/A	3	N/A	1761	109	35	241	771	1369	132	N/A	144
Sum	**2320**	**2318**	**2315**	**2314**	**2312**	**2312**	**2305**	**2292**	**2313**	**2311**	**2305**	**2315**

	Item 13	Item 14	Item 15	Item 16	Item 17	Item 18	Item 19	Item 20	Item 21	Item 22	Item 23	Item 24
A	844	134	181	2021	251	97	1608	427	524	970	699	451
B	233	341	679	33	1417	179	289	364	627	769	908	1149
C	1090	261	1344	135	333	200	346	92	250	261	490	471
D	135	1135	77	41	181	322	N/A	1304	241	137	N/A	N/A
E	N/A	418	25	72	74	1435	N/A	58	537	N/A	N/A	N/A
Sum	**2302**	**2289**	**2306**	**2302**	**2256**	**2233**	**2243**	**2245**	**2179**	**2137**	**2097**	**2071**

Individual pairs and dimensions

Table II shows the results of the dimension "Conservation of matter and volume". The table presents the paths of individual pairs of items (items 1 and 2, and items 3 and 4) and the paths of the dimension. For both pairs we present paths with a percentage greater than 6.7%, since that is the probability of answering the pair of items correctly if done randomly. The table also shows the paths for the whole dimension. In this case, we present paths with a percentage greater than 2%.

Table II
Conservation of matter and volume

I1	I2	Path	%
B	D	2183	94.2%

I3	I4	Path	%
A	E	1751	75.8%
B	C	346	15.0%

I1	I2	I3	I4	Path	%
B	D	A	E	1671	72.2%
B	D	B	C	311	13.4%

The result of the pair including items 1 and 2 is simple, the only significant path is the right answer. This pair of questions show a high percentage of correct answers, since 94.2% of students got this pair correct. In the case of the pair including items 3 and 4, there are two significant paths: the correct answer and reasoning (A and E respectively) and a second path, answer B and reasoning C; 15% of students chose the latter path.

In the "Conservation of matter and volume" dimension we found two paths in which a significant number of students chose those choices. Both of the paths start with the correct pair of items 1 and 2 as expected. In items 3 and 4, the two paths are the same as before. In this case, since students have a strong ability to reason with these two pairs, then the analysis of the dimension is simple. Note, however, that the percentages for items 3 and 4 are different from the percentages of the two paths in the dimension, even though I1 and I2 have the same answers. This proves that some students who answer the first pair of items correctly do not answer the second pair correctly as well.

Table III shows the results of the dimension "Proportional reasoning". The table presents the paths of individual pairs of items (items 5 and 6, and items 7 and 8) and the paths of the dimension. For both pairs, we present paths with a percentage greater than 4.0%, since that is the probability of answering the pair of items correctly if done randomly.. The table also shows the paths for the whole dimension. In this case, we present paths with a percentage greater than 2%.

Table III
Proportional reasoning

I5	I6	Path	%
A	B	495	21.5%
B	C	1262	54.7%
B	D	162	7.0%

I7	I8	Path	%
A	A	154	6.7%
A	E	187	8.2%
B	A	179	7.8%
B	D	298	13.0%
D	A	590	25.8%
D	E	446	19.5%

I5	I6	I7	I8	Path	%
A	B	B	A	138	6.0%
A	B	B	D	268	11.7%
B	C	A	A	90	3.9%
B	C	A	E	145	6.3%
B	C	D	A	447	19.5%
B	C	D	E	395	17.2%
B	D	D	A	86	3.7%

The result of the pair including items 5 and 6 contains three paths: the correct answer pair (54.7%) and two other paths. Note that one of them has the right answer in item 5 (option B) but with an incorrect reasoning (option D); 7.0% of students chose this path. The other path starts with an incorrect answer (option A) and incorrect reasoning (option B); 21.5% of students chose this path.

The result of the pair including items 7 and 8 contains six paths, the correct answer pair (25.8%) and five other paths. One of the incorrect paths has the right answer in item 7 (option D) but with an incorrect reasoning (option E); 19.5% of students chose that path. Two incorrect paths chose option A for item 7, with one of the paths choosing the correct reasoning (option A), with 6.7% of students choosing that path, and the other answer for the reasoning choosing option E, with 8.2% of students choosing that path. The last two incorrect paths starts with the incorrect answer, option B, with one of them following with the right reasoning (7.8% of students) and the other with option D (13.0% of students).

In the “Proportional reasoning” dimension we found seven paths, in which a significant number of students chose those choices. 19.5% of students chose the right path in the dimension. Three more paths had the right answer for the first pair but an incorrect path for the second pair. One of the paths was chosen with an incorrect path for the first pair and a correct path for the second pair. The remaining two paths have incorrect paths for both pairs.

Table IV shows the results for the dimension “Control of variables”. This is a special dimension since it has three pairs instead of two. The table presents the paths of individual pairs of items (items 9 and 10, items 11 and 12 and items 13 and 14) and the paths of the dimension. For the first pair (items 9 and 11), we present paths with a percentage greater than 4.0%, since that is the probability of answering the pair of items correctly if done randomly. For the second and third pairs (items 11 and 12, and items 13 and 14), we present paths with a percentage greater than 5.0%, since that is the probability of answering the pair of items correctly if done randomly. The table also shows the paths for the whole dimension. In this case, we present paths with a percentage greater than 2%.

Table IV
Control of variables

I9	I10	Path	%
B	D	406	17.6%
E	C	1259	54.6%

I11	I12	Path	%
A	B	165	7.4%
A	D	383	17.2%
B	A	305	13.7%
B	B	339	15.2%
C	D	332	14.9%

I13	I14	Path	%
A	D	406	17.8%
A	E	189	8.3%
C	B	170	7.5%
C	D	611	26.8%
C	E	191	8.4%

I9	I10	I11	I12	I13	I14	Path	%
E	C	A	D	A	D	113	5.0%
E	C	A	D	A	E	49	2.2%
E	C	B	A	C	D	134	5.9%
E	C	B	B	C	D	138	6.1%
E	C	C	D	C	D	81	3.6%

The result of the pair including items 9 and 10 contains two paths, the correct answer pair (54.6%) and another path. The incorrect path is an incorrect answer of item 9 (option B) and an incorrect reasoning in item 10 (option D); 17.6% of students chose this incorrect path.

The result of the pair including items 11 and 12 contains five paths, the correct answer pair (13.7%) and four other paths. One of the incorrect paths has the right answer in item 11 (option B) but with an incorrect reasoning (option B); 15.2% of students chose that path. Two incorrect paths chose option A for item 11 with an incorrect reasoning in item 12 (option B), with 7.4% of students choosing that path. Another incorrect path had option A as the answer in item 11, but with option D as answer of item 12 with 17.2% of students choosing that path. The last path (14.9% of students chose it) is with option C for item 11 and option D for item 12. Note that three out of four incorrect paths were more attractive to students than the correct path.

The result of the pair including items 13 and 14 contains five paths: the correct answer pair (26.8%) and four other paths. Two of the incorrect paths have the right answer in item 13 (option C) but with an incorrect reasoning (option B and option E); 7.5% and 8.4% of students, respectively, chose those paths. Two incorrect paths have option A for item 13 with incorrect reasoning in item 14 (option D and option E), with 17.8% and 8.3% of students, respectively, choosing those paths.

In the "Control of variables" dimension we found five paths in which a significant number of students chose those choices. Only 5.9% of students chose the right path in the dimension. All the incorrect paths chose the right answers for the pair including items 9 and 10. None of the incorrect paths chose the right answers for the pair including items 11 and 12. Two of the incorrect paths chose the right answers in the pair including items 13 and 14.

Table V shows the results of the dimension "Probabilistic reasoning". The table presents the paths of individual pairs of items (items 15 and 16 and items 17 and 18) and the paths of the dimension. For the two pairs (items 15 and 16, and items 17 and 18), we present paths with a percentage greater than 4.0%, since that is the probability of answering the pair of items correctly if done randomly. The table also shows the paths for the whole dimension. In this case, we present paths with a percentage greater than 2%.

Table V
Probabilistic reasoning

I15	I16	Path	%
B	A	609	26.5%
C	A	1319	57.3%

I17	I18	Path	%
A	C	141	6.3%
B	E	1276	57.2%
C	D	242	10.9%

P15	P16	P17	P18	Path	%
B	A	B	E	252	11.1%
B	A	C	D	74	3.3%
C	A	A	C	63	2.8%
C	A	B	E	937	41.4%
C	A	C	D	49	2.2%

The result of the pair including items 15 and 16 contains two paths: the correct answer pair (57.3%) and one other path. The incorrect path is an incorrect answer of item 15 (option B) and the correct reasoning in item 16 (option A); 26.5% of students chose this incorrect path.

The result of the pair including items 17 and 18 contains three paths: the correct answer pair (57.2%) and two other paths. The other two paths have an incorrect answer for item 17 and an incorrect reasoning for item 18. 6.3% of students chose a path consisting of option A for item 17 and option C for item 18. 10.9% of students chose a path consisting of option C for item 17 and option D for item 18.

In the “Probabilistic reasoning” dimension we found five paths in which a significant number of students chose those choices. 41.4% of students chose the right path in the dimension. Two incorrect paths have the correct answers for the pair including items 15 and 16, and an incorrect path for the pair including items 17 and 18 (2.8% and 2.2%). An incorrect path has incorrect answers for the pair including items 15 and 16, and correct answers for the pair including items 17 and 18 (11.1%). The last incorrect path had incorrect answers for the two pairs (3.3%).

Table VI shows the results of the dimension “Correlational reasoning”. This is a special dimension since it consists only on one item. The table presents the paths of the individual pair of items 19 and 20. For this pair, we present paths with a percentage greater than 6.7% since that is the probability to answer the pair of items right if done randomly.

Table VI

Correlational reasoning

P19	P20	Path	%
A	B	269	12.1%
A	D	1225	55.1%
B	A	191	8.6%
C	A	188	8.4%

The result of the pair including items 19 and 20 contains four paths: the correct answer pair (55.1%) and three other paths. One of the paths has the correct answer for item 19 and an incorrect reasoning for item 20; 12.1% of students chose that path. The other two incorrect paths have different incorrect answers for item 19 (options B and C) but the same incorrect reasoning (option A), with 8.6% and 8.4% of students choosing these paths, respectively.

Table VII shows the results of the dimension “Hypothetic-deductive reasoning”. The table presents the paths of individual pairs of items (items 21 and 22 and items 23 and 24) and the paths of the dimension. For the first pair (items 21 and 22), we present paths with a percentage greater than 6.7%, since that is the probability of answering the pair of items correctly if done randomly. For the second pair (items 23 and 24), we present paths with a percentage greater than 5.0%, since that is the probability of answering the pair of items correctly if done randomly. The table also shows the paths for the whole dimension. In this case, we present paths with a percentage greater than 2%.

Table VII

Hypothetic-deductive reasoning

I21	I22	Path	%
A	A	278	13.1%
A	B	192	9.0%
B	A	288	13.5%
B	B	217	10.2%
C	A	120	5.6%
D	C	127	6.0%
E	A	253	11.9%
E	B	236	11.1%

I23	I24	Path	%
A	B	488	24.1%
B	B	461	22.8%

P21	P22	P23	P24	Path	%
A	A	A	B	108	5.2%
A	A	B	B	62	3.0%
A	B	A	B	53	2.6%
B	A	B	B	60	2.9%
B	B	A	B	49	2.4%
B	B	B	B	51	2.5%
E	A	B	B	59	2.8%
E	B	A	B	61	2.9%
E	B	B	B	64	3.1%

The result of the pair including items 21 and 22 contains eight paths: the correct answer pair (with 13.1%) and seven other paths. There is an incorrect path with the correct answer for item 21 (option A) but with an incorrect reasoning for item 22 (option B), with 9.0% of students choosing that path. There are two paths with option B as the answer for item 21 and two different reasoning (option A, the correct reasoning, and option B), with 13.5% and 10.2% of students choosing the paths, respectively. There are two paths with option E as the answer for item 21 and two different reasoning (option A, the correct reasoning, and option B), with 11.9% and 11.1% of students choosing the paths, respectively. Another path has option C for the answer for item 21 and the correct reasoning for item 22, with 5.6% of students choosing that path. The last incorrect path consists of option C for the answer for item 21 and option C as an answer for item 22, with 6.0% of students choosing that path.

The result of the pair including items 23 and 24 contains only two paths: the correct answer pair (24.1%) and another path. That incorrect path has an incorrect answer for item 23 and the correct reasoning for item 24, with 22.8% of students choosing that path.

In the "Hypothetic-deductive reasoning" dimension we found nine paths in which a significant number of students chose those choices. Only 5.2% of students chose the right path in the dimension. One incorrect path has the correct answers for the pair including item 21 and 22 and an incorrect path for the pair including items 23 and 24 (3.0%). Three incorrect paths chose incorrect answers for the pair including items 21 and 22 and correct answers for the pair including items 23 and 24 (2.6%, 2.4% and 2.9%). The remaining four incorrect paths have the feature that for the pair including items 23 and 24 the answers are option B and option B, respectively. That combination is the only significant incorrect path in the pair including items 23 and 24.

Discussion

As previously mentioned, we will focus on the discussion of only two dimensions in this study.

Conservation of matter and volume

Figure 3 shows items 1 and 2 of conservation of matter. 94.2% of students answered the right pair, which is that the two pieces weigh the same, clay is not added or taken away. The rest of the answers correspond to small percentages smaller than the probability of answering the pair of items correctly if done randomly. This pair of items is the easiest one for these students.

1. Suppose you are given two clay balls of equal size and shape. The two clay balls also weigh the same. One ball is flattened into a pancake-shaped piece. *Which of these statements is correct?*

   a. The pancake-shaped piece weighs more than the ball
   b. The two pieces still weigh the same
   c. The ball weighs more than the pancake-shaped piece

2. *because*

   a. the flattened piece covers a larger area.
   b. the ball pushes down more on one spot.
   c. when something is flattened it loses weight.
   d. clay has not been added or taken away.
   e. when something is flattened it gains weight.

Figure 3. Items 1 and 2 from the Lawson Test. This pair of items inquires about the conservation of matter.

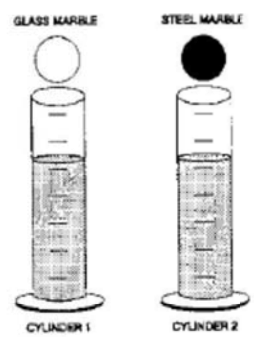
Figure 4 presents items 3 and 4 of conservation of volume. 75.8% of students answered this pair correctly by choosing that the water level will rise to the same level because the marbles are of the same size. In this pair, a significant number of students chose a second path. 15% of students answered that the steel marble will make the water level rise higher because the steel marble is heavier than the glass marble. Those students might believe that heavier objects occupy more volume.

3. To the right are drawings of two cylinders filled to the same level with water. The cylinders are identical in size and shape.

   Also shown at the right are two marbles, one glass and one steel. The marbles are the same size but the steel one is much heavier than the glass one.

   When the glass marble is put into Cylinder 1 it sinks to the bottom and the water level rises to the 6th mark. *If we put the steel marble into Cylinder 2, the water will rise*

   a. to the same level as it did in Cylinder 1
   b. to a higher level than it did in Cylinder 1
   c. to a lower level than it did in Cylinder 1



4. *because*

   a. the steel marble will sink faster.
   b. the marbles are made of different materials.
   c. the steel marble is heavier than the glass marble.
   d. the glass marble creates less pressure.
   e. the marbles are the same size.

Figure 4. Items 3 and 4 from the Lawson Test. This pair of items inquires about the conservation of volume.

In the “Conservation of matter and volume” dimension, there are two paths with a significant number of students. 72.2% of students correctly answered the conservation of matter item and the conservation of volume item, that is, the right path. However, there are 13.4% of students who answered the conservation of matter correctly and the conservation of volume incorrectly. We can also observe that almost all students who answered the conservation of volume with the path composed of options B and C correctly answered the pair of conservation of matter.

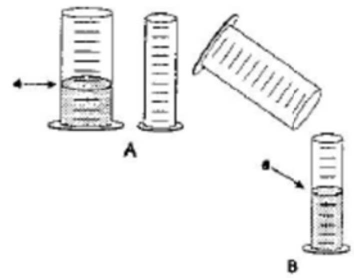
Proportional reasoning

Figure 5 presents items 5 and 6, which is the first pair of the “Proportional reasoning” dimension. There are three significant paths: the correct path and two incorrect paths. 54.7% of students answered that the mark in the narrow cylinder will be about 9, since it goes up 3 in the narrow cylinder for every 2 in the wide. This is a typical question and reasoning of proportional quantities.

21.5% of students chose an incorrect path, in which they picked the mark in the narrow cylinder that will be about 8 because it went up 2 more before, so it will go up 2 more again. Even though the answer and reasoning are incorrect, they are consistent, that is, the students are really thinking that no matter how much water is poured into the wide cylinder, when that water is poured into the narrow cylinder the water level will rise 2 more marks.

There is still another incorrect path. 7.0% of students answered item 5 correctly (the water level will rise to about the mark 9); however, the reason they chose it was because the second cylinder is narrower. This path is not entirely incorrect in the sense that students choosing option D in item 6 are mentioning something that is correct but it does not answer the question accurately, and they might be trying to avoid a more sophisticated reasoning.

5. To the right are drawings of a wide and a narrow cylinder. The cylinders have equally spaced marks on them. Water is poured into the wide cylinder up to the 4th mark (see A). This water rises to the 6th mark when poured into the narrow cylinder (see B).



Both cylinders are emptied (not shown) and water is poured into the wide cylinder up to the 6th mark. *How high would this water rise if it were poured into the empty narrow cylinder?*

a. to about 8
b. to about 9
c. to about 10
d. to about 12
e. none of these answers is correct

6. *because*

a. the answer can not be determined with the information given.
b. it went up 2 more before, so it will go up 2 more again.
c. it goes up 3 in the narrow for every 2 in the wide.
d. the second cylinder is narrower.
e. one must actually pour the water and observe to find out.

Figure 5. Items 5 and 6 from the Lawson Test. This pair of items inquires about proportional reasoning

Figure 6 presents items 7 and 8, the second pair of the "Proportional reasoning" dimension. This pair of items inquires for a more advanced proportional reasoning, in which the proportion is not calculated with integer numbers. The eleventh mark in the narrow cylinder has three times three marks plus two more marks. The proportion of water between the wide and the narrow cylinders is the same, for each three marks in the narrow cylinder, there are two marks in the wide cylinder, then nine marks in the narrow, will be 6 marks in the wide cylinder. However, there are still two more marks in the narrow cylinder. Keeping the proportion, then two marks in the narrow cylinder correspond to 4/3 marks in the wide cylinder, which is one mark and 1/3 of a mark, making 7 1/3 marks in total. Looking at the reasoning options, the students have to choose option A, that the ratios must stay the same. 25.8% of students did this reasoning or something similar. There are five incorrect paths. 19.5% of students chose the correct answer for item 7 (option D) but an incorrect reasoning, option E. Choosing that reasoning might be due to an effect that the correct reasoning is an open statement (the ratios must be the same). Although it is correct, there must be students who might have expected another kind of reasoning, i.e. mathematical reasoning. Item E has that effect. That answer includes the numbers involved: for each three marks in the narrow cylinder, there are two marks in the wide cylinder. However, it is not a subtraction.

7. Water is now poured into the narrow cylinder (described in Item 5 above) up to the 11th mark. *How high would this water rise if it were poured into the empty wide cylinder?*

a. to about 7 1/2
b. to about 9
c. to about 8
d. to about 7 1/3
e. none of these answers is correct

8. *because*

a. the ratios must stay the same.
b. one must actually pour the water and observe to find out.
c. the answer can not be determined with the information given.
d. it was 2 less before so it will be 2 less again.
e. you subtract 2 from the wide for every 3 from the narrow.

Figure 6. Items 7 and 8 from the Lawson Test. This pair of items inquires about advanced proportional reasoning

There are two paths, in which the answer for item 7 is B but the reasonings are option A (7.8%) and option D (13.0%). Option B for item 7 is: “to about 9”. To obtain that answer, students could have subtracted 2 marks from eleven marks in the narrow cylinder. That is, the expected reasoning for option B in item 7 is option D in item 8. About 65% of students who chose option B in item 7 chose item D in item 8. The rest chose option A (the ratios must be the same). We would argue that those students, indeed, subtracted two from eleven but answered with a general correct answer, that the ratios must be the same, even if they do not understand ratios.

The last two paths are from students who chose option A in item 7. From those, 6.7% chose option A in item 8 and 8.2% chose option E in item 8. Option A in item 7 is a sophisticated answer, just like the correct answer. We would argue that those students tried to get the answer with an adequate proportional reasoning but failed with their calculations. An evidence of that claim is that the two reasoning in those two paths are precisely the two reasoning for those who obtained the right answer in item 7.

The third part of Table III presents the reasoning paths for these two pairs in proportional reasoning. Only 19.5% of students answered both pairs correctly. The great majority of students who answered items 7 and 8 correctly answered items 5 and 6 correctly (75.8%). There were 86 students (14.6% from the ones who have items 7 and 8 correct) who went with the answers option B in item 5 and option D in item 6, which strictly speaking is not incorrect.

Another interesting path is BCDE for items 5, 6, 7 and 8, respectively. Those students answered with the path of option D in item 7 and option E in item 8. They also answered items 5 and 6 correctly. We said before that those students might have expected another correct answer. Observe that 88.6% of students who chose to answer items 7 and 8 in this way, answered items 5 and 6 correctly.

The other two paths for students who answered items 5 and 6 correctly were students who chose option A for item 7. Those students, we mentioned, did complicated calculations but failed to get the correct answer.

The last two paths are interesting to analyze. These students answered option A in item 5 and option B in item 7. The reasoning (option B for item 6 and option D in item 8) are similar: a low level of proportional reasoning. However, they are very consistent: observe that 82.0% of students who answered option A in item 5 and option B in item 6, are in these two paths. These students are consistently employing a proportional reasoning in terms of maintaining the ratios quantities but adding and subtracting from a fixed quantity, missing that the initial quantity is also part of the ratio.

Conclusions

Students' reasoning can be measured in different ways. We decided to use the Lawson test, which is one of the most commonly test used in Physics Education Research. The test is designed to prove the reasoning skills of students and the results could tell us in which level of reasoning our students are. It is common that the majority of entering university students or even first year university students are in the transitional level or reasoning, with a relatively low percentage of them in the formal level.

In this contribution we analyzed the answers of 2,321 students taking introductory classes in a large private university in Mexico, with the sole objective of analyzing students' paths in their reasoning. We presented the results for all dimensions but discuss only two of them.

What we found is that students are consistent in their answers. Some of them follow the right path in the dimensions (correct answers and correct reasoning), but others follow paths that are not correct but consistent, which is interesting to take into account for instruction.

In the dimension of proportional reasoning, there were three groups of students. The first group of students answered the first pair (a simple question) correctly, but when asked with a more sophisticated question (items 5 and 6), they started to make mistakes. However, the great majority made an effort and still made use of proportional thinking to answer the questions. The second group of students answered the first item of the first pair (item 5) correctly but their reasoning is a general one, with no intention of putting an effort. Over half of those students, when faced with the second pair, answered that pair correctly, which is evidence that those students have a robust structure of conceptual understanding of proportional reasoning. The third pair is the group of students who have a vague structure of conceptual understanding of proportional reasoning. These students answered the questions taking into account the proportions but adding and subtracting from a fixed quantity, instead of taking the whole quantity as part of the ratio.

One interesting finding from this work is that we have evidence to argue that item 8 (reasoning for the second pair in proportional reasoning) could be misleading students. Having a correct reasoning "the ratios must stay the same" after solving a sophisticated item that involves fractions (item 7) could lead students to answer another close reasoning. This pair is opposite to the first pair (items 5 and 6) in which the calculation is rather easy and the reasoning is sophisticated: "it goes up 3 in the narrow for every 2 in the wide". There is another choice in the reasoning item that is simple but not incorrect: "the second cylinder is narrower." We believe that these items could be revised.

The results of this work could be used by instructors and researchers. Reasoning skills are important for engineering students and we have to take into account that some of our students are starting their courses with low abilities even in proportional reasoning. We can create activities to further develop those abilities. Researchers, on the other hand, can use this work to investigate students' reasoning paths and/or to improve the Lawson test itself.

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