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Analysis of STEM Students Accumulating Calculus Knowledge to Graph a Function

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Abstract.

It is important for Science-Technology-Engineering-Mathematics (STEM) educators to find out about STEM students' success in answering calculus questions, particularly the questions that involve more than one calculus concepts that require to know other calculus concepts. Designing appropriate questions for assignments and exams that involve calculus concepts are critical in measuring student success. Effectiveness analysis of the method used for designing such questions is also important. Efforts have been made in understanding and improving engineering students' ability to respond calculus questions in (STEM) fields that require knowledge of more than one calculus concept [1-11] and more research results are added every year to these results for understanding students' approach to solve these problems. In this work, 26 undergraduate engineering students' written and oral responses to a calculus question that involves multiple calculus concepts are recorded after Institutional Review Board (IRB) approval. Triangulation method [1] and Action-Process-Object-Schema (APOS) theory [10] are used for analysis of the collected data. The students are tested on their capability to use sub-concepts as building blocks to answer the question completely and correctly. APOS classification resulted in most of the participants Intra and Inter level classification. The Triangulation method appeared as a strong method that can be applied for analysis of the participants' calculus knowledge as it was observed in [1].

1. Introduction

Observing engineering students' success in responding to calculus questions, particularly investigating the details on their conceptual understanding that involve multiple calculus concepts that require knowledge of other calculus concepts has been an interest of engineering educators. Pedagogical efforts have been made in understanding and improving engineering students' ability to respond to calculus questions in Science-Technology-Engineering-Mathematics (STEM) fields that require knowledge of more than one calculus concept [1-11] and more research results are added every year to these results for understanding students' approach to solve these problems. New question evaluation methods have been proposed in [1] and development of new teaching styles are recommended to educators to serve STEM students better by using these results. These results build on empirical data that are likely to be the key to measuring university students' success in answering conceptual calculus questions with multiple underlying calculus concepts. For instance, sketching the graph of a function requires conceptual knowledge of first and second derivatives along with limit calculations, horizontal and vertical asymptotes, and the ability to apply all these conceptual responses to be able to correctly graph the function.

APOS theory is introduced in [12] to extend the work of Piaget [13]. APOS theory is used by researchers to explain students' combined knowledge of a specific mathematical topic. It is used to observe the conceptual construction of students on sub-concepts and schemas [1-4]. The theory analyzes students' ability to build on prior existing knowledge. APOS theory cannot always be used for data analysis of pedagogical research [14].

APOS theory can be appropriately applied to the collected research data due to the involvement of certain mathematical concepts such as limits, derivatives, and asymptotes. The participants of this research are expected to use multiple calculus concepts to correctly sketch a graph by integrating calculus sub-concept knowledge.

Triangulation methodology is introduced in [1] and it is used for analysis of a data set based on fill-in-the-blank questions that summarize the research participants' responses to all questions on a single spreadsheet. The data is organized in a way to contain questions and participant ID numbers with the output summarized. The participant responses during the analysis of the Triangulation method are redesigned in a way to summarize all responses in a triangle structure within the matrix representation: The correct responses are organized by clustering them in a triangle structure within the matrix representation of the output and the percentage of correct responses to the questions are calculated within this triangle form. This percentage represents the strength of the triangulation clustering of the participant classification [1]. Similar to the work conducted in [1] and [2], we investigate the STEM students' responses to a graphing question that requires limit, vertical and horizontal asymptotes, first and second differentiation, and vertical and horizontal axes knowledge to be explained in the next section along with the details on participant information.

Next section is devoted to the research methodology and the data collection details. What follows is the qualitative and quantitative data analysis using the APOS theory. Qualitative and quantitative analysis is transformed into Triangulation of the participant responses in Section 4. Section 5 is reserved for concluding remarks and future work.

2. Research Methodology & Data Collection

Institutional Review Board (IRB) approval was attained, and the following protocol was followed to collect the data analyzed and presented in this work at a university located on the Northeastern side of the United States. The participants were 26 STEM undergraduate volunteers from a variety of disciplines and backgrounds. The data was collected over a two-year span and included oral and written responses of the research participants. Each participant was compensated for both written data collected to the questionnaire and the video-recorded oral interviews. The quantitative data analysis was based on the written responses of participants while qualitative data analysis was based on the transcription of the participants' video recorded follow-up interviews; the purpose of the follow-up interviews was to explore the depth of students' conceptual knowledge on the research question. Action-Process-Object-Schema (APOS) theory and Triangulation method developed in [1] were used to analyze the participants' responses to the following calculus research question that has multiple parts requiring the conceptual knowledge of first and second derivatives, limit calculations, and horizontal and vertical asymptotes.

2. Please draw a graph of a function that verifies all of the given information below. Write the necessary values on the coordinate axis and explain the details if you think they are necessary.

$$
\begin{aligned}
\lim_{x\to-\infty} f(x) &= 0,\ \lim_{x\to\infty} f(x) = 0,\\
\lim_{x\to-3^-} f(x) &= -\infty,\ \lim_{x\to 2^+} f(x) = \infty,\\
\textit{Vertical Asymptotes at } x &= -3 \textit{ and } x = 2,\\
\textit{Horizontal Asymptote at } x &= 0\\
f'(-2) &< 0, f'(1) < 0,\\
f''(x) &< 0 \text{ when } x < -3,\\
f''(x) &> 0 \text{ when } x > 2,\\
f''(c) &= 0 \text{ for a } x = c \text{ such that } -1 < c < 1
\end{aligned}
$$

The participants were required to complete the second course in a 3-course calculus sequence with each course consisting of 4 credits. Due to the IRB protocol, each participant was assigned an "RP" number and their names were hidden. Each student was individually called for responding to the written questionnaire with a follow-up interview scheduled to explain the written responses and allowed to edit the written information if they realized a need for it for discussion purposes.

3. APOS Theory Application - Qualitative and Quantitative Results

Action-Process-Object-Schema (called APOS) theory is applied to mathematical topics (mostly functions) in [15], and they explained this theory as the combined knowledge of a student in a specific subject based on Piaget's philosophy [16]. The theoretical method in this work utilizes APOS theory along with the Triangulation method for analysis of collected responses of participants designed for measuring success per participant per question. The quantitative analysis of the research question consisted of probabilistic results as well as the correlation analysis of the correct responses attained for all parts of the question.

Scheme idea of Piaget in the 1970's, and its development by Piaget and Garcia in the 1980's, influenced researchers of undergraduate mathematics education curriculum in the 1990's. Conceptual view of the function is defined in [17] that formed the action-process-object idea in mathematics education for the undergraduate curriculum. Action, process, object, and schema theory (called APOS theory) is applied to mathematical topics (mostly functions) by Asiala et. al in [15], and explained this theory as the combined knowledge of a student in a specific subject based on Piaget's philosophy. The categories of APOS theory can be briefly described as below [12].

- *An action is a transformation of objects perceived by the as essentially external and as requiring, either individual explicitly or from memory, step-by-step instructions on how to perform the operation...*
- *The individual reflects upon an action when the action is repeated, and he or she can make an internal mental construction called a process by which the individual can think of as performing the same kind of action without an external support...*
- *An object is results from individual's awareness of the process' totality and realizes that transformations can act on it...*
- *A schema is a linkage of collected actions, processes, objects, and other schemas that help to form a framework by using general principles in individual's mind...*

Every concept can be constructed on different concepts and schemas in the APOS theory. We can also say that every concept requires concept knowledge, and the construction of a specific concept depends on knowledge of the other concepts. APOS theory observed to be inapplicable in [14] to analyze the data. The Triad stages Intra, Inter, and Trans are introduced in [18] and used in [19]. In [20], work of [19] is used by focusing on the thematization of the schema with the intent to expose those possible structures acquired at the most sophisticated stages of schema development. In their study, the problems were structured in a way that participants were required to respond to the first eight questions and continue with the ninth question only if they succeeded in answering the first eight questions (please see [20] and the appendix, pg. 391 for further details). The detailed analysis of the collected data indicated participants' success in answering a complex

graphing problem, thus schema thematization was possible in their study.
In the last decade, APOS theory is used in several educational research areas. It is used to lead the students towards constructing the vector space concept in [21], to observe mean, standard deviation, and the central limit theorem knowledge of successful students who completed an elementary statistics course with a grade of "A" in [22], and to observe students' obstacles in the learning of two variable functions in calculus in [23].

Data is collected in [24] by observing a student and authors concluded that incorrectly created derivative images can result in mistakes of analytical reasoning of the student. Given the graph of a function, participating students' difficulty in sketching the derivative graph of the given function is observed in [25] noting that many students first tried to find an algebraic representation of the given function. Senior mathematics undergraduate and graduate students' weak rate of change concept knowledge observed to result in weak understanding of the integration concept in [26].
Students' ability to construct and develop two-variable functions by using APOS theory is observed in [23] and [27]. It is concluded in [23] that in two-variable calculus settings, students had difficulty in domain, range, and the graphs of two-variable functions. For a comprehensive coverage of the APOS theory we refer to [28].
A Scheme is an action which is repeated and can be generalized where the actions are derived from sensory-motor intelligence [29]. The coordination of schemes forms actions which are logical structures. Combination of systems and schemes can form the scheme. The similarity between the schemes in a larger combination of schemes is similar to the set inclusion in mathematics where subsets form the set. The concept knowledge can be formed in a larger combination of schemes.

The schema classification in [19] is based on the following triad classification:

- **Intra-Interval:** Ability to answer questions regarding the independent intervals where the participant can be confused by the union or intersection of other intervals.
- **Inter-Interval:** Ability to answer questions regarding only sub-domains which consists of two or more intervals but not the entire domain.
- **Trans-Interval:** Ability to answer questions regarding the entire domain.
- **Intra-Property:** Ability to interpret every analytical property independently one at a time.
- **Inter-Property:** Ability to interpret two or more analytical properties simultaneously but not all of them together.
- **Trans-Property:** Ability to interpret all the analytical properties simultaneously.

The schema classification in this work is structured by observing post-interview student responses. The data collected in this study suggested following a similar theoretical triad classification to that in [2]. The design of the question and detailed analysis of the post-interview student responses suggested a three-level triad classification:

- **Intra-level:** Responses reflected only elementary level of sub-conceptual knowledge with mistakes made in two or more analytical properties on two or more intervals. This level of students couldn't demonstrate correlated calculus conceptual knowledge indicating that they cannot apply two or more calculus sub-concepts simultaneously.
- **Inter-level:** Participants were able to apply one or two calculus sub-concepts correctly on several places but not at all places. The responses in this category indicate application mistakes or not ability to respond to the question due to the lack of conceptual knowledge, possibly for one or more analytical properties on a certain interval.
- **Trans-level:** The participants in this category made no mistakes in the application of the analytical properties throughout the entire domain of the question.

For example, a participant is categorized into the intra-level if the second derivative and the asymptote information are not applied correctly on two or more intervals. This is a result of participant's confusion by the union or intersection of other intervals and the failure to interpret every analytical property independently one at a time. If there is only one analytical property application mistake, such as the first derivative information on a certain interval that cannot consist of the union of independent intervals, then the response is categorized as inter-level. Students' trans-level triad classification is based on their ability to answer the question correctly in the entire domain.

Table 1 below displays the intra, inter, and trans level classifications of the participants using the above-mentioned descriptions. 47.83% of the participants fell into the intra level with at least one participant making any one of the concepts covered in Table 1. Inter level has 30.43% of the participants with first and second derivatives to be the only misconceptions seen in the applications. Trans level has 21.74% of the students with second derivative displaying the major challenge to the participants with only one participant having difficulty in one of the first derivative applications.

Research Question	Percentage	Missing sub-concept knowledge
Intra-Level	47.83	All Table 1 concepts
Inter-Level	30.43	First and Second Derivatives
Trans-Level	21.74	Second Derivatives

Table 1. Intra, inter, and trans level classification of the participants with missing sub-concepts.

Response of one of the Inter-level participants is displayed in Figure 1 below. This participant attempted to make changes and edit the response (marked in red) during the video-recorded interview, however they could not attain the expected answer.

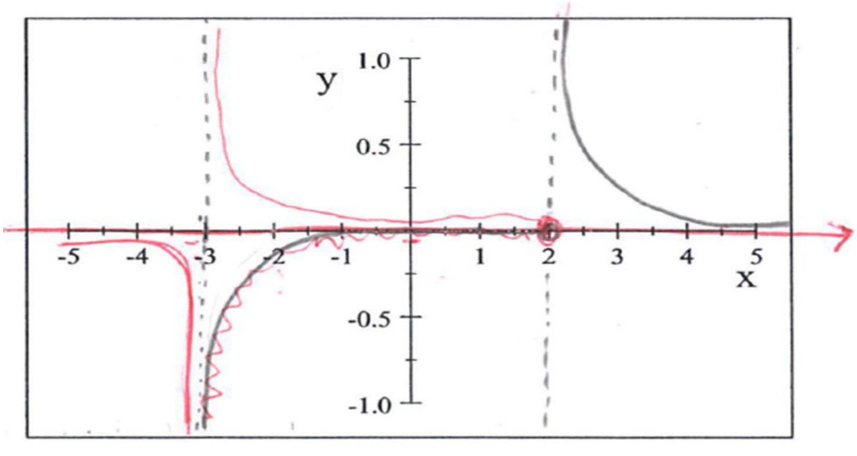

Figure 1. A response of one of the Inter-level participants to the research question.

Response of one of the Intra level participants is displayed in Figure 2 below. Unlike the majority of the other responses, this graph did not include vertical and horizontal asymptotes.

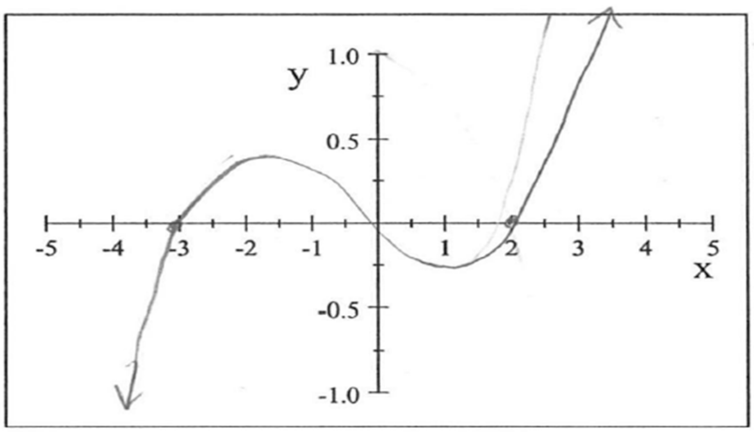

Figure 2. One of the Intra-level participant responses that excluded VA and HA.

4. Triangulation Method Application - Qualitative and Quantitative Results

Triangulation methodology introduced in [1] can be useful for organizing student responses to a set of questions in a way that can help to detect students' major misconceptions or incomplete knowledge. It can be helpful to the educators and researchers to quantitatively measure students understanding of multiple mathematical concepts. Analysis of participants' construction of conceptual knowledge starting with basic pre-calculus concepts and advancing to the use of several calculus concepts such as limits, first derivatives, and second derivatives simultaneously can be complicated. Table 2 below displays a reflection of research participants' ability to respond to a variety of questions in the order of participants acceptance to the study and random placement of the concepts covered; In this table HA stands for Horizontal Asymptote, VA stands for vertical asymptote, Diff stands for differential, H. stands for horizontal, and V. stands for vertical. Table 2 is redesigned into Table 3 for displaying triangulation. Table 3 is only organized into "+" and "-" signs as the conceptual understanding of research participants. A plus sign indicates that the student was able to respond to the concept related question at its entirety in all locations it applies by comprehending the need for transforming his/her knowledge into the corresponding location on the curve. For instance, there are two first derivative related information given therefore a participant would be getting a "+" sign if he/she got it right for both concepts. Similarly, a "-" sign is complementary to the "+" indicating the participant could not answer the concept related questions correctly in one or more locations. This "all or none" correct response approach works well for evaluation of the research question by using Triangulation noting that a mistake made on the graph can cause the graph to deform locally and be incorrect. For instance, in a location where both second derivative and HA information need to be applied correctly, misconception of second derivative can deform the graph even if the participant had the conceptual understanding of HA.

	HA	VA	1st Diff	2nd Diff	H. Axis Knowledge	V. Axis Knowledge
RP 1	+	+	-	+	+	-
RP 2	+	+	+	-	+	+
RP 3	-	-	-	+	+	-
RP 4	-	+	+	-	+	+
RP 5	-	-	-	-	-	-
RP 6	+	+	-	+	+	+
RP 7	-	+	-	+	+	+
RP 8	+	+	+	+	+	+
RP 9	+	-	-	-	-	-
RP 10	+	+	+	+	+	+
RP 11	-	+	-	-	-	-
RP 12	+	+	-	+	+	+
RP 13	+	+	-	+	+	+
RP 14	-	-	-	+	-	-
RP 15	-	-	-	-	-	-
RP 16	-	-	-	+	-	-
RP 17	-	-	-	-	-	-
RP 18	+	+	-	+	+	+
RP 19	+	+	-	+	+	+
RP 20	-	+	-	-	-	-
RP 21	+	+	-	+	+	+
RP 22	+	+	-	+	+	+
RP 23	+	+	-	+	+	+
RP 24	+	+	-	+	+	+
RP 25	+	+	-	+	+	+

Table 2. Graph drawing analysis of research participants to the research question.

Sign formation of Triangulation displayed in Table 3 is based on the attempt to place plus signs with the maximum occurrence from left to right column to form a triangle structure as much as possible. In this table 95.35% of the plus signs fall in the formed triangulation. The weakest conceptual knowledge of the participants occurred for the first derivative questions while the same occurred for the second derivative knowledge of the participants in [1].

	VA	2nd Diff	H. Axis Knowledge	V. Axis Knowledge	HA	1st Diff
RP 10	+	+	+	+	+	+
RP 8	+	+	+	+	+	+
RP 2	+	+	+	+	+	+
RP 12	+	+	+	+	+	-
RP 13	+	+	+	+	+	-
RP 6	+	+	+	+	+	-
RP 18	+	+	+	+	+	-
RP 19	+	+	+	+	+	-
RP 21	+	+	+	+	+	-
RP 22	+	+	+	+	+	-
RP 23	+	+	+	+	+	-
RP 24	+	+	+	+	+	-
RP 25	+	+	+	+	+	-
RP 7	+	+	+	+	-	-
RP 4	+	-	+	+	-	-
RP 1	+	+	+	-	+	-
RP 3	-	+	+	-	-	-
RP 14	-	+	-	-	-	-
RP 20	+	-	-	-	-	-
RP 9	-	-	-	-	+	-
RP 11	+	-	-	-	-	-
RP 16	-	+	-	-	-	-
RP 15	-	-	-	-	-	-
RP 17	-	-	-	-	-	-
RP 5	-	-	-	-	-	-

Table 3. Triangulation of the information provided in Table 2 by using the graph drawing responses.

Table 4 below displays the number of participants that had "+" signs in the Triangulation and the corresponding percentage within the total number of "+" occurrences. Triangulation can also be a very helpful grading tool to organize questionnaires for exams and assignments. In particular, if web-based calculus questions are designated to be taken several times by students due to low grades, this methodology can be useful in measuring the conceptual strength of the participants and actions can be taken by the web-based system (or the instructor) as a balancing action to recover the major misconceptions occurring. Over time, the question responses can be collected for further analysis based on participant success and the exams can be designed to include different levels of questions. For the research question used in this work, the information given for the interval (-3,2) occurred to cause much more challenge to the students to be able to respond when compared to the rest of the information given.

Group Classification	Number of Participants with "+"	Percentage (%)
VA	18	20.93%
2nd Diff	18	20.93%
H. Axis Knowledge	17	19.77%
V. Axis Knowledge	15	17.44%
HA	15	17.44%
1st Diff	3	3.49%

Table 4. The number of participants with "+" in the Triangulation and the corresponding percentage within the total number of "+" occurrences.

As it can be seen in Table 4, the major weakness of the participants is the first derivative knowledge upon applying Triangulation similar to the results attained for triad classification. One way to improve first derivative knowledge of these participants can be asking questions that require integrated knowledge of first derivative with the other concepts listed in Table 4. It is certain that asking only first derivative-related questions can help the participants to comprehend first derivative, however helping them to learn questions that integrate first derivative with other concepts listed in Table 4 can be much more helpful to the students to advance knowledge. The solutions to these questions need to be explained to the participants to help them develop the conceptual understanding of the concepts. Application questions of calculus in engineering and sciences can also help participants to comprehend concepts better.

5. Conclusion & Future Work

26 STEM students' qualitative and quantitative responses to a calculus question were analyzed in this work that required demonstrating mental calculus sub-concept construction ability. IRB approval was attained to conduct the study, with each participant compensated for their participation. The main goals of the study were to further understand engineering students' ability to answer a calculus question that requires knowledge of multiple calculus sub-concepts and further advance STEM educators to use question evaluation methods such as Triangulation for further improvement of student assessment techniques. The qualitative data used was embedded in our analysis based on the students' oral responses to explain their written responses to the research questions while quantitative analysis was based on Triangulation and APOS

classifications. Table 5, and Figures 3 and 4 below show a summary of the APOS classification and Triangulation method used in this research.

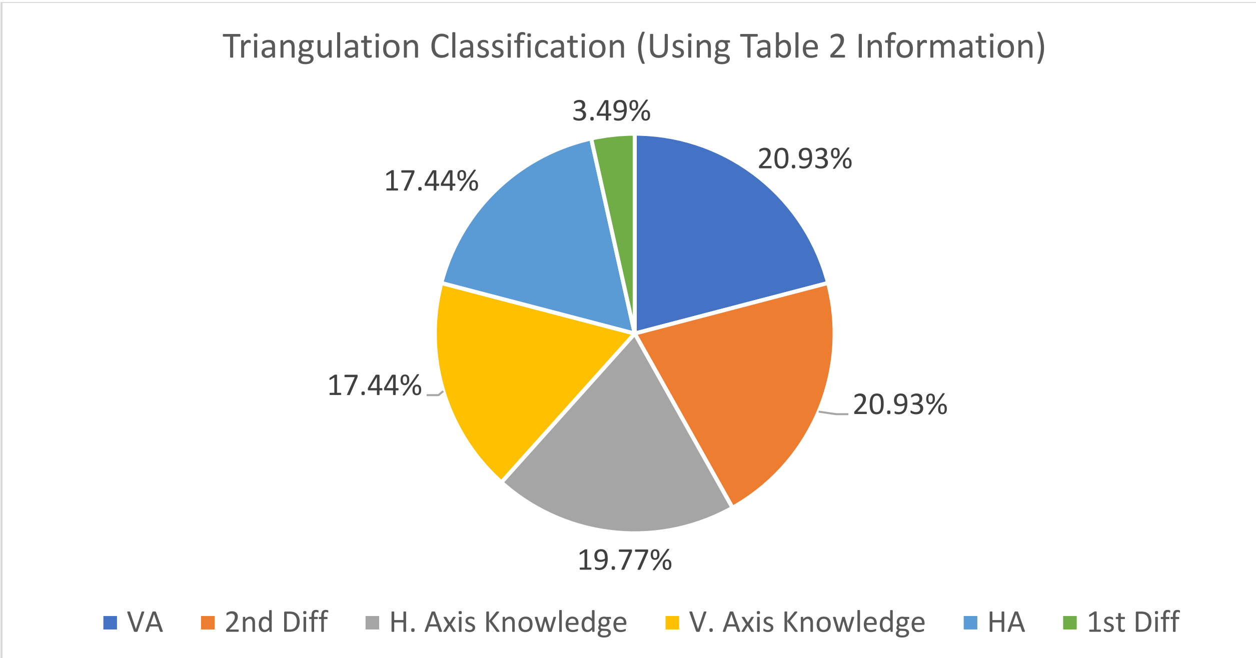

Figure 3. The triangulation classification of the participant responses

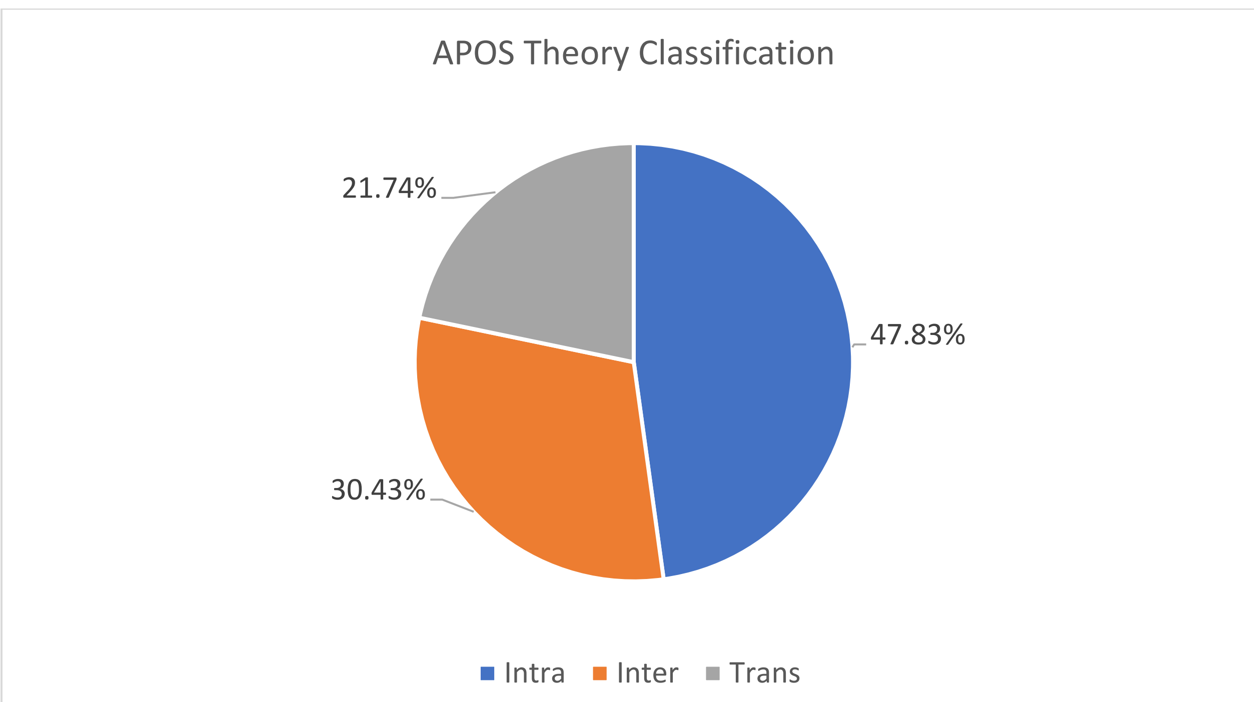

Figure 4. The Triad classification stemming from APOS application

	Triangulation Classification						APOS Classification		
Sub-classification	VA	2nd Diff	H. Axis	V. Axis	HA	1st Diff	Intra	Inter	Trans
Percentage	20.93%	20.93%	19.77%	17.44%	17.44%	3.49%	47.83%	30.43%	21.74%

Table 5. Percentage summary of participants' Triangulation and Triad classification.

Triangulation of the responses with the percentages displayed in Table 5 ranked the questions based on the difficulty levels. A triangle is structured in attempt to find an indicator of the student success to a question with multiple parts demonstrating student success in responding calculus concepts. The triangulation of the data required maximization of the correct responses to be clustered within a triangle. The Triangulation data in Table 3 displayed 95.35% of the correct highlighted responses fitting within the triangle. This Triangulation of the participants not only measures the participants' success in responding to such a calculus question but also a method to analyze weaknesses and strengths along with the possible grades that students can receive. Triangulation is shown to be an effective and strong method in [1] for a fill-in-the-blank type of question's evaluation and our current work also supports this finding for sketching the graph of a function by using provided calculus information. Therefore, Triangulation is a strong method that can be used by STEM educators for evaluating similar questions in STEM fields.

The APOS classification through Triad categorization of the participants shown in Table 5 are determined to be 47.83% at Intra level, 30.43% at Inter level, and 21.74% at Trans level. Compared to prior APOS classification provided in [3] and [4], the participants in this work showed a stronger calculus sub-concept knowledge at Trans level. The results indicated a similar trend to [2] with strength of students' knowledge using APOS.

Triad and Triangulation combined results indicate Intra and Inter level categorization of the participants due to the first and second derivative knowledge that agrees with prior research results attained in [1-3]. Triangulation analysis appears to indicate a much better classification when compared to Triad noting that the measurable outcomes of participant responses are much more transparent. The weakness of the Triangulation classification applied in this work is the "all or none" approach of the participant responses. One way to improve Triangulation methodology is by incorporating heat map approach to it by using percentages instead of "all or none" classification and incorporating correlation analysis within the triangle formed. Additionally, the use of Triangulation method and APOS theory along with the use of Artificial Intelligence and gamification can be very useful in improving STEM students' educational experiences noting that the two pedagogical methods can help with the design of advanced software applications to teach mathematics concepts.

The techniques used in this work can be used by researchers on empirical data sets for attaining measurable outcomes; Educators can measure student strengths and weaknesses on different calculus questions and other areas of interest in STEM. We encourage other STEM field researchers and educators to apply APOS and Triangulation methods in other types of questions.

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