

## **Analytic Framework for Students' Cognitive Mistakes in Studying Electromagnetic Fields**

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## Analytic framework for students' cognitive mistakes in studying electromagnetic fields

### Abstract

Students at various levels often experience cognitive difficulties when learning electromagnetic (EM) fields and waves. This can be attributed to the intensive mathematical reasoning and invisibility of physical EM phenomena. Students' common misconceptions and alternative conceptual frameworks of EM concepts have been extensively studied. Students' difficulties with mathematical thinking and manipulation in the contexts of EM have also been reported. However, most of the previous research studies were conducted using either multiple-choice surveys, or in-depth interviews conducted outside of class. There have been few efforts, however, to address students' performance on the topic of EM fields and waves through examination of the artifacts that students create as part of their course i.e. homework.

To get a broader and systematic understanding on students' learning difficulties and knowledge development in EM, we conducted a comprehensive study in a junior-level EM fields class in the department of electrical engineering, with 53 students. An analytical framework for written response analysis was proposed based on knowledge associated with structured problem solving include situation, conceptual and procedural. The framework was used to inform the development of a coding scheme to analyze students' weekly homework. Our results indicate students make two main types of mistakes on their homework: conceptual and procedural mistakes. Each of these categories contain a range of specific examples of common mistakes students demonstrate in their problem solving of course homework. Because of the unfamiliarity with EM learning context, students may have conceptual difficulty in activating the declarative knowledge needed to solve the problems. Also, consistent with other research, we find that students face significant procedural challenges in combining their physics knowledge with mathematical strategies at their early stage of learning. Common homework mistakes for learning topics Coulomb's law, Gauss's law, electric potential and energy, current flow and density, polarization in dielectrics, boundary conditions, capacitance, Ampere's law and etc. are also reported. Finally, we discuss the implications of this study for increasing instructor's awareness of students' difficulties, and also providing in-time feedback for instructors to design new learning activities or adjusting instructional strategies to improve teaching efficiency.

### Introduction

Electromagnetic (EM) fields are an important foundation of modern society. A course introducing EM fields is a common core course for electrical engineering and physics students. In the study of EM fields, as with any topic that requires intensive mathematical reasoning of invisible physical phenomena, students at various levels often experience cognitive difficulties when first facing problems that use abstract notions and operators varying with time or/and space<sup>1-3</sup>. We provide a brief background of research on student difficulties with EM phenomena. We first discuss the conceptual difficulties and later the difficulties in mathematical problem solving.

*Students' Conceptual Difficulties with EM*

Research on students' conceptual difficulties with EM phenomena in introductory courses has utilized multiple choice concept inventories. The Conceptual Survey of Electricity and Magnetism (CSEM) was created for introductory physics students<sup>4</sup> to probe their conceptions before and after instruction. The initial version of the survey was drafted by a group of experienced college physics instructors, and the refined versions combined multiple data, such as students' responses, and instructors' feedback. In the study of a general physics course for non-engineering or physics college students, Maloney<sup>5</sup> found that, most students had the misconception that magnetic poles were positive/negative charged in a similar manner to the electrostatic charges.

Student understanding in upper level EM was probed using the Electromagnetics Concept Inventory (EMCI) that was developed<sup>6-7</sup> to primarily target the conceptions of undergraduate junior, or graduate level students. The EMCI is composed of three main tests: EMCI-Fields, EMCI-Waves, and EMCI-Fields and Waves, each of which includes 23-25 multi-choice questions. Taking the EMCI-Fields exam as an example, the inventory topics cover not only conceptual physical laws (e.g., Coulomb's law, Gauss' law, Ampere's law and etc.), but also problem-solving strategies (e.g., using symmetry and etc.) and applicable facts (e.g., a conducting body is equipotential, no field in a conductor, etc.).

In addition to studies of conceptual understanding based on conceptual inventories, other studies have probed the impact of student understanding. Bagno and Eylon<sup>8</sup> conducted a diagnostic study among students of the first year college level equivalent. Their analysis results from students' written questionnaires indicated that, students' misconception in mechanics, i.e., force and motion, would cause transferrable understanding difficulties in EM studies. After examining 184 second-year engineering students' preconceived knowledge on fundamental concepts in circuit theory before formal instruction on electromagnetism, Periago and Bohigas<sup>9</sup> reported around 80% of students mistook a battery as a constant source of current resulting in the potential difference. And about 50% of students confirmed the current in a close circuit would diminish as it flows round.

In addition to circuits, studies have also reported student difficulties with fundamental principles such as Gauss's and Ampere's Laws. Singh<sup>10</sup> examined the learning difficulties in identifying charge distribution and applying Gauss's law among 541 students in the introductory calculus-based physics class, upper-level electricity and magnetism class, and graduate level teaching assistant seminar class. Many students had difficulty in using the superposition principle, specifically, they could not differentiate the electric field generated by individual point charges from the electric field at any point, the latter of which is the vector sum of fields generated by all charge sources. This finding confirmed Rainson et. al's results<sup>11-12</sup> that in a situation involving multiple field sources, students had difficulty in analyzing the overall causality. Singh<sup>10</sup> and Pepper et al.'s<sup>13</sup> work also found that the misunderstanding of electric flux through a closed surface was a crucial obstacle to the study of Gauss's law. Some students could not distinguish between the concepts of electric flux and electric field, some students failed to choose a Gaussian surface, and some students ignored the closed surface as the requisite when applying Gauss's law. Students also seemed to have difficulty with Ampere's law both in introductory level<sup>14</sup> and in the upper division<sup>15</sup>. The difficulties included inappropriately choosing an Amperian loop and

incorrectly deciding enclosed current were common mistakes reflected from students' quantitative written responses<sup>15</sup>.

### *Students' Difficulties with Mathematical Problem Solving in EM contexts*

Research studies discussed in the previous section have provided insights on students' common misconceptions of EM phenomena. However, conceptual understanding of EM phenomena is not independent of sophisticated mathematical analysis, especially in upper level EM courses<sup>13</sup>. In these situations, the vector nature of the EM fields and the use of abstract operators can make the study of EM phenomena significantly more challenging. The mathematical formalism used to model EM phenomena can be much different than straightforward but abstract application of mathematical rules, rather it requires students translate physical phenomena into mathematical representation or extract physical information from mathematical representations<sup>13,16</sup>.

Pepper et. al<sup>13</sup> examined the common mathematical mistakes in the context of EM. They found that the most significant learning challenge was that students tended to separate physics understanding and mathematical problem solving in their study of EM phenomena. Students' had difficulty in choosing appropriate mathematical tools and doing correct mathematical calculations. Specifically, students struggled with the concept of divergence and the choice of integral element when doing vector calculus. Incorrectly identifying the integral limits, that is, wrongly determining the potential difference, was a common mistake for the calculation of electric potential at a certain point<sup>13</sup>.

### **Motivation and Research Question**

Much of the research described above has focused on the use of concept inventories, clinical interviews, and observations of students. Students' written homework is typically considered to be of limited value as a research artifact that sheds light on students' understanding. However, in most science and engineering classes, homework is regarded as an opportunity for students to reinforce the concepts taught during class, and to practice desired problem solving skills<sup>17</sup>. Research has shown that homework completion and feedback can have a positive influence on students' learning<sup>18-19</sup>. The time students spent on homework is a good predictor of their academic performances as measured by exam scores<sup>18</sup>. Especially, for students with average lower ability, practice with homework can help them compensate for their performance gap with average higher ability students<sup>20</sup>. Therefore, we focus this study on the analysis of student homework in an EM class for upper-division electrical engineering majors. Specifically, we seek to answer the following question: *What student common mistakes of EM phenomena can we characterize through analysis of their paper-and-pencil homework in an upper-division class for electrical engineers?*

### **Theoretical Framework**

In order to analyze student homework to investigate their learning difficulties in the context of EM phenomena, we adapt a cognitive framework by de Jong and Fergusson-Hessler<sup>21</sup>. This framework recognizes that there are several different types of knowledge a solver must possess, including knowledge of situations, concepts, procedures, and strategies<sup>21</sup>. *Situational knowledge* refers to an awareness of common problem situations or contexts that occur in a domain, which can help a solver recognize important features and form a representation of the problem. Visualization construction and other representational approaches that would support problem

statement are good reflections of situational knowledge. *Conceptual* or “*declarative*” knowledge includes the major definitions, concepts, and principles of a subject, whereas *procedural knowledge* refers to any action appropriate for solving the problem under a particular context. *Strategic knowledge* can include general or domain specific heuristics that are necessary to solve the problem. Different knowledge characterizes learning activities to different levels of education objective and students’ “*science achievement*”<sup>22</sup>. In the Data Analysis section of the paper we describe the categories of the kinds of knowledge that we have created based on de Jong and Ferguson-Hessler’s<sup>21</sup> science knowledge classification framework.

### **Research Context and Participants**

The intended class ECE 311 *Electromagnetic fields* is a required class for junior ECE students in a Midwestern University and offered in every Spring and Fall semester. The average attendance is about 120 students each semester, and two similar sections with different instructors are provided simultaneously. Textbook references are *Elements of Electromagnetics* by Matthew Sadiku, and *Schaum’s Outlines Electromagnetics* by Joseph Edminister. The main topics covered in this class include vector analysis, electrostatics, magnetostatics, Maxwell’s equations and electromagnetic wave propagation. Due to the large enrollment and intensive content, traditional lecture is employed. Since there is no introductory EM related course offered by the department, many students attended the class with few relevant background. Considering students’ limited background knowledge and the heavy course load of junior ECE students, paper-based homework and exams are the optimized choice as the main practice and assessment approach.

Homework was optional, but was counted as extra credits up to 30 points (totally 450 points for the class, including 3 midterms and 1 final exam). Homework was assigned weekly, and collected in class one week later. Students were allowed to work in a team or attend office hour to discuss the homework problems. But all turned in solutions should be completed individually. All problems are representative and carefully designed by the third author, who has rich experiences in teaching junior level EM fields class. Generally, ECE 311 homework problems were basic questions, including short answers, multiple choices, and computational questions. The difficulty level of which was adequate for most of the students. But the completion of these problems required students’ intellectual efforts, such as reviewing the contents delivered in class, consulting course resources (text books, notes, online videos, and etc.), and perhaps discussion with classmates, and the instructor or teaching assistant.

### **Data Collection and Analysis**

Our research was conducted in one section of ECE 311 involving 53 students in 2015 Fall. The 1-hour class met every Monday, Wednesday and Friday. The instructor assigned weekly homework and provided a detailed solution for grading. Grading and qualitative analysis were separated. The research team evaluated each problem once the homework was graded. The grading results provided us a good reference on quantitatively understanding students’ overall performances, which will be elaborated in the following section. But we didn’t use scaling scores to measure individual student’s understanding level. The efforts involved in problem analysis was much more than grading itself because the data we cared about were not right or wrong quantitative solution results, instead, we focused on what kind of mistakes students made and how to characterize these mistakes to further infer students’ potential learning difficulties during EM studies.

Adapted from de Jong and Ferguson-Hessler's<sup>21</sup> science knowledge classification, a general analytical framework was created and the main information elements examined by the framework included:

- *Situational knowledge*—if any assumption, related parameter, known or unknown quantity was identified from the problem statement;
- *Conceptual knowledge*—if any governing principle, theory, concept was applied to a certain problem;
- *Procedural knowledge*—if any mathematical manipulation or step-by-step calculation was provided to support the solution;

The analytical framework was flexibly applied to rubric design for individual homework topic, which will be introduced in the following paragraphs. For example, for some problems not requiring complex calculation, the procedural element would not be evaluated. In our first draft, the element to strategic knowledge was also included, which was defined as students organizing “*their problem-solving process by directing which stages they should go through to reach a solution*”<sup>21</sup>. Strategic knowledge is knowing when, where and how to apply the knowledge. Some researchers also include a self reflection component<sup>23</sup>, which indicates the strategic knowledge is making the general plan of action and representing a higher level of inquiry. However, it was found difficult to differentiate the strategic step or strategic mistake from the first three procedures during our homework analysis. On one hand, students’ problem solving strategy was already implied in their formula choice and quantitative solution. All the above three knowledge elements were parts of the strategic plan. On the other hand, the purpose of homework in this class was to help students familiarize contents and practice skills. Therefore, homework problems were well defined and the sequence of problem solving process were quite similar. From this point of view, the requirement on strategic knowledge development was not the main focus of homework design.

The data analysis and rubric design was an iterative process. Explicitly, the rubric design experienced three steps:

1. Create a general rubric.

The research team drafted a general rubric for each topic based on the analytical framework and researchers’ subjective knowledge.

2. Detail descriptive criteria.

The research team checked students’ solutions, marked and categorized all mistakes. These identified mistakes were used to detail the descriptive criteria of rubric check items. From our experiences, a revised rubric could be developed after analyzing 15 students’ homework.

3. Finalize the rubric.

A rubric developed from step 2 was then used to compare with the rest of homework mistakes to ensure the descriptive criteria could fit with each specific problem under the same topic. New criteria could be added, and old criteria could be revised in this step.

Therefore, the finalized rubric had two main features. First, the rubric summarized all mistakes that appeared on students’ homework responses in our class. Second, all mistakes were categorized. The identified mistakes are not equal to learning difficulties directly. Instead, mistakes are final products resulting from learning difficulties. Therefore, we could infer potential cognitive difficulties that students might have during their EM studies from the categorized mistakes. To classify students’ mistakes, we used situational mistake, conceptual

mistake and procedural mistake to correspond different knowledge categories. Using the analysis and categorization approach introduced above, all mistakes for each homework problem were categorized and recorded on a spreadsheet, and then summarized as homework analysis report. A sample analytical rubric and how it was fitted to a specific homework problem of finding electric flux density via Gauss's law are shown in Table 1 and 2, respectively.

Topic: Gauss's law		Check item
Inquiry Requirement	Problem statement (Situational knowledge)	1. Identify charge source (point/line/surface/volume charge)
		2. Identify symmetrical or asymmetrical structure
		3. Identify appropriate coordinate
		4. Identify or decide correct units
	Formula or theory applied (Conceptual knowledge)	1. Identify Gauss's law or Coulomb's law
		2. Choose appropriate Gaussian surface
		3. Specify the relation between $\mathbf{D}$ and $\mathbf{E}$
	Step-by-step solution (Procedural knowledge)	1. Calculate $Q_{enc}$
		2. Apply the principle of superposition
		3. Express the differential element ( $dS$ )
		4. Compute the integral
		5. Completed discussion in spatially various regions
		6. Specify the $\mathbf{D}$ direction

Table 1. Sample analytical rubric on topic of Gauss's law.

<p><b>HW3 Problem 2</b>            There is a -1C point charge at the origin. On the surface <math>r = 10\text{m}</math>, there is a surface charge density of <math>\rho_s = \frac{1}{200\pi} \frac{\text{C}}{\text{m}^2}</math>. Find the electric flux density everywhere.</p>
<p><b>Problem statement (Situational knowledge)</b>            1. A point charge and surface charge are given;            2. Appropriate symmetrical charge distribution;            3. Spherical coordinates are appropriate for a radially symmetrical charge distribution;</p>
<p><b>Formula or theory applied (Conceptual knowledge)</b>            1. Apply Gauss's law for symmetrical charge distribution <math>\oint \mathbf{D} \cdot d\mathbf{S} = Q_{enc}</math>;            2. Choose spherical Gaussian surface;</p>
<p><b>Step-by-step solution (Procedural knowledge)</b>            1. For <math>r &lt; 10\text{m}</math>, only the point charge is enclosed, <math>Q_{enc} = -1\text{C}</math>; <math>r &gt; 10\text{m}</math>, total charge enclosed includes two parts: point charge+surface charge, <math>Q_{enc} = (-1) + \rho_s \cdot 4\pi r^2 = 1\text{C}</math>;            2. For <math>r &lt; 10\text{m}</math>, <math>D \cdot 4\pi r^2 = Q_{enc}</math>, <math>\mathbf{D} = \frac{Q_{enc}}{4\pi r^2} \mathbf{a}_r = -\frac{1}{4\pi r^2} \mathbf{a}_r</math>  <math>r &gt; 10\text{m}</math>, <math>\mathbf{D} = \frac{Q_{enc}}{4\pi r^2} \mathbf{a}_r = \frac{1}{4\pi r^2} \mathbf{a}_r</math>.</p>

Table 2. Problem analysis example.

Three representative incorrect answers to the above problem are listed in Table 3. Mistake descriptions are examples of how we categorized and interpreted mistakes.

Mistake example	$2) -1C @ (0,0,0) \quad r=10m$ $D_0 = \epsilon_0 E = \frac{Q}{4\pi r^2} \hat{r} = -\frac{1}{40\pi} \hat{r}$ $D_1 = \frac{1}{2} \rho_0 \hat{r} = \frac{1}{2} \frac{1}{600r} = \frac{1}{600r} \hat{r}$ <p>Inside sphere = <math>-\frac{1}{1200r}</math></p> <p>Outside sphere = <math>600r</math></p>	$\int \vec{D} \cdot d\vec{s} = Q_{enclosed} = \rho_0 A \quad D = \epsilon_0 E = \frac{Q}{4\pi R^2} \hat{r}$ $2) DA = \rho_0 A$ $D = \frac{\rho_0}{2}$ $D = \frac{1}{400\pi} \quad r > 10$ $= -\frac{1}{400\pi} \quad r < 10$	$r < 10 \quad \oint \vec{D} \cdot d\vec{s} = Q_{enclo} = -1$ $\vec{D} 4\pi r^2 = -1$ $\vec{D} = -\frac{1}{4\pi r^2} \hat{r}$ $r > 10 \quad \oint \vec{D} \cdot d\vec{s} = Q_{enclo} = -1 + 2 = -1$ $\vec{D} 4\pi r^2 = -1$ $\vec{D} = -\frac{1}{4\pi r^2} \hat{r}$
Description	<p><b>Conceptual mistake</b>—student failed to determine the application of Gauss’s law for symmetrical charge distributions.</p>	<p><b>Conceptual mistake</b>—student wrote down Gauss’s law, but the following step-by-step solution showed the student didn’t understand how to use Gauss’s law.</p> <p><b>Procedural mistake</b>—incorrect calculation of distance quantity <math>r</math>.</p>	<p><b>Procedural mistake</b>—student made mistake on radius boundary. Since the student used correct number to calculate the enclosed charge, we didn’t categorize this mistake as a situational mistake.</p>

Table 3. Example incorrect answers.

## Results

In the semester 2015 Fall, 7 homeworks have been analytical reviewed. The topics of Coulomb’s law, Gauss’s law, electric potential and energy, current flow and density, polarization in dielectrics, boundary conditions, capacitance, Ampere’s law, etc. were covered. Since homework counted as extra credits to the final, not all students submitted homework each time. Overall, the average submission rate was 74%, as shown in Figure 1. Therefore, the analysis results were representative, which should reflect most students’ mistakes with homework problems. The accuracy rate, which was counted as (students’ average grading scores)/(full homework scores), was also included in Figure 1.

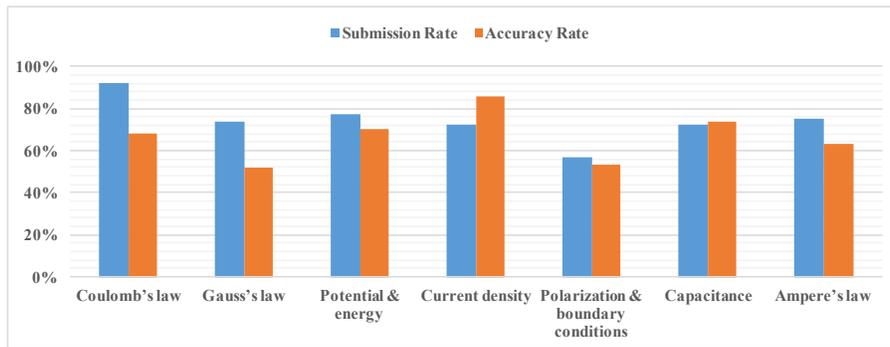


Figure 1. Average submission rate and accuracy rate for each homework.

From the statistic result, lower accuracy rates appeared in topics of Gauss’ law (52%), polarization & boundary condition (53%), Ampere’s law (63%) as well as Coulomb’s law (68%), which are important knowledge in real EM problem solving. The overall poor performances in these four topics suggested, nearly half of the students might experience learning difficulties in these topics. To systematically examine students’ learning problems, detailed qualitative analysis results on each topic were summarized below.

### 1. Coulomb’s law.

Situational mistakes:

- Missed or mistook units;

- Incorrectly identified the coordinate system implied in the problem statement.

Conceptual mistakes:

- Incorrectly calculated the field intensity  $E$  with reciprocal of distance ( $r^{-1}$ );
- Made mistake in force law, and failed to decide the field intensity  $E$  at one point under given force;
- Mixed up the concepts of point charge and electron charge.

Procedural mistakes:

- Made mistake with adding vectors when applying superposition principle;
- Incorrectly decided differential element to do surface/volume integration;
- Calculus mistake.

## 2. Gauss's law.

Conceptual mistakes:

- Failed to determine the application of Gauss's law;
- Inappropriately chose Gaussian surface;
- Mixed up the concept of electric field intensity ( $E$ ) and flux density ( $D$ ).

Procedural mistakes:

- Failed to decide and calculate the enclosed charges by using superposition principle;
- Didn't make complete conclusion when discussing the electric flux density at various locations.

## 3. Potential & energy.

Situational mistakes:

- Missed or mistook units.

Conceptual mistakes:

- Failed to understand that the electric potential is a relative value between a reference point and a test point—the potential difference  $V_{AB}$  is defined as the potential at B with reference to A ( $V_B - V_A$ ), so incorrectly decided the integral path;
- Incorrectly determining the electric potential caused by point charges when the value of  $V(\infty)$  was chosen as something other than zero;
- Incorrectly calculated the potential with reciprocal of distance ( $r^{-2}$ );
- Failed to identify the equipotential surface or misunderstanding on electric field distribution of line charges.

Procedural mistakes:

- Calculus mistake;
- Failed to use integration to calculate continuous charge distribution;
- Incorrectly analyzed  $E$  field of continuous charge distribution.

## 4. Current density.

Situational mistakes:

- Forgot the minus sign of electron charge;
- Failed to analyze the current flow in a spherical system.

Conceptual mistakes:

- Misunderstood the relation between current flow and electron velocity.

Procedural mistakes:

- Misunderstanding in the current areas so failed to calculate total currents from two different current density areas;
- Miscalculation of total resistance;
- Incorrectly decided differential element to do surface/volume integration.

## 5. Polarization & boundary conditions.

Situational mistakes:

- Made mistake in recognizing the unit of dipole moment  $p$ .

Conceptual mistakes:

- Failed to use Gauss's law to calculate electric flux  $\mathbf{D}$ ;
- Misunderstanding on the relation of charge density vs. flux/polarization: ignored the consideration of field direction, and mistook the surface charge density directly as the  $\mathbf{D}$ , and mistook the bound surface charge density directly as the  $\mathbf{P}$  for non-planar surfaces;
- Unclear about the relation between polarization and electric dipole moment;
- Failed to use boundary condition to separately discuss tangential component of  $\mathbf{D}$  and normal component of  $\mathbf{E}$  in two different materials;
- Didn't apply Poisson's equation correctly.

Procedural mistakes:

- Incorrectly determine the direction of  $D_n$  components;
- Didn't make complete conclusion when discussing the potential and  $\mathbf{E}$  field at various locations.

## 6. Capacitance.

Conceptual mistakes:

- Forgot  $\epsilon_0$  when calculating  $\mathbf{D}$  or  $\mathbf{E}$ ;
- Didn't correctly understand the total charge  $Q$ ;
- Had problems in recognizing different capacitor connection.

Procedural mistakes:

- Didn't correctly apply boundary condition.

## 7. Ampere's law.

Situational mistakes:

- Mixed up concepts of  $\mathbf{H}$  and  $\mathbf{B}$ ;
- Couldn't differentiate line current source vs. volume current source, so had trouble in deciding the correct coordinate system.

Conceptual mistakes:

- Didn't know to apply Ampere's law to simplify symmetrical problem;
- Didn't identify the correct direction of  $\mathbf{H}$ .

Procedural mistakes:

- Incorrectly decided the enclosed current. Some students thought the enclosed current should be zero outside the cylindrical current region;
- Failed to decide and calculate the total  $\mathbf{H}$  using the superposition principle;
- Didn't make complete conclusion when discussing the  $\mathbf{H}$  field at various locations.
- Failed to calculate the  $\mathbf{H}$  generated by a sheet current source.

## Discussion

The instructor designed problems for students to familiarize the topics covered every week. Therefore, each homework had a central main topic, but not exclusively. Under some main topics, several important principles or concepts were also included, such as superposition, Poisson's equation, etc. Common mistakes for homework have been listed in the results session, from which several mistake themes across topics could also be identified.

### 1. Situational mistakes.

In our analysis, situational mistakes were defined as problem statement misunderstanding. Any students' mistake in identifying declarative or factual information was attributed to this category. Homework problems are usually well structured to help students integrate new knowledge into their existing knowledge network<sup>23</sup>. Therefore, the complexity of the problem is reduced to students recognizing the relevant constitutive laws influencing the phenomena, assumptions, and the relevant parameters and quantities from problem statements. The removal of narrowing an open ended problem to a specific point quantitative analysis increases the potential for students to demonstrate their conceptual and procedural skills to apply one of the governing laws of EM. In our homework sets, students were not required to specifically write down all the known or unknown parameters. We inferred students' situational knowledge from their plug-in numbers, sketching and written descriptions.

- Unit.

The most common situational mistakes found in students' homework were unit errors. Physics unit is a particular type of scientific notation, with which physical quantities could be scaled, compared and converted. Developing the ability to express scientific units and to be able to convert the values of a physical quantity in different units was one of the learning objectives in the EM class. Some students didn't include or wrongly converted units. For example, one student forgot the  $10^{-6}$  when converting the unit from  $\mu\text{C}$  to  $\text{C}$ . Calculating the quantitative result is not the final solution. For most physical quantities, the interpretation of numerical values depends on the unit in which these quantities are expressed. However, many students didn't have unit awareness in mind while solving problems. The unit error also revealed some of the students' conceptual mistakes in understanding a physical quantity. In the question asking for dielectric constant, 2 students made a mistake in recognizing the unit of the dipole moment  $p$ , the unit of which was given as  $\text{Cm}$  (Coulomb·m). However, these students misread the unit as  $\text{cm}$  (centimeter), which led to the wrong solutions.

- Common constants.

Another notable mistake was found in the sign of the electron charge. Around half students forgot that the fact that electron is a particle with negative elementary electric charge. In the problem statement, the density of conduction electrons was given as  $n = 8.48 \times 10^{28}$  *electrons*/ $\text{m}^3$ , students were expected to plug in the value of electron and to figure out the flowing current with other known parameters. However, we found from students' written responses that most students incorrectly used the number  $1.6 \times 10^{-19}\text{C}$ . There was also one student mixed up the concept of point charge and electron charge. This student misused  $e$  to calculate the electric field under a certain force, even though a point charge of  $-5\mu\text{C}$  was already given. In classical EM problems, the concept of electron is usually regarded as a quantity with constant value, and the above mistakes could be interpreted as students new to the EM area were unfamiliar with this

quantity. People could also argue that, if the exact value of the electron charge was provided in the problem statement, most students would obtain quantitatively correct answers. But these mistakes still reminded us, we should find a better way to help students understand and then memorize some common physics constants by emphasizing the physical meaning.

- Charge or current source configuration.

Some students had difficulties in identifying the charge or current source configuration, or choosing the wrong coordinate system for spatial representation and mathematical calculation. We read these situational mistakes from students' sketches in their homework. Cognitive researches have suggested that, the understanding of a physical or engineering fundamental concept usually leads to the building of mental models. Students will understand a physical phenomenon if they are able to build a mental model adequate to the physical model<sup>24</sup>. As an external representation product, a sketched figure would reflect a students' mental model. Students who made the wrong sketching representation might have difficulty in relating relevant physical quantities as a whole functional system.

## **2. Conceptual mistakes.**

Students' mistakes in choosing the governing principle, theorem, or concept were considered as conceptual mistakes. In our homework analysis, it was not easy to probe how students made their conceptual decisions but to examine what formulas or equations students chose. One concern for weekly homework analysis was that, each homework was assigned right after some topics were taught, so there were not major inquiry challenges for students to practice their conceptual knowledge. However, making the learning targets explicit to students is important at the beginning stage of knowledge construction<sup>23</sup>. Increasing students' awareness of how intended physical concepts would be introduced under a physical context is beneficial for students. In the category of conceptual mistake, we mainly cared if students correctly wrote down an appropriate formula for solving a problem. And in the multiple choice or short answer problems, if students made the correct choice or provided correct answers.

- Gauss's law and Ampere's law.

Gauss's law is an important theorem in electrostatic field, which provides mathematical convenience to calculate electric flux density/electric field when the source charge distributions are symmetric. The analysis results from different homework indicated that, students had confusion on making the choice between Coulomb's law and Gauss's law. Coulomb's law and how to use Coulomb's law to calculate the electric field was the first fundamental EM concept taught in class, and then following with Gauss's law. Some students who were not proficient at Gauss's law tended to apply Coulomb's law to solve all problems, particularly the problems with symmetrical charge configurations, when asking for the electric flux density/electric field. Theoretically, any results from Gauss's law could be derived from Coulomb's law because Gauss's law is a special alternative statement of Coulomb's law. But in many cases, it is not easy for our students to make that derivation without strict mathematical training. Therefore, students were usually stuck halfway if they tried to use Coulomb's law to solve a Gauss's law applicable problem. A similar mistake was also found in the application of Ampere's law. Some students had the problem in identifying the applicable situation for Ampere's law or Biot-Savart's law.

Another problem with Gauss's law is how to decide an appropriate Gaussian surface shape and therefore an appropriate coordinate system for calculation. For example, to find out the electric field generated by a volume charge whose charge density is  $\rho$  in the region of  $-2\text{m} < z < 2\text{m}$  and 0 elsewhere, some students chose a spherical surface and tried to solve the problem under a spherical system which greatly increased the complexity of calculation. Some students misunderstood a Gaussian surface as the actual surface of the charge region. This mistake was common when students were asked to discuss the electric flux density/electric field generated by a spherical surface charge—students used the fixed radius of surface charge  $r_{charge}$  to calculate the Gaussian surface area incorrectly. There were also students not able to decide the total charges in an enclosed region, and this problem often lead to superposition calculation failure.

- $E$  vs.  $D$ ,  $B$  vs.  $H$ ,  $E$  vs.  $\Phi$  (electric potential).

During the homework analysis, we found several groups of similar concepts that students often mixed up. Here a summarized discussion is made. It is not easy for new learners to differentiate some close concepts, especially electric field intensity  $E$  vs. electric flux density  $D$ , and magnetic field intensity  $H$  and magnetic flux density  $B$ . Even though the two quantities in each group, for example  $E$  and  $D$  are constitutive related via the simple relation  $D = \epsilon E$ , the unclear definitions of the two concepts would impede students' learning on other relevant concepts, e.g., boundary condition.

The concept of electric field intensity  $E$  and electric potential  $\Phi$  are compared because of their similar expression.  $E$  was introduced when explaining the action-at-a-distance force between point charges based on Coulomb's law. Any electric charge (a source charge) would create an electric field, which affects other charged object (a test charge) entering into the field. Therefore, the vector  $E$  is defined as the electric force per unit charge when placed in an electric field, so  $E = \frac{Q}{4\pi\epsilon_0 r^2} \rightarrow$ . The potential difference  $\Phi$  is the potential energy per unit charge with the consideration of location dependency. From the definition formula  $\Phi_{AB} = -\int_A^B E \cdot d\mathbf{l}$ , we would have a simple expression for the potential at any point  $r$  if we choose infinity as reference location ( $\Phi$  at infinity as zero), that is,  $\Phi = \frac{Q}{4\pi\epsilon_0 r}$ . Therefore,  $E$  and  $\Phi$ , the former is inversely proportional to the square of the distance ( $r^{-2}$ ) while the latter to the distance ( $r^{-1}$ ). However, we found many students applied the formula in an opposite way—used  $r^{-1}$  for calculating  $E$  or used  $r^{-2}$  for  $\Phi$  by mistake in homework. We would infer, students who made these mistakes didn't have a good understanding of the two physics quantities and the underlying physical relations.

### 3. Procedural mistakes.

Procedural mistakes were identified from students' step-by-step solution. Compared with the situational and conceptual mistakes in "knowing what", the procedural mistakes reflect students' errors in "knowing how"—how to analyze vector calculus, and how to apply strategic algorithm to complete the problem solving.

- Superposition.

Students' mistakes in using the superposition principle were identified in many different topics. Our findings are consistent with previous researches<sup>11-12</sup>, that is, students didn't effectively use the superposition principle to decide the vectorial sum of all individual components. When calculating the magnetic field intensity of two planes with different sheet current densities, some students didn't think about applying superposition principle to analyze the spatial distribution of

magnetic fields. In the study of Gauss's law, some students made mistakes in calculating total charges enclosed in a Gaussian surface.

- Boundary condition.

Boundary condition was introduced when discussing the electric fields in two different media. The separating boundary sets additional constraints resulting from discontinuity of the materials on each side. Students were found struggling with the continuous or discontinuous components of  $\mathbf{E}$  or  $\mathbf{D}$ . In the problem of capacitance calculation, some students incorrectly decided the tangential component of  $\mathbf{E}$  because they failed to identify the electric field direction. Some students didn't realize the applicable condition of  $\mathbf{D} = \epsilon\mathbf{E}$ , but simply applied the formula without considering the media differences.

- Symmetrical arguments.

Identifying the symmetrical configuration would usually facilitate the mathematic calculation. Two representative topics where symmetry mattered are Gauss's law and Ampere's law. We have discussed students' conceptual failure on deciding the application of Gauss's law or Ampere's law when a symmetrical source (charge or current) configuration was given. However, for those students who thought about the appropriate theories, many of them still experienced problems in how to correctly apply the theories. Two main observations were summarized. Firstly, students were sensitive to geometrical symmetry. In the homework "Gauss's law", two configurations—spherical surface charge and semi-infinite rectangular volume charge (with  $x, y$  directions extending to infinity) were tested. The former problem had a typically symmetrical structure, and 14 students (36%) made mistakes on this problem. The mistakes identified included incorrectly using Coulomb's law (15.4%), inappropriately choosing and discussing Gaussian surface (15.4%), and incorrectly calculating the total enclosed charge (5.1%). However, in the volume charge case, 32 students (82%) students failed to solve the problem. Around 20% of students didn't use Gauss's law, over 30% of students discussed an inappropriate Gaussian surface or incorrectly calculate the enclosed charge, some students couldn't differentiate  $\mathbf{E}$  and  $\mathbf{D}$ , and a few students made mistakes in mathematical calculation. Similar mistakes were made with Ampere's law. We compared students' accuracy rates of two problems—one was asking to find the  $\mathbf{H}$  generated by a cylindrical volume current, the other was to figure out the  $\mathbf{H}$  generated by a semi-infinite rectangular volume current. The comparison results again showed us an uneven distribution. 16 students (40%) made mistakes related to Ampere's law with a cylindrical volume current, 4 of which had problems in identifying the enclosed current. Surprisingly, 35 students (87.5%) failed the problem with a semi-infinite rectangular volume current. 7 students misunderstood the current source configuration, which was actually a situational mistake. 18 students (45%) students didn't know how to appropriately choose an enclosed region and correctly calculate the total current inside.

The mistakes found in Ampere's law implied another concern involving symmetrical argument. For instance, the students were shown the  $\mathbf{H}$ -field was zero inside a hollow cylinder with current flowing in the axial direction. They then incorrectly concluded the  $\mathbf{H}$ -field was zero in a solid cylinder with current flowing in the axial direction because all radial components added up to zero "by symmetry".

- Incomplete conclusion.

In EM problems, the understanding of spatial distribution is critical. However, our analysis results showed that many students only had partial considerations and incomplete discussions on the intended problem. For example, in homework “potential”, students were asked to calculate the potential and electric field intensity of the whole system consisting of two semi-infinite planes (shown in Figure 2). Around half students only discussed potential and electric field distributions in the region of  $(0 < \phi < 3/4\pi)$  but neglected the region  $3/4\pi < \phi < 2\pi$ . Similar results were also found in discussions of  $E$  and  $H$  field distribution.

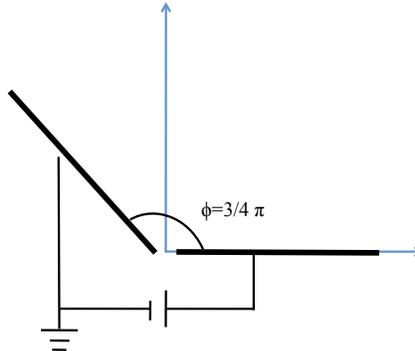


Figure 2. System with two semi-infinite conductors.

- Calculus mistakes.

Incorrectly decided differential element to do line/surface/volume integration, and integral calculation mistake with continuous charge/current source are two main mathematical mistakes that appeared on students’ homework. For example, in the topic of electric potential, many students failed to understand that when determining the electric potential, it must be with respect to a reference. The electric potential is a relative value between a reference point and a test point—the potential difference  $V_{AB}$  is defined as the potential at B with reference to A ( $V_B - V_A$ ). With concept misunderstanding, students had problems in deciding the correct integral path. And similar results have also been reported by Pepper et al.<sup>13</sup>.

Overall, we show the proportion of each type of mistake in figure 3. Conceptual and procedural mistakes are main mistakes that could be identified from students’ homework. Because of the unfamiliarity with EM learning context conceptually, students might have difficulties in applying EM concepts to correctly solve problems. Also, consistent with other research results, how to combine physics considerations with mathematical strategy is a big challenge for students at their early stage of learning.

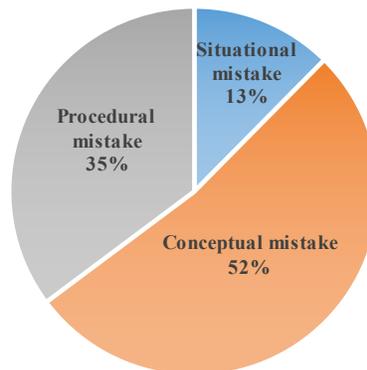


Figure 3. Overall statistics of mistakes.

## **Limitation**

Homework is a good lens through which students' potential learning difficulties in coordinating their math and conceptual resources could be noted. However, we should also emphasize that homework analysis will not reveal students' dynamic reasoning, decision making, or reflection such kind of high level cognitive activities, but provide static evidence, for example common mistakes, to suggest the potential learning difficulties in knowledge assimilation.

Another concern with the homework analysis is, students may not pay much effort on homework problems. Further, students were able to work in study groups where peers could provide additional cognitive support to students' generation of solutions. So in our next step, exam mistakes will be included for correlation analysis.

## **Implications for Teaching**

Some researchers argue that, the study of most EM concepts is the process of new knowledge acquisition<sup>25</sup> because students lack experiences with those EM phenomena or familiarity with the “*concepts, language, principles, and relations*”<sup>4</sup>. Therefore, there will be ontological or epistemological difficulties when students develop their EM conceptual profile<sup>25</sup>. From this point of view, students' learning difficulties in the concept of Gauss's law<sup>10</sup>, superposition<sup>10</sup>, electric potential, electromagnetic induction and so on are because of the cognitive conflict or reasoning failure when acquiring EM knowledge in new problem situations<sup>11</sup>. It is important to help students avoid and correct misconceptions from the beginning as they learn these concepts. Our homework analysis aims to diagnose and document students' mistakes and potential learning difficulties from their daily practice. In the meanwhile, homework analysis could provide in-time feedback for instructor to be aware of students' learning progress and design remedy activities if necessary.

Timely feedback based on homework analysis will be an in-time pedagogical strategy, which bridges students understanding and instructors teaching in an indirect way of interaction. On the one hand, students would receive in-time feedback on their homework, which will benefit their new knowledge construction; on the other hand, the teacher would have enough time to incorporate insights gained from students' homework into the upcoming lesson. Teachers will have a good sense of the learning pace of the class, and therefore revise the lesson flow—which part should be emphasized more; which part should be slightly mentioned. Therefore, teachers would begin the new class with an awareness of the most common misconceptions of the class, and effectively use the classroom time to review mistakes so that the incorrect understanding won't be woven into new content study. For teachers who are new to teach EM classes, the mistakes identified would become documentary evidence to inform instructors students' potential learning difficulties, and therefore to help instructors preempt these difficulties by careful instruction design.

## **Conclusion**

In this paper, an analytical framework as well as the coding scheme for written response analysis were introduced. Common homework mistakes for learning topics Coulomb's law, Gauss's law, electric potential and energy, current flow and density, polarization in dielectrics, boundary conditions, capacitance, Ampere's law, etc. were reported. The way and perspective we examined homework were only from our teaching focus and based on our curriculum design. We

didn't mean to propose a standard or definitive analytical framework that broadly adapted to all EM class, but to introduce a practical, easy-to-use analysis framework that would help practicing instructors to better understand students' learning performances and progress. We have identified small portion of problem. Homework analysis might have its own limitation, but it will contribute as part of multiple data that could reflect students' learning difficulties at the initial learning stage.

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