2006-179: ANIMATION SOFTWARE FOR THE TEACHING OF ELECTRICAL TRANSMISSION LINES

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Introduction

Electrical transmission lines are conduits for the transport of both information and electrical energy. As such, they are an important subject for undergraduate students in electrical engineering to master. With pressure to include more topics in the curriculum, most electrical and computer engineering curricula have limited the required coverage of electromagnetics to a single three or four semester hour course. With this development, the allotted space in the single course must be used wisely and many topics formerly covered in a multi-course sequence must be omitted. This forces some difficult curricular decisions as some pet topics of various faculty cannot be covered in the allotted time. At many institutions, the topic of transmission lines was in itself a separate course which has long ago been deleted from the required curriculum. With new emphasis on communication systems and integrated circuits, the topic of transmission lines is more important than ever, although the context has changed. In many cases, the topic of transmission lines has become part of the remaining required electromagnetics course.\(^1\) Usually this coverage is limited to sinusoidally driven lossless lines and propagation of suddenly applied D.C. signals on lossless lines.

Since transmission lines are distributed parameter systems, it is difficult to visualize the voltage and current on the line from analytical solutions. Many of our students are graphical learners and thus computer graphics may provide enhanced student learning. The authors have developed MATLAB-based animation software to handle the following cases:

- sinusoidally driven lossy lines (both resistance and conductance are present),
- sinusoidally driven lossless lines (no series resistance or shunt conductance),
- propagation of suddenly applied d.c signals on a lossless line, and
- propagation of a rectangular pulse on a lossless line.

The software discussed here may employed for educational purposes in two distinct ways. The first is use by the instructor in the classroom to illustrate the concepts as they are discussed. The second is use by students in the computer lab to solve problems assigned by the instructor. This may require careful design of problem sets by the instructor to ensure that some hand calculations are required. A typical problem for the application of the lossless transmission line simulator is given in Appendix A. The software is not intended as a substitute for the understanding gained early in the process by solution of problems using a hand calculator. The concepts of load and source impedance matching, wave reflection, and standing wave ratio can be explored experimentally by students in the setting of the computer keyboard.

The concept of using MATLAB for the animation of lumped parameter dynamic systems was demonstrated by Watkins et al.\(^2\) Recently there have been a number of papers describing the graphical interpretation of partial differential equations. The transport of pollutants in groundwater has been described using web-based graphics\(^3\) and another paper reports a virtual laboratory for teaching quasistationary electromagnetics.\(^4\) Another recent paper discusses the solution of groundwater problems using a spreadsheet.\(^5\) Still another paper employs a
spreadsheet to examine the topic of electromagnetic wave propagation. The authors previously reported the use of animation to clarify a variety of partial differential equation solutions and one example given in that paper was the sinusoidally driven lossless transmission line which will also be discussed briefly here.7,8

**TEM Transmission Lines**

Transverse electromagnetic (TEM) transmission lines are those in which the electric and magnetic fields are orthogonal to the direction of wave propagation. Consider here the transmission line arrangement shown here in Figure 1 with a Thevenin equivalent signal source and a linear load.

![Figure 1. The general case of a TEM transmission line.](image)

The line is governed generally by the telegrapher’s equations

\[
\frac{\partial v}{\partial z} = -L' \frac{\partial i}{\partial t} - R' i \\
\frac{\partial i}{\partial z} = -C' \frac{\partial v}{\partial t} - G' v
\]

where \(L', C', R'\) and \(G'\) are respectively the inductance, capacitance, resistance and conductance per unit length. The boundary conditions are

\[
v_g(t) - A[i(-L,t)] = v(-L,t) \\
v(0,t) = B[i(0,t)]
\]

where \(A[f]\) and \(B[f]\) are respectively linear integro-differential time domain operators. It is also assumed that the initial functions for voltage \(v(z,0)\) and current \(i(z,0)\) are known functions of \(z\). It is clear at the outset that the analytical solutions for these equations are difficult to understand because the line voltage and current are each a function of both location \(z\) and time \(t\).

We discuss below the four special cases of the problem posed above. We then elaborate on the software for each of these cases, pointing out the appropriate use of each.
Sinusoidally Driven Lossy Line

In this case, the source is sinusoidal at frequency $\omega$ so in order to get the steady-state solution the phasor method will be employed. The telegrapher’s equations (1) are therefore reduced to a pair of ordinary differential equations in phasor voltage and current. In this work the phasor representation of a sinusoidal signal has a magnitude equal to the sinusoid amplitude and an angle equal to the phase shift from a pure cosine wave.

\[
\frac{dV}{dz} = -(j\omega L' + R') I
\]
\[
\frac{dI}{dz} = -(j\omega C' + G') V
\]

(3)

The phasor solution to these equations for the phasor line voltage is

\[
V(z) = V_g \left[ \frac{Z_{in}}{Z_g + Z_{in}} \right] \frac{e^{-\gamma z} + \Gamma_L e^{\gamma z}}{e^{j\theta} + \Gamma_L e^{-j\theta}}
\]

(4)

where $\gamma = \alpha + j\beta = \sqrt{(j\omega L' + R')/(j\omega C' + G')}$ is the complex propagation constant and $\Gamma_L = (Z_L - Z_0)/(Z_L + Z_0)$ is the reflection coefficient. Here $V_g$ is the phasor source voltage.

The input impedance in relation (4) is given by the solution process to be

\[
Z_{in} = Z_0 \left[ \frac{Z_L + Z_0 \tanh \gamma L}{Z_0 + Z_L \tanh \gamma L} \right]
\]

(5)

where the characteristic impedance is $Z_0 = (R' + j\omega L')/(G' + j\omega L')$. The time domain solution is

\[
V(z, t) = V(z) \cos(\omega t + \angle V(z))
\]

(6)

A specific example will serve to clarify the software developed. Data for an RG58A/U coaxial instrumentation cable is given below in Table 1.

<table>
<thead>
<tr>
<th>$L'$ / H/m</th>
<th>$C'$ / F/m</th>
<th>$R'$ / $\Omega$/m</th>
<th>$G'$ / $S$/m</th>
<th>$L$ / m</th>
<th>$f$ / $Hz$</th>
<th>$V_g$ / V</th>
<th>$Z_L$ / $\Omega$</th>
<th>$Z_g$ / $\Omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5x10^{-7}</td>
<td>1.01x10^{-10}</td>
<td>0.028</td>
<td>5.9x10^{-14}</td>
<td>1000</td>
<td>3x10^6</td>
<td>10</td>
<td>100</td>
<td>50</td>
</tr>
</tbody>
</table>

Figure 2 illustrates the line voltage as a function of location on the line for twenty values of time. It is easy to note the attenuation of the voltage signal as it propagates down the line toward the load due to the lossy nature of the line. There are several ways of presenting the solution to this problem in addition to that of employing $t$ as a parameter as in Figure 2. Figure 3 illustrates the line voltage as a function of $t$ and $z$.

The authors find neither of the presentations of Figures 2 or 3 particularly satisfying. However, an animation of voltage as a function of $z$ for closely spaced values of time is very informative to the student but cannot be presented in the static form of a written paper.
Figure 2. Line voltage as a function of location for several times. Note that the load is located at $z = 0$.

Figure 3. Three dimensional rendition of line voltage as a function of $t$ and $z$.

**Sinusoidally Driven Lossless Line**

The most often discussed case in the classroom is that of the transmission line with no energy dissipation ($R' = G' = 0$) driven by a sinusoidal signal. This is the case for which the well-known Smith chart is applicable to investigate reflection, load matching, and standing waves. This is also the case that the authors have presented previously.\textsuperscript{7,8} Since this case is so
frequently discussed in the classroom setting, the authors developed a software module with a graphical user interface (GUI) to handle the input, output and graphical data presentations. The solution for the phasor line voltage for this situation is

\[ V(z) = V_0 \left[ \frac{Z_{in}}{Z_{in} + Z_g} \right] e^{-j\beta L} + \Gamma_L e^{j\beta L} \]  

(7)

where \( Z_0 = \sqrt{L/C} \) is the characteristic impedance of the line and the line input impedance is

\[ Z_{in} = Z_0 \left[ \frac{Z_L + jZ_0 \tan \beta L}{Z_0 + jZ_L \tan \beta L} \right] \]  

(8)

and the reflection coefficient \( \Gamma_L \) is as defined in the previous section. The phase constant \( \beta = \omega \sqrt{L/C} = \omega / u_p \), where \( u_p \) is the wave propagation velocity on the line. The time domain form for the line voltage is

\[ v(z,t) = |V(z)| \cos(\omega t + \angle V(z)) \]  

(9)

Our GUI driven program handles all the numerical inputs and outputs and presents the graphical output. The GUI, shown in Figure 4, makes the changing of the input variables an easy task.

![GUI driven program](image)

**Figure 4.** Graphical user interface for the lossless transmission line simulator.
This graphical output can be the standing wave pattern; the line voltage as a function of location and time; a plot of source, input and load voltages as functions of time; or an animation of line voltage as a function of location with varying time. In this environment students can quickly explore the concepts of wave reflection standing wave ratio (VSWR), line-load matching and the effects of frequency and line length. Also illustrated in Figure 4 is line voltage as a function of time and location. Note that in this case there is no attenuation with distance.

In the Introduction reference was made to the careful design of problem sets to use these programs. The calculations for the solution of such problems should be accomplished by hand calculator and checked against results from the programs, in this case, tls.m. A problem designed for the lossless line concepts is given in Appendix A.

**Transient Behavior of a Line to a D.C. Source**

Another important class of problems is those of signals of constant level suddenly applied to a lossless transmission line with no initial current or voltage. This situation is shown in Figure 5. The line has characteristic impedance $Z_0$ and propagation velocity $u_p$. It will be assumed that the source has internal resistance $R_g$ and that the line is terminated with a resistive load $R_L$ so the reflection coefficients at the load and source ends of the line are respectively

$$
\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0}
$$

$$
\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0}
$$

(10)

![Diagram of loaded transmission line with a suddenly applied D.C. source.](image)

Figure 5. Loaded transmission line with a suddenly applied D.C. source.

Since the resistance looking into the non-energized line is the characteristic impedance $Z_0$, the initial voltage on the source end of the line is given by

$$
V_0 = V_g \left( \frac{Z_0}{Z_0 + R_g} \right)
$$

(11)

Immediately after the switch closes a wave of constant value $V_0$ propagates down the line with velocity $u_p$ until it arrives at the load whereupon the wave is reflected with magnitude $\Gamma_L V_0$ superimposed on the already present level $V_0$. The situation is often most easily described by the so-called bounce diagram of Figure 6 where the diagonal lines represent the times and locations of voltage level transitions on the transmission line. If the load is matched to the line ($R_L = Z_0$)
then there is no initial reflection from the load and the line voltage remains at $V_0$ for all time greater than $L/u_p$. As a specific case, consider the numerical values given in Table 2.

Table 2
Variable values for a lossless line carrying a D.C. waveform

<table>
<thead>
<tr>
<th>$V_g$ = 10 V</th>
<th>$Z_0$ = 50 Ω</th>
<th>$R_g$ = 25 Ω</th>
<th>$R_L$ = 75 Ω</th>
</tr>
</thead>
</table>

![Figure 6. The bounce diagram for constant value wave propagation.](image)

For the values given, the initial input voltage to the line is $V_0 = 6.67$ V and the time domain voltage at two specific points on the line is illustrated in Figure 7.

![Figure 7. Voltage waveform at two points on the line for a D.C. applied voltage.](image)
The line voltage as a function of location and time is illustrated in Figure 8; it is interesting to note that the top view of this figure is the bounce diagram of Figure 6.

![Line voltage as a function of location and time.](image)

**Figure 8.** Line voltage as a function of location and time.

**Pulse Propagation on a Lossless Line**

As a final example, the problem of the propagation of a rectangular pulse on a lossless line is to be considered. The situation is the same as illustrated in Figure 5 except the switch is closed at \( t = 0 \) and opens after a period of \( fL/u_p \) seconds where \( f \) is a positive fraction and \( u_p \) is the wave propagation velocity. The reflection coefficient at the load is calculated from the first of equations (10) and the second of equations (10) indicates that the reflection coefficient at the source end of the line for \( t > fL/u_p \) is negative unity because the switch is open for that time. The modified bounce diagram for this case is shown in Figure 9 and the dashed lines represent transitions of opposite sign from that of the nearby solid line.

The data for the problem discussed here are those given in Table 2 and the two resulting reflection coefficients are respectively \( \Gamma_L = 0.2 \) and \( \Gamma_g = -1 \) because the source circuit is open. Data for seven frames of the animation are illustrated in Figure 10 and it is interesting to note that due to reflection the voltage at the load is increased over that of the propagated wave.

For the given data the line voltage as a function of location and time is illustrated in Figure 11. The animation of the pulse propagation is particularly instructive because the idea of reflection at the load is clarified. In the case of a matched load (\( R_L = Z_0 \)), the arriving pulse is not reflected so there is no left moving reflection (i.e. toward the source) on the line.
Figure 9. Modified bounce diagram for rectangular pulse propagation.

Figure 10. Seven animation frames for pulse propagation on the line.
Figure 11. Line voltage as a function of distance and time for pulse propagation.

Conclusion

The authors have discussed a series of MATLAB programs written to assist in the teaching of electrical transmission lines. Both sinusoidal steady-state and transient behaviors are examined graphically with dynamic animations being the most instructive. The programs can be used in the classroom or in the computer laboratory, although some careful thought by the instructor on how they will be used is required to get the maximum benefit. These and other partial differential equation animation programs are available at the University of Wyoming MATLAB animation resource website at

www.eng.uwyo.edu/classes/matlabanimate

These programs may be used freely for educational, non-commercial purposes.

References

Appendix A---A Problem for Application of the Developed Software

The problem presented below represents a rather complete analysis of a sinusoidally driven lossless transmission line beginning with the line geometry. Students will find it arduous and may need to be given intermediate numerical answers.

A 300 meter coaxial cable used in instrumentation applications has an inner conductor radius of 0.0005 m and the radius to the outer conductor is 0.0015 m. The insulation between the conductors has a relative dielectric constant of $\varepsilon_r = 2.5$ and the relative magnetic permeability is unity. The cable is driven by an ideal sinusoidal source in series with a source resistor of 30 $\Omega$. The source has a frequency of $10^7$ r/s (1.5915x$10^6$ Hz) and a peak voltage of 5 V. The line is terminated with a 50 $\Omega$ resistive load. Assume the line to be lossless ($G' = R' = 0$).

(a) Calculate the per unit length capacitance and inductance $C'$ and $L'$.

(b) With the answers to part (a) calculate the propagation velocity $u_p$, the phase constant $\beta$, and the characteristic impedance $Z_0$.

(c) Calculate the reflection coefficient $\Gamma_L$, the voltage standing wave ratio, and the input impedance $Z_{in}$ at the driven end of the line. (You may check these answers with tls.m.)

(d) Calculate the input voltage $V_{in}$ and the input current to the line $I_{in}$, at the driven end of the line and the voltage at the terminal end of the line $V_L$. (These can be checked qualitatively with tls.m if you realize that the line is a linear system and superposition applies.)

(e) Find the power delivered by the ideal voltage source and the power delivered to the load.

(f) Repeat parts (c), (d) and (e) if the load is matched to the characteristic impedance of the line.

Answers.

(a) $L' = 2.19x10^{-7}$ H/m, $C' = 0.1264x10^{-9}$ F/m

(b) $u_p = 1.897x10^8$ m/s, $\beta = 0.0527$ rad/m, $Z_0 = 41.7$ $\Omega$

(c) $\Gamma_L = -0.1631$, VSWR = 1.389, $Z_{in} = 30.15 + j2.047$ $\Omega$

(d) $V_{in} = 2.51 \angle 1.936^\circ$ V, $I_{in} = 0.083 \angle -1.948^\circ$ A, $V_L = 2.498 \angle 176.8^\circ$ V

(e) $P_g = 0.2076$ W, $P_L = 0.104$ W

(f) $Z_{in} = 41.7$ $\Omega$, $V_L = V_{in} = 2.908 \angle 0^\circ$ V, $I_{in} = 0.0697 \angle 0^\circ$ A, $P_L = 0.1014$ W, $P_g = 0.1774$ W