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Application-Centric Math Curriculum for Electrical Engineering Majors

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Abstract

Inadequate math-preparedness among freshman engineering students has been an enduring challenge for Historical Black College and Universities (HBCUs), where due to various reasons, most of the students spend an average of 1-2 years clearing the math prerequisites and consequently show poor performance in math-intensive engineering courses. It is observed that the students are not able to link their math knowledge to the engineering applications covered in their subsequent engineering courses. At Norfolk State University (NSU), the Engineering and Mathematics departments worked together to address this issue by embedding supplementary teaching resources into mathematics courses that are original, engaging, and interesting to students. Hands-on application modules based on Electrical, Electronics, Biomedical, and Optical Engineering topics were created for pre-calculus, calculus, and differential equations courses for engineering majors. Examples of application modules include graphical analysis of electrocardiogram (ECG) signals, first- and second-order differential equations using electrical circuit responses, wireless power transfer, and ellipsometry. Several novel delivery methods were employed such as hands-on sessions in engineering labs using portable hardware kits, team-teaching by engineering and mathematics faculty, and co-teaching freshman (calculus) and sophomore (differential equations) classes to enable peer interaction and networking. The introduction of discipline-specific content in an engaging way in prerequisite math courses provides a unique learning experience to the students that will empower them to transfer their math knowledge to subsequent advanced engineering courses.

Introduction

It is observed among all the HBCUs that most of the black students do not have sufficient academic preparation for college-level mathematics [1]. The issue of under-preparation in freshmen students is in fact much wider which spans majority of the minority-serving institutions. Recent data published by the US Department of Education Office for Civil Rights [2] provide evidence that along with Latino students, African-American students are less likely to pass Algebra 1 and less likely to attend high schools that offer advanced classes in science, technology, engineering, and mathematics (STEM) and prepare them for the rigor of college mathematics. The same study also pointed out several reasons on this issue, including, funding inequities, lack of experienced STEM teachers, and low expectations, among others. It is observed that the students with engineering majors in HBCUs demonstrate poor arithmetic skills while enrolled in pre-calculus courses and lack mastery in basic algebraic knowledge while enrolled in calculus courses. Consequently, a big portion of classroom discussions and homework assignments are spent on reviewing prerequisite knowledge from arithmetic, algebra, geometry, and trigonometry. As a result, the students postpone enrolling in higher-level mathematics courses due to their inability to succeed in lower-division mathematics courses which lengthens their academic tenure at the university and adversely affects the student retention rates at HBCUs [3].

While various teaching initiatives and creative pedagogies are being carried out to solve these problems (for example, just-in-time teaching, co-requisite approach, active learning, meta-cognition of mathematics learning, etc), this report highlights a national effort by a consortium of colleges and universities across the nation, called Synergistic Undergraduate Mathematics via Multi-institutional Interdisciplinary Teaching Partnerships (SUMMIT-P). With a general aim of helping our STEM students become successful in their mathematics courses, the SUMMIT-P leadership team at NSU is guided by a driving force of improving our mathematics curricula by highlighting applications of mathematics in engineering majors. We accomplished this through a teaching partnership between the mathematics and engineering departments. Several small teams of faculty, comprising of two or three faculty from both departments, created and designed supplementary teaching resources that highlight the utility of mathematics in various engineering applications. These resources are then carefully embedded in existing mathematics curricula.

This paper is organized as follows. In Section 2, we present the processes that we have employed in order to support faculty conversation and collaboration between the two departments with a goal of improving the curriculum and creating teaching resources. In Section 3, we feature some of our teaching modules in the precalculus, calculus, and differential equations courses that have been particularly effective in achieving our goals. In Section 4, we mention some next-steps, recommendations, and other issues to consider.

Meeting of the Minds

Norfolk State University is the only HBCU member of an NSF IUSE-funded national consortium of universities called SUMMIT-P: Synergistic Undergraduate Mathematics via Multi-institutional Interdisciplinary Teaching Partnerships. Guided by the belief that perspectives and applications from outside the mathematics classrooms will influence vital changes towards a more effective mathematics curriculum, the SUMMIT-P team created observation, conversation and collaboration opportunities between the mathematics and engineering departments. A particularly effective observation opportunity was the "fishbowl" discussion [4], where the mathematics faculty observed and passively listened to the conversations among engineering faculty about the mathematics concepts that they would like their engineering majors to master. This activity revealed that engineering and mathematics professors have vastly different expectations from their students and tend to focus on different aspects of mathematics. Indeed, while a mathematics professor is bound by its calculus curriculum to use three weeks in mastering limit computations via analytical and algebraic pencil-pushing, an engineering professor may appreciate a more firmly connected approach by using the limit concept to talk about precision and accuracy.

Another approach to create conversation and collaboration opportunity, this time on the efficiency side of things, followed a speed-dating format: eight faculty members from both departments were divided into four groups, and each group was given ten minutes to talk about their research interests and teaching experiences. The leadership team decided to impose a ten-minute time limit so that the faculty participants would focus on the salient points of their research and teaching experiences. After ten minutes, the teams were rearranged to allow for meeting other participants. This activity ended with a grouping of four teams, each team consisting of at least one math and at least one engineering faculty, with a task of creating supplementary teaching resources that can be embedded in existing precalculus, calculus, and differential equations curricula. The criteria used for developing the teaching modules were that they should highlight engineering applications, should be original (that is, cannot be found in the usual mathematics textbooks), and should be interesting and engaging to the students. The main task for the engineering faculty member in each team was to suggest various engineering applications in which mathematical concepts play pivotal roles in the modeling process; the topics may come from their own engineering research or from the engineering major courses that they teach to higher-level students. The main task of the mathematics faculty member in the team was to make sure that the teaching modules were written and planned in such a way that they would complement and supplement the existing mathematics curriculum and syllabi of the course the application would be used in.

Sample Teaching Modules

In this section, we present some novel examples of engineering applications that were created by our faculty teams and have been particularly effective in engaging our students.

Precalculus: Parameter Estimation using Observed Refractive Indices

Even among college-aged kids, who would resist a short laboratory trip to an actual working optical laboratory with state-of-the-art equipment? The NSU Center for Materials Research houses a Variable-Angle Spectroscopic Ellipsometry (VASE), an equipment that is capable of providing highly-accurate measurement of various properties of thin films. Due to the COVID-19 pandemic, a short video of an engineering professor providing a tour of the laboratory replaced an in-person lab visit. While still a freshman, NSU's future engineers are gently introduced to the VASE capability of collecting permittivity indices of different kinds of materials.

Table I shows a representative example of observed permittivity values of thin films using the VASE, with varied wavelengths. Students were asked to compute the refractive index n using a formula that relates the real ϵ_1 and imaginary ϵ_2 permittivity indices for each of the various wavelengths given in Eqn. (1).

Table I. Representative readings of thin film permittivity values measured by VASE equipment.

wave_length	real_permittivity	imaginary_permitivity
350	6.31926655	1.46669569
370	6.10840677	0.342721901
440	4.83578974	0.011457119
550	4.37286874	0.001967483
710	4.13551023	0.000341121
920	4.00262169	0.001533069
1190	3.90588312	0.019329345
1280	3.88256249	0.031816157
1740	3.82928079	0.124451167
1970	3.83613928	0.169550951
2190	3.85447881	0.204530818
2300	3.86617499	0.218740434

$$n = \sqrt{\frac{\epsilon_1 + \sqrt{\epsilon_1^2 + \epsilon_2^2}}{2}} \qquad (1)$$

The formula for the refractive index given by Eqn. (1) involves straightforward complex analysis computations of comparing the real and imaginary parts. In this way, this standard precalculus skill of analyzing the components of a complex number is given engineering context and emphasizes the value of the computational approach. Moreover, from the precalculus professor's perspective, the Eqn. (1) is a great example to illustrate the use of a computing technology. Even though the students are performing a plug-and-chug activity, it reinforces the observation that students should not expect textbook examples (involving integers only) when they are dealing with real observed data.

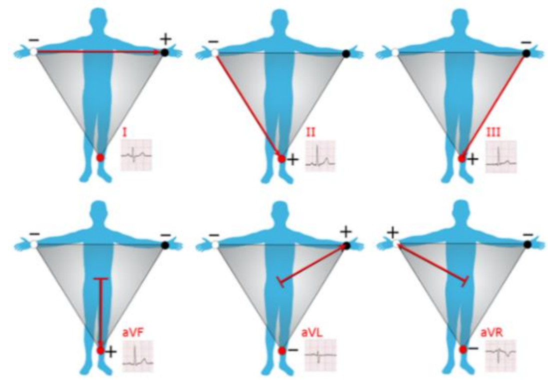

Figure 1. The Einthoven triangle is formed by taking three limb points with the heart in the approximate center. (Image Source: Wikipedia).

Continuing the precalculus computations, this engineering application can be further taken to the next-level by introducing parameter-estimation or data-fitting skills. For example, a simple rational function is traditionally used to obtain a Cauchy approximation of index n and wavelength λ, namely, $n(\lambda) = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}$. With this closed-form relation between the index and wavelength, the students can then enjoy their first foray in the area of fitting observed values to a theoretical relationship. Indeed, a straightforward spreadsheet code (e.g. Microsoft Excel or Google Sheets) will be able to compute the indices n as the fourth column and a method of least squares to compute the approximate values for $A, B,$ and C. With the computed values of the parameters $A, B,$ and C, the students now have a concrete example of a rational function that they can explore. With a specific context behind the rational function, the mathematics tasks of analyzing the domain, sketching the graph, and finding the asymptotes provide more value to the students.

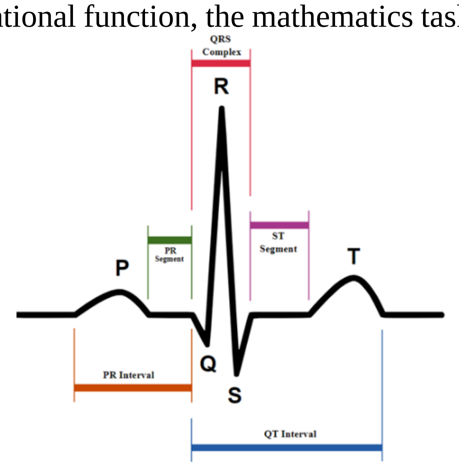

Figure 2. A schematic of a PQRST wave segment in an ECG. The P bump captures the atrial contraction, the QRS complex captures the ventricular contraction, while the T bump captures the re-polarization of the ventricles. Image source: Wikipedia.

Calculus: Graphical Analysis using ECGs

Throughout any undergraduate STEM student's academic training in the university, two-dimensional graphical analysis methods are used to understand various phenomena that illustrate relationships between the variable represented on the horizontal axis with the variable represented on the vertical axis. From the perspective of the precalculus and calculus professor, graphical analysis includes computing domain, range, intercepts, period, symmetries, asymptotes, intervals of increase/decrease, intervals of concavity, and points of inflection. Here, we propose an engineering application that can be used to strengthen students' appreciation and develop conceptual understanding of graphical analysis techniques. The application comes from signals analysis in biomedical engineering. An electrocardiogram (ECG or EKG) is a graphical visualization of heart's electrical signals in two-dimensional coordinate plane formed by two hands and left leg. The horizontal axis represents time while the vertical axis represents voltage recordings while the heart beats periodically. The discovery of electrocardiography is attributed to Willem Einthoven, a Dutch physician, who was awarded the 1924 Nobel Prize in Physiology and Medicine "for the discovery of the mechanism of the electrocardiogram." He proposed that if we connect electrodes on left arm, right arm and left leg, then these three limb points form an imaginary equilateral triangle (also called the Einthoven Triangle; illustrated in Figure 1), and the heart can be assumed to lie in the center of the triangle. An iconic segment of an ECG signal is called the PQRST wave, illustrated in Figure 2. A mathematics professor looking at a PQRST wave may be able to see possibilities of using ECG waves to illustrate relative extrema and cuspidal points, among others.

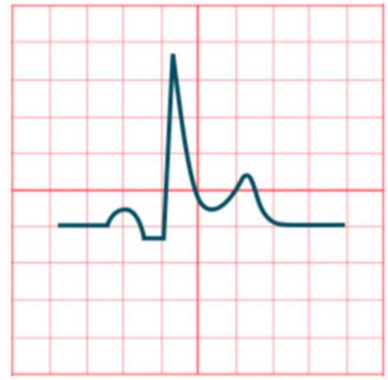

Figure 3. An example of ECG segment from a patient suffering from pericarditis.

Moreover, underlying principles used in ECG analysis can be applied to other bioelectrical signals from human body such as EEGs (electroencephalograms– signals from the brain), EMGs (electromyograms– signals from muscles), and EOGs (electrooculograms– signals from the eyes). The internet has a plethora of unlicensed images of segments of ECGs, EEGs, EMGs, and EOGs that can be used to create various questions and worksheets for precalculus and calculus 1 courses. For example, Figure 3 represents an ECG segment of a patient suffering from pericarditis, which is a condition where the pericardium (outer lining in the heart) has inflammation. From a calculus perspective, a worksheet activity question may be framed as “Sketch the derivative of this function.” To see more examples and possibilities, refer to [5].

When segments of ECGs are used in explaining graphical analysis concepts, students are reminded that graphs are not just abstract mathematical objects that mathematics professors seem to randomly sketch on the board. In the pericarditis example, the graph comes from a real-life application that physicians, nurses, and EMTs are expected to spot quickly and easily from an ECG reading.

Differential Equations: Laplace Transforms in Wireless Power Transfer

Differential equations are mathematical tools that can be used to model many engineering systems, including wireless power transfer systems (WPT), which has significantly changed our daily life by enabling a large variety of wireless charging applications. Although the design implementation of a WPT system is complex, the underlying principle is simple. Since WPT transfers power without the use of cables or wires, this engineering breakthrough creates a lot of advantages and has large-scale

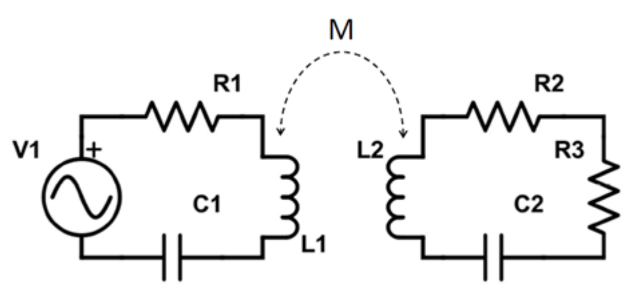

Figure 4. A simplified equivalent circuit of a WPT. Here, L1, L2, M are the inductors, R1, R2, R3 are the resistors, C1, C2 are the capacitors, and V1 is the voltage power source.

applications in agriculture, biomedical engineering, aerospace engineering and modern transportation systems. A simplified circuit diagram of a simple WPT system is illustrated in Figure 4.

The equations that govern this equivalent circuit is given by a system of two second-order linear differential equations with constant coefficients as given by Eqn. (2), one of which is non-homogeneous while the other one is homogeneous.

$$\begin{cases} L_1 \dfrac{d^2\, I_1(t)}{dt^2} + M \dfrac{d^2 I_2(t)}{dt^2} + R_1 \dfrac{dI_1(t)}{dt} + \dfrac{I_1(t)}{C_1} = \dfrac{dV_1(t)}{dt} \\ L_2 \dfrac{d^2\, I_2(t)}{dt^2} + M \dfrac{d^2 I_1(t)}{dt^2} + (R_2 + R_3) \dfrac{dI_2(t)}{dt} + \dfrac{I_2(t)}{C_2} = 0 \end{cases}$$

Where $I_1(t)$ is the instantaneous current in the primary circuit (consisting of R1, L1 and C1), $I_2(t)$ is the instantaneous current in the secondary circuit (consisting of R2, R3, L2 and C2), and M is the mutual inductance between the two linked circuits. A picture of a WPT system that we use in class is illustrated in Figure 5.

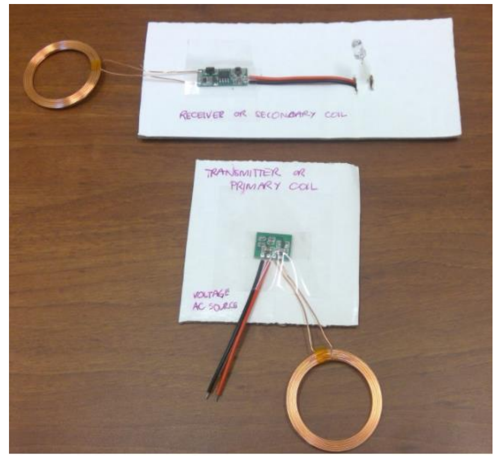

Figure 5. A basic WPT system consists of a receiver circuit and a transmitter circuit. When the black and red cables are connected to a power source, the LED bulb will light when the two circular coils are brought in a close proximity.

In traditional differential equations textbooks, mechanical examples of a single second-order differential equations include pendulums and masses connected to strings. These examples are fundamental and important, unfortunately, these examples may not keep the 21st engineering student engaged and interested. Imagine the challenge for some students when more than one second-order differential equations are coupled together! With the goal of preserving student interest using real-life examples, we propose that the example of WPT system be used as a practical, engaging, and interesting example of a system of two second-order differential equations [6]. This engineering breakthrough can be seen in various applications that students see in their everyday lives, including, wireless toothbrushes, cellphone chargers, and even charging electric cars. From the perspective of the mathematics professor, the WPT system is a great example to illustrate many solution methods in the differential equations course, namely, Laplace transforms, method of systematic elimination, method of undetermined coefficients, and variation of parameters.

Classroom Implementation and Assessment

These application modules were administered in in-person (pre-pandemic) as well as online (during and post-pandemic) math classes. During in-person classes, the hands-on module activities were conducted in engineering labs. During online classes, the activities were taught virtually and video recording on hands-on demonstrations were shown in class. The student curiosity and engagement were evident during the activities. For each application module, lab worksheets were prepared for the students to complete during the activities. The worksheets were designed to ensure student participation and engagement during the modules. The effectiveness of the modules was assessed through pre- and post-activity survey instruments. The student feedback was also recorded for each module through the post-activity survey. This is an ongoing activity, and the survey responses and feedback are being analyzed for quantifying the effectiveness of these modules. Further assessment of the application modules by tracking the student cohort's performance in advanced engineering courses is planned.

Summary and Conclusions

Various teaching and technology initiatives have been suggested to improve engineering education in HBCUs. In this report, we highlighted the efforts of the NSU SUMMIT-P team in rejuvenating the mathematics curricula in order to support our students' path to success in higher-level engineering courses. We created carefully crafted applications of precalculus, calculus, and differential equations skills and concepts to interesting and engaging engineering examples. This ongoing contextualized teaching and learning approach is grounded on a solid collaboration and partnership between engineering and mathematics faculty.

Acknowledgement

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