
AC 2012-3202: APPLICATIONS OF MODERN PHYSICS: A SOPHOMORE-LEVEL PHYSICS COURSE AND LABORATORY FOR ELECTRICAL ENGINEERING STUDENTS

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Applications of Modern Physics: a sophomore-level physics course and laboratory for electrical engineering students

Abstract

The transition from lower-level to upper-level physics courses is difficult for many students as the course material becomes more abstract, and the mathematics more sophisticated. In this paper, we describe the development of a sophomore-level *Applications of Modern Physics* course that bridges the lower-level and upper-level curriculum for electrical engineering and physics students. The course starts from the atom and quantum mechanics, building up to nano-scale systems, and finally solids and devices. Applications, such as lasers, quantum dots, diodes, and superconductors, are interwoven throughout.

The laboratory for the course is closely tied to the class and illustrates complex concepts such as quantized energy levels and probabilities in classical and quantum physics. It follows the theme of “particles in a box.” Laboratories, which are under development, consist of tutorials using simulations, computational modeling using Matlab, and brief, illustrative experiments. Our primary goal in this course is not to develop strong experimental skills, as this is addressed in several other courses in the curriculum, but rather to use experiments to engage students with the material as they test the validity of the computational models they develop. The laboratories will feature the interplay between modeling and experiment that is central to the advancement of scientific knowledge, and they will give students the theoretical background, mathematical and computational skills they need. Course development will be guided by established best-practices in Physics Education Research.

Our goals in presenting this paper are to receive feedback from the engineering education community on the design and development of this course, and to inform the engineering education community of this effort, as we hope elements of this course can be adapted for use at other institutions.

Introduction

The School of Engineering at the University of St. Thomas was established in 2004. Students in the Electrical Engineering (EE) program are required to take several Physics courses: the calculus-based introductory *Classical Physics* sequence, *Applications of Modern Physics*, and a two-semester *Electricity and Magnetism* (E&M) sequence. The introductory physics and E&M sequences are well established. However, it became clear after the EE program was started that we needed to redevelop the *Applications of Modern Physics* course so that it bridges the introductory and advanced courses, is relevant and useful to both Physics and EE majors, and motivates further study in both fields. Our goal is to develop a sophomore-level *Applications of Modern Physics* course based on Physics Education Research proven best-practices that:

- (1) as an allied requirement for Electrical Engineering, provides students the necessary physics background to understand semiconductor devices and nano-scale systems, and

- (2) as a required course for EE and Physics majors, serves as a bridge between the introductory and advanced physics sequences and motivates the advanced courses through real-world examples.

At the University of St. Thomas, the introductory Physics sequence is taught as a modified workshop course, with laboratory, problem-solving, and lecture combined into a longer class period. *Applications of Modern Physics*, on the other hand, was traditionally taught as a regular lecture course with an associated laboratory. The laboratory was taught by a different instructor, and did not closely parallel the material in lecture. Students struggled with the transition from introductory physics to modern physics, given the more difficult concepts and mathematics involved, and the lack of guided problem-solving. The lack of proficiency and confidence in mathematical skills has been highlighted in the Institute of Physics' (IOP) report "Physics – Building a Flourishing Future",¹ and confirmed by our own experiences. In the redeveloped course, in-lab tutorials are used to address this issue, as discussed in the "Course Development" section, below.

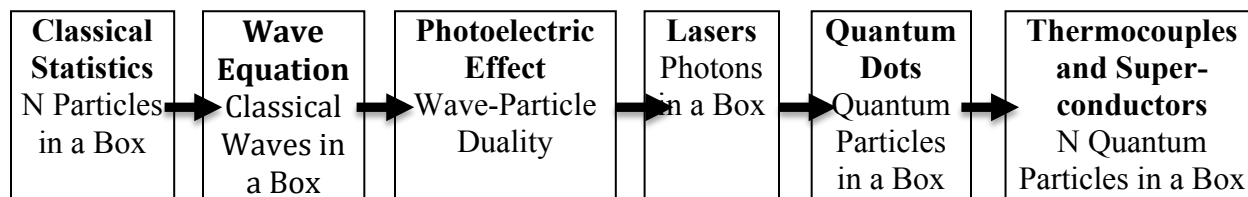
We expect that students in upper level courses have competent programming skills so that they can begin attacking more complex real-world physics problems, but the original course did not contain enough computational training to fit this need. In the redeveloped course, laboratories center around computational modeling, while experiments serve to engage students with the material, illustrate the physics behind the computational models, and to tie back to concepts discussed in the course. Because our students gain complex experimental skills in several other courses, experimental skills are not emphasized in the laboratory associated with *Applications of Modern Physics*. A diagram showing how *Applications of Modern Physics* fits into the Electrical Engineering curriculum, as well as the relative emphasis of theory, experiment, computation, and technical communication in each physics course, is included in Appendix A.

Course Development

The first stage of course development has happened over the past three years. For spring 2010, we decided to have the same instructor teach both the *Applications of Modern Physics* lecture and laboratory, and use part of the laboratory period to coach students in problem-solving. Working in groups, students completed several tutorials (modified from those developed at the University of Colorado-Boulder) that use PhET (Physics Education Technology)² simulations. This proved successful in making students more at ease with the mathematics while also addressing conceptual difficulties. As an added benefit, it allowed the instructor to build a better relationship with the students, given the smaller laboratory sections. Students were more comfortable asking questions and participating in class, and often had prolonged discussions about the material during the laboratory period, both with each other and with the instructor. The same model for the course was used in 2011, but with further modifications to the tutorials as the laboratory was designated a *Writing to Learn* course in the University of St. Thomas *Writing Across the Curriculum* initiative.

The redeveloped course covers modern physics from a materials science perspective, i.e. building from the properties of single atoms to collections of atoms (quantum dots and other nano-scale systems), and then to solids, with applications interwoven throughout. Time in the

laboratory is used to reinforce concepts learned in class through the use of simulations and tutorials, while also introducing students to computational modeling using Matlab, a high-level computer language that is widely used in scientific and engineering applications. Both Physics and Electrical Engineering students use Matlab throughout their upper-level courses. Furthermore, competency in Matlab is an in-demand skill on the jobs market. The computational modeling is followed by brief, illustrative experiments to test the validity of the models. The common thread in the laboratories is the “particle-in-a-box,” as shown in the following diagram:



This is a daunting task that would not be possible in a traditional Modern Physics course that included relativity and other topics found in our *Foundations of Modern Physics* course (which complements *Applications of Modern Physics* and is a required course for our Physics majors), or with a traditional Modern Physics textbook. We have picked a textbook³ that reflects our philosophy for the *Applications of Modern Physics* course. After reviewing the literature and course materials available online, we decided to adapt the Physics-Education-Research-based course materials developed at the University of Colorado-Boulder,⁴ which consist of lecture slides, tutorials and the use of on-line simulations, to fit the *Applications of Modern Physics* course.

The objectives we have established for course development are:

- I. Adapt and expand course materials developed by the University of Colorado – Boulder Physics Education Research group in order to increase students’ conceptual understanding of modern physics topics, and
- II. Develop laboratories that (1) target important conceptual issues, (2) improve students’ computational skills, and (3) demonstrate the interplay between theory, modeling and experiment in physics.

The redeveloped *Applications of Modern Physics* course consists of three distinct pieces:

(1) **Simulations:** guided inquiry using PhET (Physics Education Technology) interactive simulations, which have been shown to facilitate conceptual understanding,⁵ followed by brief, illustrative experiments.

As Steinberg et al.⁶ point out, it is important that students have “a strong conceptual base when learning more advanced topics in physics” and it is also important to “continuously and systematically probe student learning by using the tools of physics education research.” The goal of the tutorials in the *Applications of Modern Physics* course is to have the students work in groups while being coached by the instructor, therefore making the mathematics more accessible

and providing time for prolonged discussion of the concepts. The use of simulations in physics courses has been shown to increase students' conceptual understanding by focusing students' attention to relevant details.⁵

Students will take an online quiz after each laboratory, modeled after the "Weekly Reports" developed at Rutgers University,⁷ to assess their conceptual understanding of the topic covered in lab that week. These post-lab assessments will help the instructor tailor the following lecture(s) to address any remaining difficulties, and will also guide the revision of the laboratories. Students' responses to exam questions will then be used to assess students' understanding once again.

(2) **Modeling + Experiment:** Computation is now a ubiquitous tool in science and engineering, complementing theory and experiment. There have been several successful efforts to introduce computation in the introductory physics sequence and upper-level curriculum (for example, Matter and Interactions,⁸ the course developed by J. Tobochnik and H. Gould,⁹ and projects at Lawrence University¹⁰ and Brigham Young University,¹¹ among others¹²), using VPython, Maple, and other computational languages. However, as pointed out by Patterson,¹³ "the interests, backgrounds, and abilities of students taking physics courses vary widely . . . and it is a challenge to provide appropriate courses for diverse student populations," such as Physics and Electrical Engineering majors who need to acquire proficiency in Matlab. Although the University of St. Thomas is one of a few liberal arts universities with a school of engineering, this is the norm at R1 institutions, and many physics graduates at liberal arts schools go on to graduate programs in engineering, so the course materials we develop could be transferred and adapted to those institutions.

Laboratories, some of which are still under development, consist of computational modeling using Matlab, followed by brief, illustrative experiments. It is not our goal in this course to develop strong experimental skills, as this is addressed elsewhere in the curriculum, but we will use experiments to engage students with the material as they test the validity of the computational models they develop. Students will write a laboratory summary where they analyze their experimental data and discuss the connection between the theory, the computational model, and the experiment.

(3) **Exams:** Given both the qualitative and the quantitative nature of the material in the *Applications of Modern Physics* course, we have found that the 65-minute lecture period is not enough time to adequately test students' knowledge. Furthermore, for students with test anxiety, the limited time is an important factor in exacerbating the anxiety. Having exams during the 3-hour laboratory period will help relieve test anxiety due to the time constraint, allow us to incorporate a much richer variety of questions into the exam to target students with different learning styles, and give us time to test the students on some of the more applied aspects of the course. For example, students could be asked to write a simple simulation using Matlab or to take the spectra of a discharge tube and explain, by using an energy-level diagram, why only a few lines are present.

Laboratory development

Plans for the laboratory are outlined below. Preliminary versions of most laboratories have been used in the course in the past two years, but all laboratories are still in development. A full description of the laboratories can be found in our course development website.¹⁴ Course materials can be obtained by contacting the author.

Laboratory #1: Classical Statistics: A Classical Particle in a Box

- *Conceptual goal:* Develop familiarity in applying statistics to classical physics.
- *Modeling goal:* Develop an understanding of how a computational model is developed and implemented.
- *Computational goal:* Introduction to Matlab, for loops and if statements, basic plotting.

As no prior exposure to Matlab is assumed in this course, the first week is dedicated to familiarizing the students with Matlab through the tutorials found in reference.¹⁵ We have developed a handout that then guides students in using Matlab to model a classical particle in a two-dimensional box and to track statistics for the particle (time spent on the left half vs. the right half of the box, for example). In the laboratory, students understand what it means to discretize an equation, and have to decide on an appropriate time-step and how long to run their model in order to get adequate data.

Laboratory #2: Classical Statistics Experiment: A Boltzmann Machine

- *Conceptual goal:* Apply a statistical analysis to a classical system.
- *Experimental goal:* Use a Boltzmann machine to determine the fraction of time a ping-pong ball spends on the upper level of a two-level system.

When students are first exposed to quantum mechanics they face several difficulties. First, as suggested by L. Bao and E. F. Redish,¹⁶ it is traditionally assumed that students understand probability and energy level diagrams, yet they cannot relate a probabilistic representation to an actual observation. In this laboratory, students will use a Boltzmann machine, as suggested by J.J. Prentis¹⁷ to simulate canonical statistics. The Boltzmann machine consists of a two-level system with a motorized ball confined to each level, and a ping-pong ball that is free to move between levels. The time that the ping-pong ball spends in each level is an exponential function of the energy gap of the system (mgh , where m is the mass of the ping-pong ball, g is the gravitational constant, and h is the height difference of the two levels). The concept of degeneracy of a level is introduced, as it is proportional to the surface area of each level. If a_0 is the area of the lower level and a_1 the area of the upper level, then the ratio of the time the ping-pong ball spends at each level is

$$t_1 / t_0 = (a_1 / a_0) \text{Exp}[-mgh/kT]$$

where the temperature factor, kT , corresponds to the average kinetic energy of the ping-pong ball.

Students will be asked to make a prediction for the ratio of the time the ping pong ball spends on the upper and lower levels of the Boltzmann machine. They will then observe patterns on the Boltzmann machine, and modify their predictions in order to explain the experimental results.

Laboratory #3: Classical Statistics Simulation: A Boltzmann Machine

- *Conceptual goal:* Apply a statistical analysis to a classical system.
- *Modeling goals:* Gain an understanding of what is an appropriate time-step and how long to run a model. Develop an awareness of the interplay between theory, modeling and experiment.
- *Computational goal:* Nested for loops, animations.

The computational component of this laboratory will consist of modeling the Boltzmann machine. We will guide the students through the modeling process, but will leave enough room to allow for students to make decisions about important physical aspects such as initial conditions, what data to record, and how to analyze the data.

Laboratory #4: Wave Equation: Classical Waves in a Box

- *Conceptual goals:* Become comfortable with the mathematical formalism of differential equations. Understand how applying boundary conditions leads to quantization.
- *Experimental goal:* Use standing waves in a microwave oven to estimate the speed of light.

We have adapted the Wave Equations tutorial from the materials developed at the University of Colorado-Boulder.⁴ In the wave equation tutorial, students solve the wave equation for electromagnetic radiation inside a microwave oven, and find that the boundary conditions lead to only certain wavelengths being allowed (a first glimpse of quantization). The tutorial developed at UC-Boulder will be extended to include visualization of the stationary waves in two dimensions by plotting in Matlab. Students will use paper moistened with a CoCl_2 solution to reveal the pattern of microwaves in an oven, following reference¹⁸. They will then estimate the speed of light from the distance between nodes on the paper and the known operating frequency of the microwave.

Laboratory #5: Photoelectric Effect

- *Conceptual goals:* Understand how the photoelectric effect experiment leads to wave-particle duality. Understand that when a particle tunnels through a barrier, energy is conserved.

In this tutorial, students explore the photoelectric effect experiment using a PhET simulation.² By changing the settings (intensity, wavelength of light, battery voltage) in the simulation, students are guided through the reasoning that lead to the proposal of the wave-particle duality for electromagnetic radiation.

Laboratory #6: Lasers: Photons in a Box

- *Conceptual Goal:* Develop a basic understanding of the relationship between energy levels in an atom and how lasers work.
- *Experimental Goals:* Use of a spectrometer to record intensity vs. wavelength. Relate results to an energy level diagram.

Both Physics and EE students should understand the basic operation of Lasers since they are widely used in research and industry. Students will discover the requirements for building a laser while following this tutorial and exploring the associated PhET simulation.² We have extended the tutorial developed at UC-Boulder to include rate equations and Einstein coefficients. We plan to develop an experimental portion for this laboratory in which students will use spectrometers to record the spectra of the side (non-lasing) light of a clear-case HeNe laser. Students will analyze the photon counts for different wavelengths and develop an energy level diagram that includes the different electronic transitions involved.

Laboratory #7 Quantum Dots Experiment: Quantum Particles in a Box

- *Conceptual goals:* Learn about a system whose properties can only be explained by using quantum mechanics. Relate the size-dependent band gap of the quantum dots to their emission spectra.
- *Experimental goals:* Use of a spectrometer to record intensity vs. wavelength for four different vials. Compare the spectra obtained and relate results to the different sizes of the quantum dots.

The drive to understand the nano-scale (systems of size $\sim 10^{-9}$ m) originated, in part, because of the electronics industry. Electronic circuits are approaching length scales of 100 nanometers. The challenge for researchers, and for our students, is to understand the fundamental physics involved.

An atomic cluster is a nano-scale system that contains only a few to a few thousand atoms. Much research is currently being done on atomic clusters, as it was discovered that, because of quantum confinement, their properties (for example, at what wavelengths they absorb or emit light) can be size dependent. It is thus possible to tailor these properties, which might lead to new technological applications. For the experimental portion of this laboratory, we have adapted the handout that accompanies CENCO Physics' quantum dots.¹⁹ Students learn how to use a spectrometer to record emission data for a set of CdSe quantum dots, and how to relate the peak emission wavelength to the semiconductor's gap by modeling the dots as semiconductor spheres with a particle and a "hole" (the absence of an electron).

Laboratory #8 Quantum Dots Simulation: Quantum Particles in a Box

- *Conceptual goals:* Learn about a system whose properties can only be explained by using quantum mechanics. Relate the size-dependent band gap of the quantum dots to their emission spectra.
- *Modeling goal:* Learn how differential equations are discretized and solved in a computer.
- *Computational goal:* Eigenvalues and eigenvectors in Matlab.

Students will be asked to model the system in Matlab by numerically solving for the first few eigenvalues and eigenfunctions of the three-dimensional Schrödinger equation for a particle in a box with infinite potential outside the box. Our students are generally enrolled in differential equations, but have not taken linear algebra by the time they take *Applications of Modern Physics*. We will develop a tutorial (adapted in part from references²⁰ and²¹) where students will be guided through how to discretize functions, use the finite difference approximation for the second-order derivative, write the Schrödinger equation as a Hamiltonian matrix, and finally diagonalize the matrix to obtain the eigenvalues and eigenfunctions.

Laboratory #9: Thermocouples and Superconductors Experiment: N Quantum Particles in a Box

- *Conceptual goals:* Obtain a qualitative understanding of the significance of the Fermi energy, the physics behind thermocouples, and the difference between a perfect conductor and a superconductor.
- *Experimental goals:* Calibration of a thermocouple.

Thermocouples are ubiquitous temperature sensing devices. Physics and Electrical Engineering students often use thermocouples without fully understanding the physics behind their operation—they are deceptively simple devices. This lab introduces thermocouples as a direct application of topics covered in lecture, particularly emphasizing the importance of the Fermi energy level in metals. When a temperature gradient is established across a metal, there is a net potential difference across the system. To measure this potential difference, the connecting wires need to be of a different material as the sample; otherwise, an identical but opposite voltage would develop across the wires. When two different metals (the sample and connecting wires) are brought together, their Fermi levels align by moving electrons from one metal to the other such that the total energy of the system is lowered. In this case, it is possible to measure a net potential difference, which will depend on the temperature difference between the two points where the sample is measured. This property is exploited in a thermocouple, where one junction between two metals is held at a reference temperature, and the other used to sense the temperature of interest.³ In this way, temperature from tenths to hundreds of degrees Kelvin can be measured.

For the experimental portion of this laboratory, we have developed a handout that guides students in designing and carrying out an experiment to calibrate a T-type thermocouple using known boiling, condensation, melting and sublimation temperatures of different substances such as water, dry ice, acetone, and liquid nitrogen. Students fit their voltage vs. temperature data to a polynomial using Matlab, and use their results, along with the Meissner effect, to measure the critical temperature of a superconductor. The Meissner effect relates to the fact that a superconductor will expel all magnetic fields by developing a surface current.³ A magnet will levitate above a superconductor cooled below its critical temperature. As the superconductor's temperature rises, superconductivity is gradually lost until, at the critical temperature, the levitating magnet comes to rest on the superconductor's surface.

Laboratory #10: Thermocouples and Superconductors Simulation: N Quantum Particles in a Box

- *Conceptual goals:* Obtain a qualitative understanding of the significance of the Fermi energy, the physics behind thermocouples, and the difference between a perfect conductor and a superconductor.
- *Modeling goal:* Further the students' understanding of the interplay between theory, modeling, and experiment (what happens when a model does not fit the experimental results? Is the theory wrong? Is the model too simple?).
- *Computational goal:* Use of random numbers, polynomial fits.

We will develop a laboratory in which students explore how a thermocouple works by first making a computational model of a conductor with a temperature gradient as N particles in a box whose velocities depend on the local temperature. This is a good model for materials with negative Seebeck coefficient, as explained in reference ³, but does not explain how some materials, such as copper, can have positive Seebeck coefficients. Students will be asked to brainstorm possible modifications to the model that would account for the latter type of materials. Energy-dependent mean free scattering times (due to scattering from lattice vibrations) will be added to the model as random forces on the particles to yield positive Seebeck coefficients.³

Course and Laboratory Assessment Plan

The course and laboratory will be assessed in a number of different and complementary ways:

1. Piece-wise: Following the model of Weekly Reports (WR) developed at Rutgers University,⁷ students will take an online quiz after each laboratory to assess their conceptual understanding of the topic covered in lab that week. The instructor will use the Weekly Reports to tailor the following lecture(s) to address any remaining conceptual difficulties, and to guide revisions of the laboratories. Responses to exam questions will then be used to assess students' understanding once more.

2. Globally:

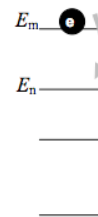
(a) A comprehensive qualitative final exam with multiple choice and short-answer questions will be given focused on testing students on the learning goals established for the course: knowledge of the concepts of energy levels and probability densities, and how they apply to physical devices. We will model exam questions after the ones shown in reference ⁶.

(b) Pre- and post-tests using the QMCS²⁴ (Quantum Mechanics Conceptual Survey) and CLASS²⁵ (Colorado Learning About Science Survey), along with the students' Weekly Reports and answers to exam questions will be used to measure conceptual understanding of basic quantum mechanics and attitude about science with the goal of being able to compare the course pre and post-reform (QMCS pre and post-tests have been given to students since 2009; CLASS was given for the first time in 2011), as well as compare results across institutions.

A relevant sample question from the QMCS is shown below:

1. The diagram at right shows the electronic energy levels in an atom with an electron at energy level E_m . When this electron moves from energy level E_m to E_n , light is emitted. The greater the energy difference between the electronic energy levels E_m and E_n ...

- A. ...the more photons emitted.
- B. ...the brighter (higher intensity) the light emitted.
- C. ...the longer the wavelength (the more red) of the light emitted.
- D. ...the shorter the wavelength (the more blue) of the light emitted.
- E. More than one of the above answers is correct.



And sample statements from the CLASS:

- 8. When I solve a physics problem, I locate an equation that uses the variables given in the problem and plug in the values.
- 13. I do not expect physics equations to help my understanding of the ideas; they are just for doing calculations.
- 21. If I don't remember a particular equation needed to solve a problem on an exam, there's nothing much I can do (legally!) to come up with it.
- 26. In physics, mathematical formulas express meaningful relationships among measurable quantities.

Concluding Remarks

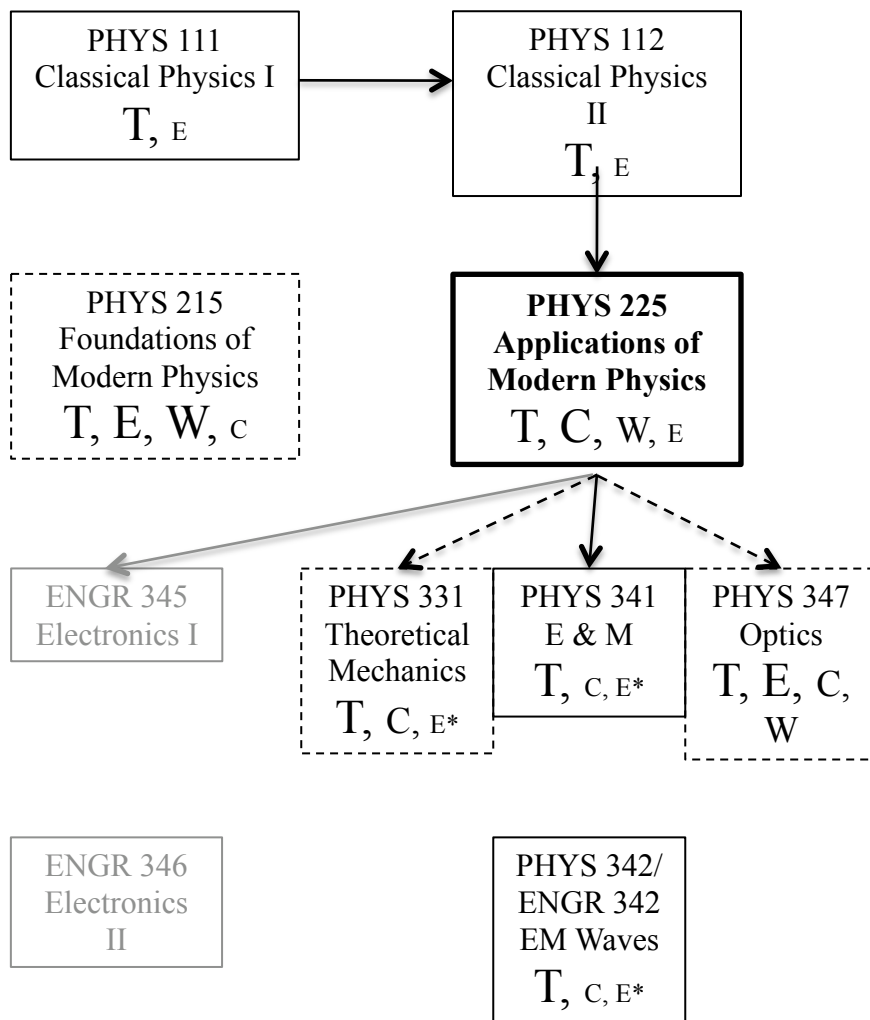
The *Applications of Modern Physics* course presented in this paper is meant to bridge the introductory and advanced physics courses, be relevant and useful to both Physics and Electrical Engineering majors, and motivate further study in both fields. The laboratory associated with the course is used to reinforce concepts learned in class through the use of simulations and tutorials, while also introducing students to computational modeling using Matlab. The computational modeling is followed by brief, illustrative experiments to test the validity of the models.

We welcome feedback on the course development plan we have presented. As laboratories are developed, we will add them to our course development webpage. Please contact us if you have any questions or would like to use our materials in your own course. We hope to present results of our assessments at a future ASEE conference.

Acknowledgements

We are grateful to the University of St. Thomas Department of Physics, the University of St. Thomas Faculty Development Center, and MathWorks Inc. for supporting this project.

APPENDIX A: Diagram showing how *Applications of Modern Physics* fits to the Electrical Engineering curriculum



Note:

T = theory, E = experiment, C = computation, W = writing/technical communication

Font size indicates relative emphasis. Three approximate levels: **A**, A, A

Dashed lines indicate elective courses for EE majors.

Applications of Modern Physics is also an elective for Mechanical Engineering majors.

* In development

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