Apply Second Order System Identifications

Dr. Tooran Emami, U.S. Coast Guard Academy

Tooran Emami is currently a faculty member in the Department of Engineering at the U. S. Coast Guard Academy (USCGA). She received her M.S. and Ph.D. degrees in Electrical Engineering from Wichita State University (WSU) in 2006 and 2009, respectively. Her research interests are in control systems and particularly are dynamic positioning, autonomous vessel, Proportional Integral Derivative (PID) controllers, robust control, time delay, compensator design for continuous-time and discrete-time systems, and analog or digital filter design.

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Abstract

This paper presents a 2^{nd} order system identification of a linear time invariant system in an undergraduate junior level control systems laboratory. In this laboratory students identify a system transfer function from the parameters of cascade Resistor-Inductor-Capacitor (RLC) circuit by computer programming and analyze the output response. Electrical Engineering students use MATLAB software to determine the relationship between the standard 2^{nd} order system transfer function with the simple RLC circuit parameters. This laboratory reinforces students' learning about the effect of tuning the system parameters and demonstrates a wide range of system performance in time and frequency domains. Students learn about the relationship between the locations of poles and the system output responses. They learn about the relationship between the damping ratio and natural frequency response with the RLC circuit parameters in time domain and frequency responses.

In the second part of this laboratory students estimate the system parameters from a given time domain step response of a vessel output response. A continuous time transfer function of the vessel is identified from the measurement data. The vessel roll dynamics has been defined as a transfer function of roll-angle and the disturbance torque input. In this lab students apply the principle of standard 2^{nd} order system identification to the vessel motion about its roll axis. Students in this lab demonstrate achievement of numerous a-k ABET criteria.

Introduction

The United States Coast Guard Academy (USCGA) like many institutions around the world enhances an active teaching procedure in the Autonomic Control Systems course. One of the challenges in engineering education today is to motivate students to learn about the fundamental control systems concepts in an undergraduate control course. This paper present a process of teaching the concept of integration from the 2^{nd} order transfer function of a simple cascade Resistor-Inductor-Capacitor (RLC) circuit to the standard 2^{nd} order system transfer function in control course. This laboratory also teaches students about an application of standard 2^{nd} order transfer function that they would see in their career. Students determine the 2^{nd} order modeling for a linear time invariant system. They exercise how the location of poles can be changed based on the variation of damping ratio and natural frequency parameters. These responses illustrate as over damped, under damped, undamped, and critically damped. They learn about the impact of damping ratio and natural frequency responses on the step and the frequency response performances. In the second part of this laboratory students estimate the system parameters from a given time domain response of a vessel at sea. This laboratory allows students an active learning about some of the main concepts in the text books in a 160 minutes laboratory based on computer programming.

Many institutions around the world established control system laboratory [6,7,8,9,10,11,12]. O'Brien and Watkins developed new methods in teaching controller design[6,7,8] for undergraduate students. They defined a unified approach for teaching root locus, Bode design, and then applied it to a physical system for the control system laboratory[8]. Joel Lenoir[9] created a combined course for mechanical vibrations and controls course. In this course students applied the mathematical modeling and simulation skills to a system in lab to support the theoretical concepts that they learned in class. Jack[10] supported the laboratory and project for the control course by using microcontrollers in junior level for mechanical and manufacturing students. Emami[11] developed a laboratory for teaching the process of 1st order system modeling for a DC motor system. Emami and Benin developed how the computer helped to teach the concept of Routh Hurwitz criterion in undergraduate control systems and software engineering courses[12].

The current paper presents the processes of the 2nd order system identification in one control systems laboratory. In the first part of laboratory students learn about finding the transfer function in terms of RLC circuit and the standard 2nd order system transfer function parameters. The relationship between RLC circuit with damping ratio, natural frequency, and the pole locations are studied in both time and frequency responses. In the second parts of laboratory students estimate the damping ratio and the natural frequency response from the step response data of a vessel at sea[1]. They apply the principle of standard 2nd order system identification to the vessel motion about its roll axis. The vessel roll dynamics is defined as a transfer function of roll-angle and the disturbance torque input.

Part 1: Relationship between RLC Circuit and Standard 2nd Order System

Consider a second order low pass filter shown in Figure 1; the continuous time transfer function of this cascade RLC circuit can be defined as the ratio of Laplace transform of output voltage, $V_{out}(s)$, across the capacitor and the input source voltage $V_{in}(s)$, such as: $G(s) = \dfrac{V_{out}(s)}{V_{in}(s)}$.

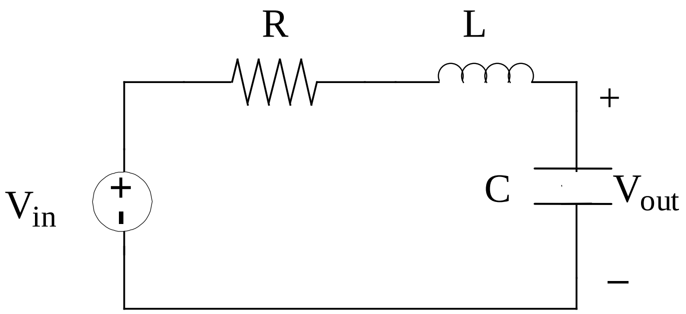


Figure 1: RLC Circuit

It is easy for students to find the transfer function in terms of RLC elements such as:

$$G(s) = \frac{V_{out}}{V_{in}} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}} \tag{1}$$

In the first undergraduate control course the standard second order system transfer function is presented to students such as[1,2,3,4,5]:

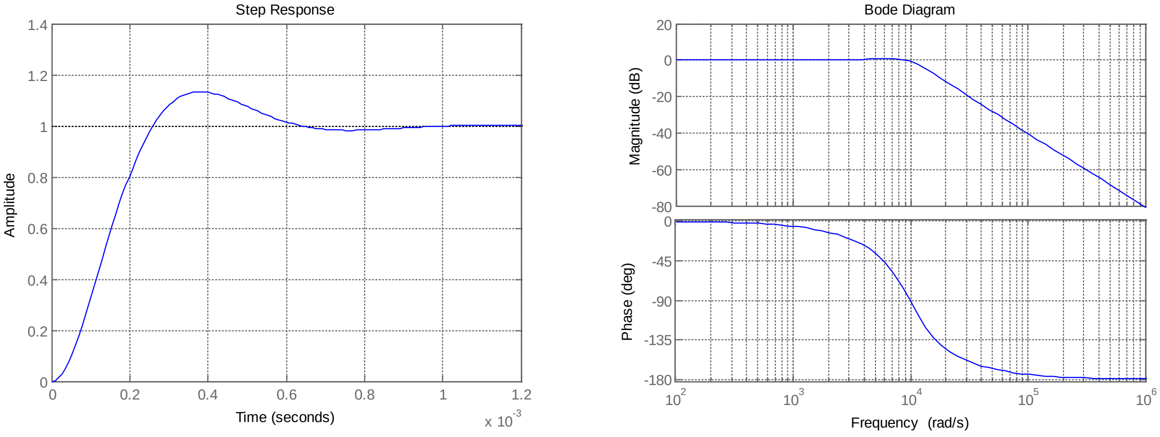
$$G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{\omega_n^{\ 2}}{s^2 + 2\zeta\omega_n s + \omega_n^{\ 2}} \tag{2}$$

where ζ is the damping ratio and ω_n is the natural frequency. One of the objectives of this laboratory is to learn about the connection between the RLC parameters as an electrical circuit with the damping ratio and natural frequency parameters based on the computer programming in addition to the traditional calculation in the text books[1,2,3,4,5]. The relationship between RLC circuit elements and ζ, and ω_n can be determine such as:

$$\omega_n = \frac{\sqrt{LC}}{LC} \text{ and } \zeta = \frac{R\sqrt{LC}}{2L} \tag{3}$$

As can be seen from equation (3) the natural frequency is a function of inductance and capacitance elements. The damping ratio is a function of all three RLC circuit elements. From these equations students can predict the variation in inductance and capacitance elements control the natural frequency and the variation of all three RLC circuit elements control the damping ratio.

One of the key concepts for students in the first classical control course is to learn about the stability and the effect of circuit elements on the time and frequency responses. Students write a program in MATLAB for a given values of L=47mH, C=0.22μF, and R=500 Ω, to find the numerical transfer function from equations (1) and (2), such as $G(s) = \frac{V_{out}(s)}{V_{in}(s)}$. They look at the step (Figure 2-a) and frequency (Figure 2-b) responses for the continuous time transfer function. The first observation of both the step and frequency responses are the system is stable.



(a) (b)

Figure 2: Step (a) and frequency (b) responses of the RLC circuit

The pole locations based on RLC variables from equation (1) can be found as:

$$s_{1,2} = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2} \tag{4}$$

The pole locations based on ζ and ω_n variables from equation (2) can be determined as:

$$s_{1,2} = -\zeta\omega_n \pm \sqrt{\left(\zeta\omega_n\right)^2 - \left(\omega_n\right)^2} \tag{5}$$

Students plot the s-plane pole locations for the continuous time transfer function for the given circuit elements as it is shown in Figure 3. The pole locations are in the left-half of Laplace plane (s-plane). The left half plane poles predict a stable system that also verifies the stable step response in Figure 2a.

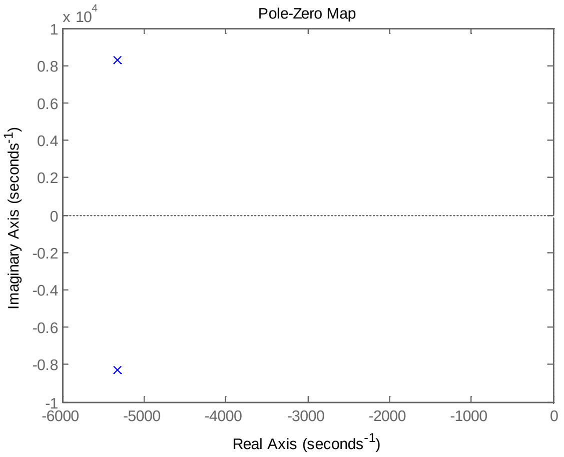


Figure 3: The pole locations of the RLC circuit

The variation of damping ratio and natural frequency impact the pole locations and the circuit elements from the equations (4) and (5). To start with the exercise students are asked to vary the damping ratio while keeping a constant natural frequency. They plot the continuous time pole locations and analyze the effect of damping ratio. The pole locations for different values of the damping ratio are shown in Figure 4. It is easy for students to see at damping ratio of zero there are two poles on the imaginary axis of s-plane. As the damping ratio increases between 0-1 the complex poles move away from imaginary axis and get close to real axis. At damping ratio equal one the two repeating poles are located on the real axis. The step and frequency responses of the system with varying the damping ratio are shown in Figure 5 and 6. Students can clearly see in Figure 5, the poles on the imaginary axis corresponds an undamped oscillation response, the repeated poles on the real axis corresponds a critically damped system response, and the rest of left-half plane complex poles gives an underdamped step responses. Students also can see as the damping ratio increases the percent overshoot of the step response of the system decreases. In Figure 6, students can analyze the frequency response with the variation of damping ratio. This response shows that the variation of damping ratio does not impact on the frequency responses at low and high frequencies. The effect of this variation is on the magnitude and phase of the frequency responses around the natural frequency as it shows in Figure 6. Students also can see that as the damping ratio increases the magnitude frequency response of the system decreases around the natural frequency response. The frequency phase responses show that the variation of damping ratio for each response is around the natural frequencies.

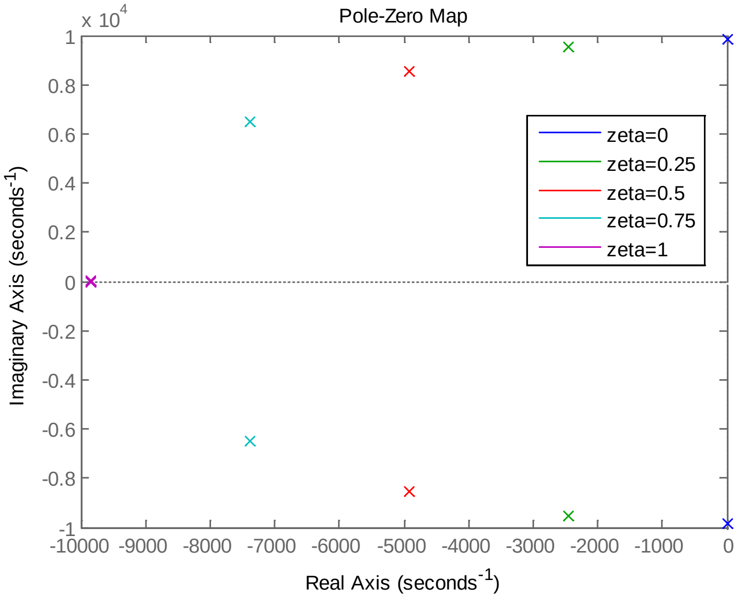


Figure 4: The pole locations with varying the damping ratio

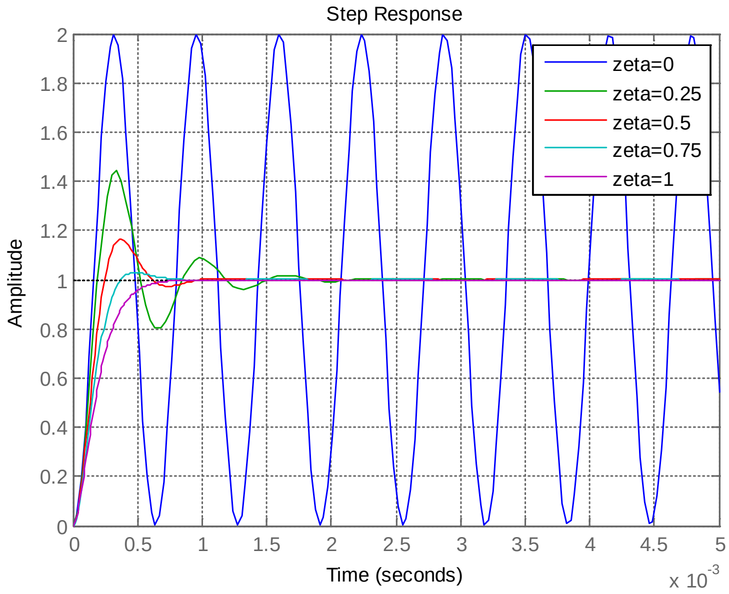


Figure 5: The step responses with varying the damping ratio

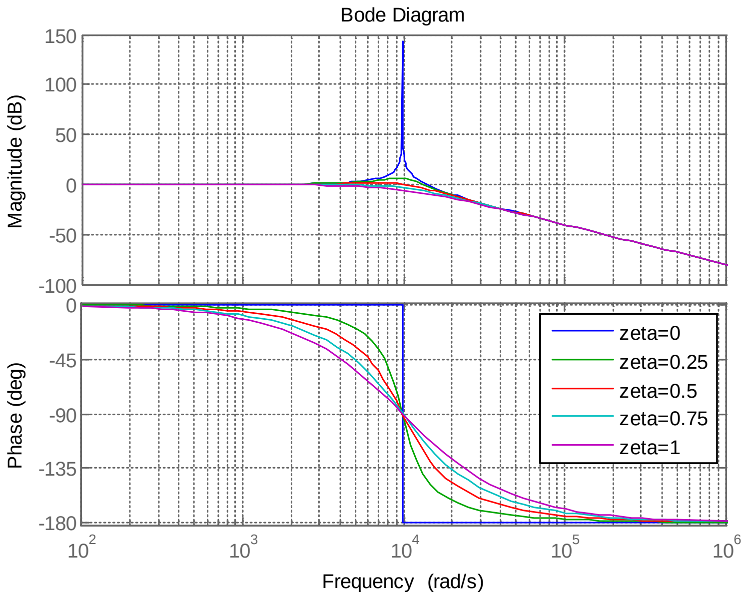


Figure 6: The frequency responses with varying the damping ratio

In next part of lab, students are asked to analyze the variation of natural frequency on the pole location and system performances, while the damping ratio is constant. They plot the pole locations, step and frequency responses for this part of lab. The pole locations for different values the natural frequency is shown in Figure 7. All of the poles have the same angle with the real axis of s-plane. As the natural frequency increases the poles are moving far from both real and imaginary axis. The step and frequency responses of the system with varying the natural frequency and constant damping ratio are shown in Figure 8 and 9. As clearly can be seen in

Figure 8 all the step responses are an underdamped response and have the same percent overshoot. The step responses also show that the settling time decreases as the natural frequency increases. In Figure 9, students can analyze the frequency response with the variation of natural frequency. The variation of natural frequency response does not impact on the frequency responses at low frequencies. This variation impacts the frequency responses after the natural frequency and at high frequencies. The frequency phase responses show that this variation is around the natural frequencies.

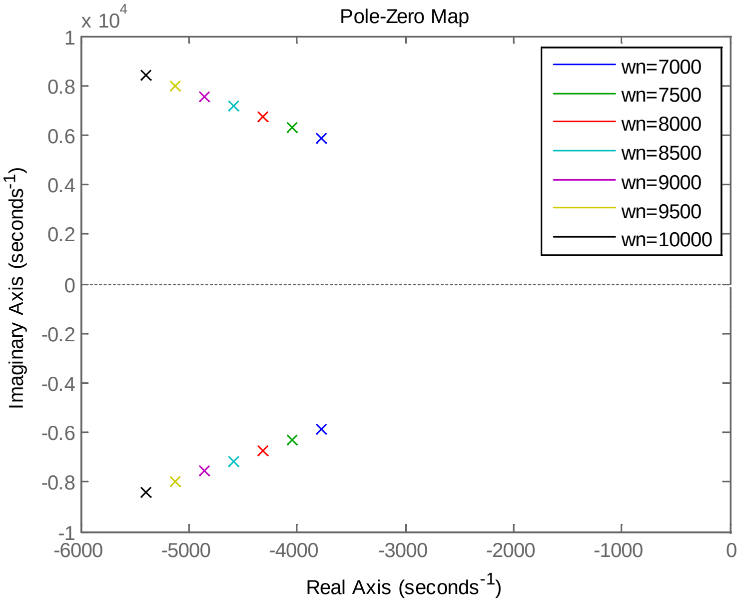


Figure 7: The pole locations for different values the natural frequency

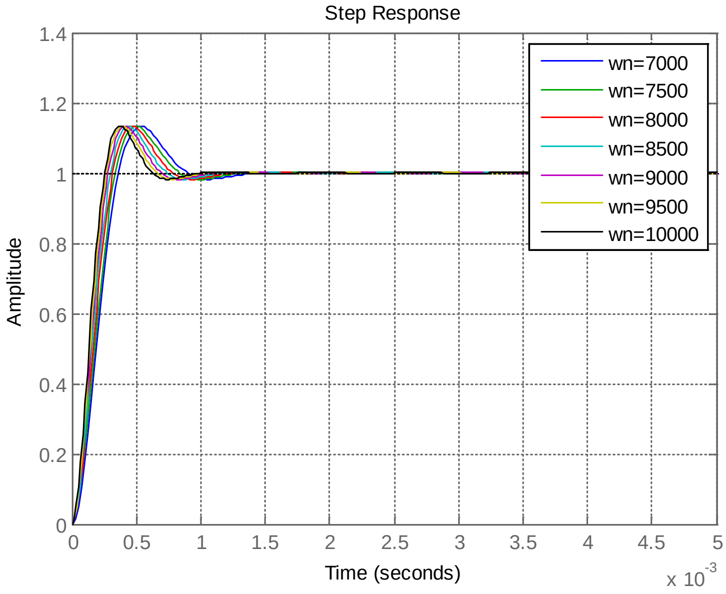


Figure 8: The step responses of the system with varying the natural frequency

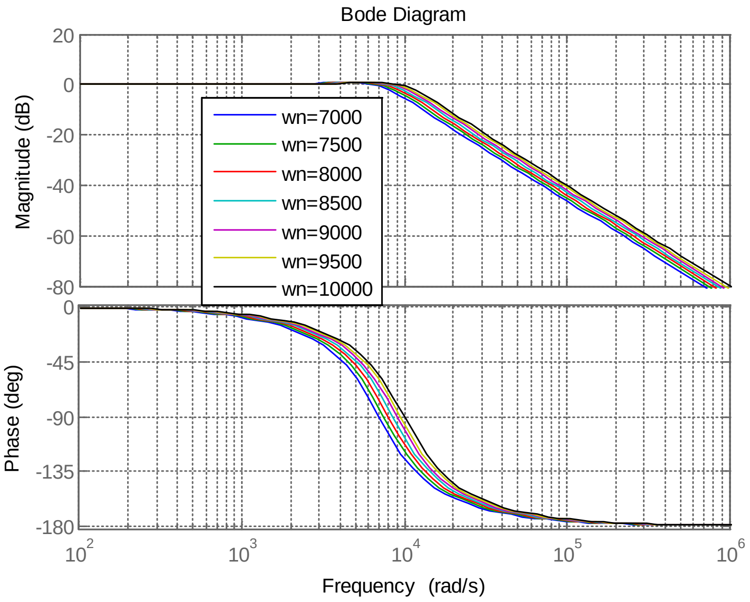


Figure 9: The frequency responses of the system with varying the natural frequency

Part 2: Application of Standard 2nd Order System Identifications

A ship at sea[1] shown in Figure 10 has been considered as an application of this lab. This application is particularly important for the USCGA students. Most of the students have been in ship prior of taking the control class and have a fair good understanding of ship heading angle. In this part of lab students, apply the same principle of the standard 2nd order system identification to the ship motion about its roll axis. The input to the ship at sea is assumed a unit step disturbance torque, T_d, and the output is the ship heading angle, i.e., the ship's roll angle, θ. The assumption here is the relationship between disturbance torque and ship's roll angle is the standard 2nd order system. The step response of the ship roll angle shows in Figure 11.

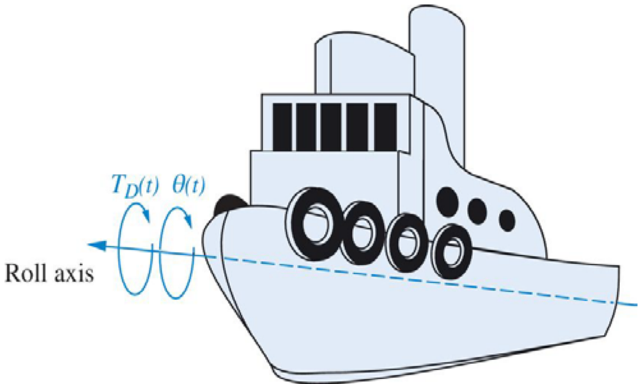


Figure 10: A ship at sea[1]

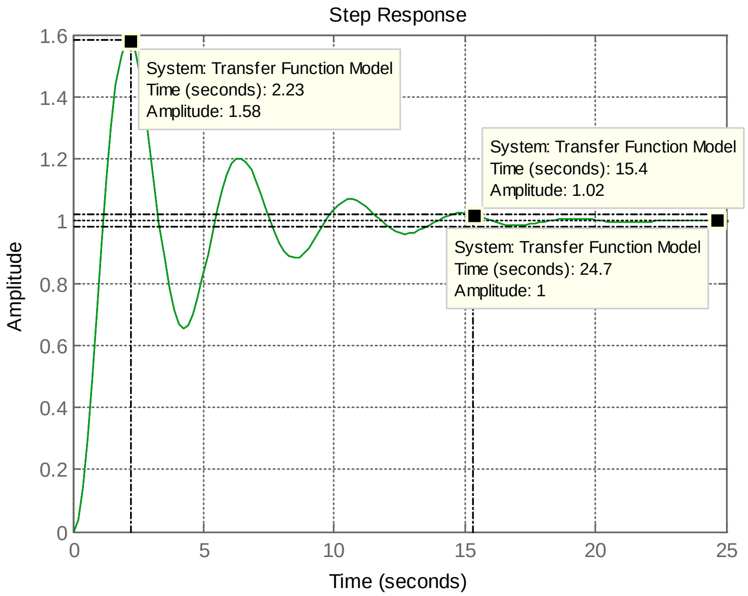


Figure 11: The step response of the ship roll angle

Students use the traditional estimation of hand calculation to estimate the percent overshoot and damping ratio for the heading angle as following:

$$\%PO = \frac{\theta_{\max\ value} - \theta_{final\ value}}{\theta_{final\ value}} \tag{6}$$

$$\varsigma = \frac{-\ln\left(\frac{\%PO}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{\%PO}{100}\right)}} \tag{7}$$

The settling time can be estimated from the heading angle step response as the time which the heading angle reaches and stays at $\pm 2\%$ of its final value. The natural frequency of the system then can be found as:

$$\omega_n = \frac{-\ln\left(0.02\sqrt{1-\zeta^2}\right)}{\zeta T_s} \tag{8}$$

The estimation of percent overshoot from equation (6) is 58% that corresponds a damping ratio from equation (7) equal to $\varsigma = 0.1708$. The estimation of settling time from Figure 1 gives a settling time of $T_s = 15.4$ seconds. Substituting the damping ratio and settling time in equation (8) gives a natural frequency of $\omega_n = 1.4925$ radian/seconds. Substituting the damping ratio and natural frequency data in the standard second order system transfer function in equation (2) gives the following transfer function for the ship heading angle facing a unit step disturbance input.

$$G_{ship} = \frac{\theta(s)}{T_d(s)} =: \frac{2.2277}{s^2 + 2.51s + 2.2277} \tag{9}$$

Students in this lab demonstrate achievement of numerous a-k ABET criteria. The following ABET outcomes can be assessed in this lab:

a. “An ability to apply knowledge of mathematics, science, and engineering.”
d. “An ability to function on multidisciplinary teams.”
e. “An ability to identify, formulate, and solve engineering problems.”
k. “An ability to use techniques, skills, and modern engineering tools necessary for engineering practice.”
m. “Possess a basic knowledge of mathematics through differential and integral calculus, basic science, and engineering science necessary to analyze and design complex electrical and electronic devices, software, and systems containing hardware and software components.”

Conclusion

This paper presented a 2nd order system identification of a linear time invariant system in an undergraduate junior level control systems laboratory. In this laboratory students used computer programming to identify the system transfer function from the parameters of cascade Resistor-Inductor-Capacitor (RLC) circuit and they analyzed the output response. Electrical Engineering students used MATLAB to determine the relationship between the standard 2nd order system transfer function with the simple RLC circuit parameters. Students learned about the relationship between the locations of poles and the system output time and frequency domain responses. They also learned about the relationship between the damping ratio and natural frequency response with the RLC circuit parameters in time domain and frequency responses.

In the second part of this laboratory students estimated the system parameters from a given time domain data of a vessel at sea output response. A continuous time transfer function of the vessel identified from the step response data based on the assumption of standard 2nd order system. The vessel roll dynamics defined as a transfer function of roll-angle and the unit disturbance torque input. They applied the principle of standard 2nd order system identification to the vessel motion about its roll axis. Students in this lab demonstrate achievement of numerous a-k ABET criteria.

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