
AC 2011-186: APPLYING KNOWLEDGE FROM EDUCATIONAL PSYCHOLOGY AND COGNITIVE SCIENCE TO A FIRST COURSE IN THERMODYNAMICS

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Introduction

The fields of educational psychology and cognitive science have done much to advance the understanding of how people learn. The overarching goal of this paper is to survey the literature from these fields to glean the most important, or promising, ideas to improve the teaching and learning of thermodynamics. Our work in this area is just beginning, and the present paper lays the groundwork for developing testable strategies and methods to improve thermodynamics pedagogy. The objectives of the paper are, first, to stimulate the thinking engineering educators in this area, and second, to propose some specific examples of thermodynamics pedagogy grounded in the findings of educational psychology and cognitive science.

The need to develop teaching methods that can improve the problem solving abilities of engineering graduates is widely accepted. These methods are most likely to be effective if they are grounded in a theoretical framework and supported by empirical evidence. In this paper, we propose instruction that can be implemented in a first course in thermodynamics. The theoretical framework that guided the development of these methods is one that highlights student cognition. This framework is organized according to three types of knowledge students must acquire to become proficient in the domain of engineering. These types of knowledge, declarative, procedural, and conditional, collectively address students' understanding of concepts and principles, application of skills and strategies, and awareness and control of learning and problem solving processes.

In this paper, we describe each of these categories of knowledge, including important characteristics, and how the knowledge relates to problem solving. We also discuss specific ideas for how instructors can impact students' acquisition of, and their ability to use, each type of knowledge. Although this paper discusses how these methods can be applied to a specific thermodynamics course, the principles of instruction can be applied to a variety of engineering courses. A graphic overview of the organization of this paper and key points from each section are shown in Table 1.

A Framework of Student Cognition: Three Categories of Knowledge

The knowledge used during problem solving can be broken into different categories.^{1,2} Pol *et al.*³, for example, identify declarative, procedural, and strategic knowledge as the types of knowledge required for problem solving. Mayer and Wittrock⁴ listed six separate categories in their description of the knowledge needed for problem solving: factual, conceptual, strategic, procedural, metacognitive, and beliefs. In our framework, we collapse Mayer and Wittrock's factual and conceptual knowledge into the single category of declarative knowledge; and we combine knowledge of both skills and strategies in the category of procedural knowledge. Finally, we use the label conditional knowledge for the category that includes both students' metacognitive knowledge and their beliefs about their learning and problem solving.

Dividing knowledge into these three category types is a matter of theoretical and practical convenience rather than a psychological reality. Although knowledge can be meaningfully categorized into these three types, it is also true that complex cognitive activity requires the coordinated use of all three. The advantage of thinking about these as distinct types of knowledge, however, lies in the direction this provides to the design of instruction. Instruction in postsecondary science courses has been criticized for an over-emphasis on quantitative or procedural knowledge while overlooking the importance of teaching students the declarative, strategic, and metacognitive knowledge that also underlies problem solving.^{5,6} Grounding the design of instruction in a theoretical framework that is organized around the range of knowledge types addresses these limitations.

Table 1. Overview of the Key Characteristics and Instructional Applications of the Three Types of Knowledge

	Key characteristics	Instructional Application
Declarative Knowledge	<ul style="list-style-type: none"> • Concepts & Principles • Comprised of elements organized in a knowledge network • Well-organized knowledge supports problem solving 	<ul style="list-style-type: none"> • Matrix Notes: Organizes knowledge around principles; draws attention to the features that distinguish deep structure of problems.
Procedural Knowledge	<ul style="list-style-type: none"> • Knowledge of skills and strategies • Effortful during initial learning • Use of procedural knowledge distinguishes strong and weak engineering students 	<ul style="list-style-type: none"> • Worked Examples: Builds procedural fluency. • Self-explanation Strategy: Applies to matrix notes, worked examples, and independent problems.
Conditional Knowledge	<ul style="list-style-type: none"> • Metacognitive knowledge and beliefs • Directs and controls application of knowledge • Supports independent use of knowledge 	<ul style="list-style-type: none"> • Explicit explanation and cognitive modeling: Builds knowledge of when and why some knowledge applies. • Card Sorting: Draws attention to similarities and differences between problems/principles.

Declarative Knowledge

Declarative knowledge includes knowledge of concepts and principles. This knowledge is comprised of elements, which can be defined as a unit of a student’s knowledge that seems irreducible to him or her.⁷ Elements are organized together through connections in a knowledge network. The pattern of connections in the network supports the meaning of individual elements by determining how elements are understood in relation to one another. The knowledge network

operates through a spread of activation. When one element in a network is thought of, or activated, that activation spreads along connections and activates other knowledge elements. Those elements that are most closely associated with, or connected to, the activated element are most likely to also become activated. In other words, when a student encounters a problem and recognizes the correspondence between a feature of that problem and a stored knowledge element, that stored element is activated and activation spreads to other, associated knowledge elements.

These mechanisms show that how a student has organized the elements of their declarative knowledge structure directly influences the probability that a particular set of knowledge elements will be jointly considered during problem solving. Moreover, it is the pattern of connections across these elements that determines a student's principled understanding. A network is organized around principles when the elements contained in that principle are organized together, and other supportive elements are organized around the principle. A student whose knowledge is organized in this manner is able to approach problem solving through the application of principles. In Figure 1, we show a depiction of one expert's organization of the main principles students must learn in thermodynamics.

ANALYSIS AND DESIGN OF PRACTICAL DEVICES AND SYSTEMS		
CONSERVATION OF MASS	CONSERVATION OF ENERGY (1 st Law of Thermodynamics)	ENTROPY AND EXERGY BALANCES (2 nd Law of Thermodynamics)
PROPERTIES OF MATTER: STATE RELATIONS		

Figure 1. Key thermodynamic principles provide the basis for the analysis and design of practical devices and systems.

Psychological research reveals significant differences in the knowledge organization of experts and novices in a domain. Expert's knowledge is fused around critical principles, whereas the novices' knowledge representation is piecemeal and fragmented.^{8,9} Experts benefit from this structure because well-organized knowledge supports problem solving,¹⁰ the flexible application of knowledge,¹¹ strategy use,^{12,13} and future learning.^{3,7}

The aspect of problem solving that may be most affected by how knowledge is organized is the construction of the problem representation.^{14,15} The problem representation corresponds to the internal representation of what the problem requires the solver to do, the elements involved in the problem, and how those elements should be related.⁴ A problem solver who accurately identifies the principles involved in a problem is representing the deep structure; one whose problem representation is based on the storyline of a problem, however, is constructing a surface feature representation. Students, particularly weak students,^{16,17} have difficulty constructing representations of the deep structure of the problem.^{9,18} Moreover, there is a significant relationship between the quality of knowledge organization and the ability to recognize the deep

structure of problems.^{2,19} A novice problem solver in thermodynamics, for example, may not be able to identify properly the system of interest. Failure to construct an accurate problem representation has been tied to problem solving errors in thermodynamics²⁰ and statics.²¹

The organization of knowledge is constructed at the time of learning and can be influenced by instruction⁸ and study behaviors.¹³ In fact, studies comparing the knowledge organization of successful and struggling students in postsecondary classrooms shows that more successful students' express an organizational structure that more closely resembles that of experts than do their less successful peers (e.g., Refs. 22, 23). These differences have been associated with performance on classroom assessments²⁴ and problem solving measures.²

Taken together, this body of research suggests that one way in which an instructor could improve students' problem solving is by helping students to develop a well-organized knowledge base and showing how this knowledge applies to specific problems. In a later section, we discuss ways to promote high quality knowledge organization amongst students in thermodynamics.

Procedural Knowledge

In our theoretical framework, procedural knowledge includes both the skills and strategies a student knows. Automated skills include knowledge such as how to apply algorithms and construct diagrams. Strategies are goal-directed cognitive activities that are not required for task completion.²⁵ Examples of strategies used during problem solving include self-questioning,²⁶ self-explanation,¹⁷ and sketching.²⁷ Strategic processing during problem solving can guide students' attention, help them to recognize the type of problem, and facilitate knowledge transfer.²⁸ Figure 2 shows examples of key procedural skills from thermodynamics.

<p>Identification of System & Boundaries Control Mass/Volume Closed/Open System</p>	<p>Identification of Interactions with Surroundings Work Interactions Heat Interactions Mass Interactions</p>
<p>Detailed Treatment of Units Identifying Units for Each Quantity Applying Conversion Factors</p>	<p>Use of Tabular or Computer-Based Property Data Identification of State Region Interpolation/Software Procedures</p>

Figure 2. Procedural skills important in thermodynamics.

One cognitive factor important for use of both skills and strategies is the effort required for their execution. The cognitive effort that can be dedicated to any task is limited by attentional resources.⁴ Effort is determined by the amount of these resources required for a task. Whereas highly effortful procedures requires a large percentage of these resources, less effortful

procedures require fewer resources. The percentage of attentional resources required by a procedure determines the percentage of these resources that remain to attend to other aspects of the task. A complex thermodynamics problem, for example, may require a student to determine a number of state properties while also deciding on a process path and applying energy conservation. The student for whom determining the properties is effortful will not have sufficient attentional resources remaining to appropriately choose a process path or apply a potentially useful problem representation strategy.

Theories of both skill²⁹ and strategy³⁰ acquisition indicate that when a procedure is initially learned, that procedure will be effortful. This effort is decreased only following extensive practice.^{53,54}

Differences in the procedural knowledge of engineering students enrolled in a statics course were demonstrated in a study by Litzinger *et al.*¹⁷ Participants in this study thought aloud as they solved two analysis problems. A group of successful students from the engineering course were compared to a group of students who had more difficulty with analysis problems and were struggling in the statics course. The analysis of think alouds and students' written work revealed several important issues with regard to students' procedural knowledge. For example, both strong and weak problem solvers were unlikely to apply moment equations, which suggests students had not strongly encoded knowledge of this procedure and how it should be applied. Comparisons between the groups of strong and weak students revealed that weak students were less likely to apply the procedure of drawing component axes to help them orient the direction of forces in the problems. One of the most important differences in the procedural knowledge of these two groups was found when comparing strategy use: Strong problem solvers used a self-explanation strategy significantly more often than did weak students. These strong students were able to use this explanation strategy to guide recognition of the problem's deep structure and to support mapping of declarative knowledge principles throughout problem solving. The findings from this study suggest that the difficulties students have with solving novel problems can be at least partially attributed to weaknesses in their procedural knowledge.

Students develop procedural knowledge through their experiences completing problems. The ability to apply these procedures fluently (see Fig. 2), with little effort required, is developed through practice. The Instructional Implications section of this paper presents ideas on how this knowledge can be developed for thermodynamics.

Conditional Knowledge

Conditional knowledge refers to students' knowledge of the situations in which particular declarative or procedural knowledge should be applied.^{4,31} This knowledge reflects the students' awareness of when, where, and why other knowledge should be used, and this awareness underlies cognitive control during problem solving. That is, a student who knows the conditions under which some other particular knowledge should be used is positioned to recognize those conditions when they are encountered and, consequently, select the appropriate knowledge. In this respect, our definition of conditional knowledge is consistent with frameworks of metacognition.³² Our view that this knowledge serves as a control mechanism is consistent with

models of self-regulation³³ and conceptions of how metacognition influences problem solving.^{11,34}

Table 2. Categories of Thermodynamics Problems

Fixed Mass: State Change	Fixed Mass: Instantaneous	Control Volume: Instantaneous	Control Volume: Steady-State, Steady Flow	Control Volume: State Change
Problems may involve finding work through knowledge of the path; the application of the 1 st law to determine heat and/or work, or to determine the final (or initial) state, depending on givens. Many possible permutations and combinations are possible. State relations usually required. 2 nd law concepts may or may not be required.	Problems typically involve application of 1 st law to determine work rate, heat-transfer, or time rate-of-change of system temperature. Examples here include the heating of a filament with an electrical current and similar lumped-parameter analyses. This problem type is less common than others.	This category is not so much a problem type, but a general framework from which all other classes of problems can be derived. Conservation of energy is frequently stated in this form and then simplified.	The simplest problems involve a single stream in and out. The 1 st and 2 nd laws are typically applied to analyze common devices: pumps, turbines, heat exchangers, etc. Various simplifying assumptions are required depending on the device. Process paths may be specified. Device efficiencies are frequently involved.	Practical problems here typically involve the emptying or filling of a tank or vessel. Application of the 1 st law requires the integration of the instantaneous form for as control volume. This class of problems is particularly difficult for students.

With respect to declarative knowledge, a student’s conditional knowledge allows him or her to determine the problem conditions in which a particular principle should be applied. A student who has accurately associated these conditions with declarative knowledge elements possess the ability to recognize the deep structure of a problem, determine the key features that distinguish problem types from one another, and consequently, select the correct principle to apply. Table 2 illustrates the types of problems encountered in a first course in thermodynamics and their distinguishing characteristics. We base these categories on the definition of the system (open or closed) and how time relates to the problem (state change, instantaneous, steady). We subsume cycle analysis as a repeated application of state-change or steady-flow processes for simplicity.

Conditional knowledge is also a critical factor in determining students’ use of procedures. In his theory of how automated skills are developed, for example, Anderson²⁹ postulates that the actions of a procedure are stored along with knowledge of the conditions in which the actions should be applied. These condition-action pairs are connected together to form a rule-based knowledge network that drives application of the procedure: When the student encounters conditions that match a stored condition-action pair, that match triggers activation of the corresponding procedural steps. When the conditions are not accurately specified and tied to the action, it is unlikely that the correct procedure will be selected. Markman³⁵ points out that there are differences across individuals in the conditions that are encoded in these condition-action

pairs and that these differences can account for the errors that are made in the actions that are executed.

Conditional knowledge is also an important determinate of students' ability to independently use strategies. Students who know *how* to use a strategy are typically able to execute the strategy when prompted but are not able to maintain or transfer the strategy. Students who know when and why the strategy could be used, however, are better able to transfer the strategy to novel situations.^{4,28}

Unfortunately, students' conditional knowledge is often poorly specified. A study by de Jong and Ferguson-Hessler² demonstrated that the ability to recognize the connections between conditional and declarative or procedural knowledge distinguished strong and weak physics students as well as predicted scores on a problem solving measure. In this study, students and experts were provided a set of cards with each card containing a statement of either declarative, procedural, or conditional knowledge. Each declarative knowledge card stated one conceptual idea, procedural cards specified a problem solving procedure, and conditional knowledge cards specified problem solving situations. Both students and physics experts sorted these cards into piles so that cards that belong together were placed in the same grouping. Weak novices in this study typically sorted cards based on surface features so that, for example, cards that identified the same variables were placed together. The organization expressed by strong students more closely resembled the categories compiled by experts. These students were more likely to associate declarative and procedural knowledge elements with corresponding conditional knowledge. The quality of sortings were related to performance on a problem solving assessment covering those principles.

Helping students to identify the conditions for applying declarative and procedural knowledge should be considered a central component of instruction. In doing so, students must be shown not only those features that identify a problem as a member of a particular class, but also those features that distinguish one class of problems from another. Likewise, students' knowledge of problem solving skills and strategies must also be developed in association with the conditional knowledge that can guide and control the application of procedures.

Summary

The three types of knowledge described here provide a useful framework for thinking about the curriculum, or content, of instruction, but instructors must also be aware that the three types develop together. There is, for example, a relationship between the quality of college students' declarative knowledge organization and the tendency to use deep, rather than superficial, study strategies.^{12,13} Strong students are also more likely to apply strategies toward both declarative and procedural knowledge than are their weaker performing peers.³⁶ During instruction then, instructors must help students to become aware of these relationships and recognize how to coordinate the various types of knowledge. In thermodynamics, for example, knowing the state principle, that two independent intensive properties define the state of a simple compressible substance, can trigger a number of procedures depending upon the problem. For example, if only one property is known, some procedure must activate to find a second property

to define the state. Alternatively, if the state is defined, application of the first law can proceed, if required.

Instructors must also be aware that, while these knowledge types speak to the content of instruction, one must also consider the methods of instruction.³⁷ The following section begins with a description and rationale for instructional methods that we believe are congruent with the findings from educational psychology and cognitive science discussed above.

Instructional Applications

We advocate instructional methods that use explicit explanation, apply cognitive modeling, and promote active student engagement. Explicit explanation refers to methods of instruction in which teachers provide direct statements explaining what students are expected to know and how they should know it.³⁸ Cognitive modeling is when the instructor talks out loud to provide explanations of problem solving processes that are primarily hidden within internal thought processes.³⁹ Explicit explanations given during cognitive modeling teach students how solution steps are related to problem solving⁴ and support the development of cognitive and metacognitive knowledge.⁴⁰

At the same time, however, we also know that students who are passive recipients of these explanations learn less effectively than do students who are actively involved in their learning.^{6,37,41} Prompting the use of deep learning strategies is one way to engage students in active learning.⁴² Highlighting the use of these strategies also draws the attention of both students and teachers to the importance of learning how to learn and solve problems. Students who attend to these processes are more likely to detect progress toward their learning goals and have higher self-efficacy relative to their peers who attend to the quality of final products.³⁹

Taken together, these instructional methods can be employed to help students acquire declarative, procedural, and conditional knowledge. In the remainder of this section, we share ideas on how this can be achieved in an introductory thermodynamics course. For our purposes, we define an introductory course to cover the content shown in Table 3.

Table 3. Topics Covered in a Typical Introductory Thermodynamics

TOPIC
Introductory Concepts & Groundwork
Thermodynamic Properties & State Relations
Mass Conservation
Groundwork for Energy Conservation – Heat, Work, & Energy
Energy Conservation – 1 st Law for Thermodynamic Systems
Energy Conservation – 1 st Law for Control Volumes & Applications
2 nd Law of Thermodynamics
2 nd -Law Properties, Property Relations, and Isentropic Efficiencies
Vapor Power Cycles
Gas Power Cycles
Vapor-Compression Refrigeration Cycle

Declarative Knowledge

Figure 1 depicts the four main principles that thermodynamics students must learn: (1) thermodynamic property relations, (2) conservation of mass, (3) conservation of energy, and (4) the second law of thermodynamics. Students must come to understand how these principles are applied to problems involving closed and open systems, and the role of time in these problems, and how these principles are associated with the features of these problem classifications (see Figure 2 and Table 2). A goal of instruction in this course is for students to learn how to distinguish problems along these classifications and to understand how the principles apply in each case.

A prerequisite to developing any deep understanding of thermodynamic principles is for students to understand the symbolic representations used to describe these principles.⁴³ Although this might seem trivial, many students require time and practice to develop the necessary vocabulary. Because of the rich content of thermodynamics, many symbols are required. Explicit building of a symbolic vocabulary can be accomplished using in-class exercises, homework, and quizzes. Figure 3 illustrates a quiz that fosters this development.

To be successful in thermodynamics, a student must develop a knowledge network in which the elements that comprise the four principles are the major organizing component of the network. The student must organize this knowledge relative to the features underlying problem classifications. Knowledge that allows the student to identify the defining features of problems must also be incorporated into this knowledge structure.

A. Using words only, define the following terms as they relate fluid mechanics and thermodynamics. Do not just recite the symbols in words; for example, $mv^2/2$ is kinetic energy, not one-half of the mass times the square of velocity. Also give the units of the quantity.

	<u>Meaning</u>	<u>Units</u>
a. \dot{W}	_____	_____
b. P	_____	_____
c. E	_____	_____
d. u	_____	_____
e. \dot{Q}	_____	_____
f. h	_____	_____
g. $\dot{m}Pv$	_____	_____
h. q	_____	_____

B. The letter vee (upper and lower case) is used to represent several quantities in this course. List these quantities and indicate which vee is used for each.

Figure 3. Quiz to assess students' understanding of symbolic representations.

These goals can be achieved through the use of matrix notes, a note taking strategy that increases the completeness of students' notes and encourages the construction of internal knowledge connections that emphasize the organizational relationships across ideas.⁴⁴ Matrix notes use a table format in which organizes information according to both unique instances and repeated categories. Figure 4 shows a completed matrix that could be used in a thermodynamics course. In this matrix, the unique instances correspond to the different classification of problems encountered in the course. These unique instances are given in the headings for each row of the matrix. Columns of the matrix contain the repeated categories that correspond to three of the four principles. We have also added an additional column labeled defining features.

The matrix acts as a framework to organize students' knowledge throughout the course. Students would start the course with a blank matrix and fill in the blanks as they proceed through the course. The structure provided by the columns draws students' attention to the features that distinguish one class of problems from another. Comparing similarities and differences in the principles across the problem classes (columns) supports understanding of how the principles apply to these problems. Once completed, a student can use the matrix as an aid to problem solving. When given a problem, the student can first inspect the rows to identify the class that the problem comes from. That student can then follow each column across that row to see how each principle should be applied. Once the student has learned the contents of the matrix so that it is part of the internal knowledge network, this well-organized representation can be accessed without having to inspect the external, physical matrix. The entries in Fig. 4 have been expressed quite generally; simpler relationships could have been used in many instances.

The matrix shown in Fig. 4 provides an organizational structure that spans the entire semester of the thermodynamics course. The matrix should be introduced early in the semester as the first problem class is encountered and completed as additional problem classes are covered throughout the semester. In keeping with our belief that the best instruction involves explicit explanation and cognitive modeling, the instructor can take responsibility for selected entries.

Matrix notes can also be used to treat selected topics. For example, a matrix could be constructed for a selection of ideal-gas processes: constant-volume, constant-pressure, and constant-temperature. Students would then fill in the matrix by writing in the simplified first-law expression, useful state relations, and expressions that could be used to calculate work and heat for the process. Figure 5 shows such a matrix. Having students complete this matrix should help students integrate their knowledge and internalize that special cases follow quite simply from a few general principles. Clearly, the use of matrix notes is not limited to the two examples shown here; a thermodynamics course is content rich and many opportunities exist for the application of matrix notes. For example, matrix notes can very naturally be applied to the topic of steady-flow devices. Developing matrix notes can also be an active learning exercise using small groups.

While the matrices described here are primarily organized around declarative knowledge, this framework can also be used to build connections with procedural and conditional knowledge. For instance, in addition to the features that define the deep structure of a problem class, the matrix can also contain information about the derivation of the equation that applies to that class. Other ideas regarding how procedural and conditional knowledge can be incorporated with the matrix are discussed within those sections below.

Procedural Knowledge

Successful problem solving in complex domains such as thermodynamics requires both component skills, such as the ability to apply formulas and construct diagrams, as well as cognitive strategies that support learning and problem solving.^{5,15} One goal for this thermodynamics course then is for students to build fluency with component skills so that these skills will become automated and require less effort. A second goal is for students to acquire knowledge of strategies that can be independently applied to support problem solving.

To achieve the first goal, we recommend using worked examples to aid development of skills that can be used fluently with little effort. In the context of cognitive science, worked examples are defined as samples of problems that show each step of how an expert would complete the problem.³⁹ Example problems are common in thermodynamics textbooks and as subjects for lectures. Students, for better or worse, depend upon example problems. A well-done worked example allows a student to get into the brain of the expert to understand the thought processes involved in the problem solution. In addition to showing the solution steps for a problem, a worked example can also explicitly document the thinking that went into the expert's solution. On the whole, research on worked examples has shown that providing students with worked examples leads to better learning of component skills than does providing sample problems that the students complete on their own.⁴⁵ One reason worked examples may be effective is because they reduce the amount of attentional resources required during learning of procedural knowledge.⁴⁶ The current practice of using worked examples in engineering instruction is congruent with the findings from cognitive science; thus, rather than disparaging students' use of examples, instructors could consider how to help students use these examples most effectively. A course instructor could, for example, annotate textbook examples with verbal explanations of the steps that were taken as a relatively painless way to provide the missing links. Further, as discussed below, students can also be assigned the task of generating their own explanations for the steps in a solution process.

We now consider our second goal: increasing students' knowledge of, and ability to apply, effective problem-solving strategies. Although several strategies could be used to support thermodynamics problem solving, the strategy we recommend is self-explanation. Self-explanation is a strategy in which students must generate causal explanations of targeted phenomena.⁴⁷ This strategy directs students to activate their prior knowledge,⁴⁸ supports the connection of known principles to problems,¹⁷ and supports anticipative reasoning during problem solving.⁴⁹ We recommend this particular strategy because self-explanation can be prompted and taught with relative ease (e.g., Bielaczyc, Pirolli, & Brown⁵⁰), and the benefits of the strategy for learning in engineering has been documented.

We recommend incorporating self-explanation with both the matrix notes and worked examples. Specifically, students can use self-explanation as they take responsibility for completing cells of the matrix. For example, when completing the row associated with control

	Conservation of Mass	Conservation of Energy	2 nd Law Entropy Balances	Defining Features
Fixed Mass: State Change	$m_2 = m_1 = m$	${}_1Q_{2,in} + {}_1W_{2,in}$ $- {}_1Q_{2,out} - {}_1W_{2,out}$ $= E_2 - E_1 \equiv \Delta E$	$\Delta S = \int_1^2 \left(\frac{\delta Q}{T} \right) + S_{irrev}$	A process occurs to change the system's state. Time is irrelevant. Equilibrium prevails at the beginning and end of the process.
Fixed Mass: Instantaneous	$m_2 = m_1 = m$	$\dot{Q}_{in} + \dot{W}_{in}$ $- \dot{Q}_{out} - \dot{W}_{out}$ $= dE_{cv} / dt$	$dS / dt = \dot{Q} / T + \dot{S}_{irrev}$	The state of the system is continually changing. System descriptions are a snapshot in time.
Control Volume: Instantaneous	$\sum_{In} \dot{m}_i - \sum_{Out} \dot{m}_i =$ dm_{cv} / dt	$\sum \dot{E}_{in} - \sum \dot{E}_{out} = dE_{cv} / dt$ <p style="text-align: center;">or</p> $\dot{Q}_{in} + \dot{W}_{in}$ $+ \sum_{In} \dot{m}_i (h_i + V_i^2 / 2 + gz_i)$ $- \dot{Q}_{out} - \dot{W}_{out}$ $- \sum_{Out} \dot{m}_i (h_i + V_i^2 / 2 + gz_i)$ $= dE_{cv} / dt$	$dS_{cv} / dt = \dot{Q} / T$ $+ \sum_{In} \dot{m}_i s_i - \sum_{Out} \dot{m}_i s_i$ $+ \dot{S}_{irrev}$	Mass enters/exits the CV, carrying energy with it. The state at any location within the CV is continually changing. CV descriptions are a snapshot in time. Relationships can be used to derive all other equations given in this matrix.
Control Volume: Steady-State, Steady Flow	$\sum_{In} \dot{m}_i - \sum_{Out} \dot{m}_i = 0$	$\sum \dot{E}_{in} - \sum \dot{E}_{out} = 0$ <p style="text-align: center;">or</p> $\dot{Q}_{in} + \dot{W}_{in}$ $+ \sum_{In} \dot{m}_i (h_i + V_i^2 / 2 + gz_i)$ $- \dot{Q}_{out} - \dot{W}_{out}$ $- \sum_{Out} \dot{m}_i (h_i + V_i^2 / 2 + gz_i)$ $= 0$	$0 = \dot{Q} / T$ $+ \sum_{In} \dot{m}_i s_i - \sum_{Out} \dot{m}_i s_i$ $+ \dot{S}_{irrev}$	Mass enters/exits the CV, carrying energy with it. The state at any location within the CV is fixed. CV descriptions apply at all times. Time is irrelevant.
Control Volume: State Change	$m_2 = m_1 +$ $\int_{In} \dot{m} dt - \int_{Out} \dot{m} dt$ <p style="text-align: center;">or</p> $m_2 = m_1 + \Delta m$	$E_{cv,2} = E_{cv,1} + {}_1Q_{2,in} + {}_1W_{2,in}$ $+ \int_{In} \dot{m}_i (h_i + V_i^2 / 2 + gz_i) dt$ $- {}_1Q_{2,out} - {}_1W_{2,out}$ $- \int_{Out} \dot{m}_i (h_i + V_i^2 / 2 + gz_i) dt$	$S_2 = S_1 + {}_1Q_2 / T$ $+ \int_{In} \dot{m}_i s_i dt - \int_{Out} \dot{m}_i s_i dt$	The state of a CV changes as a result of mass entering or exiting with the possibility of heat and work also occurring.

Figure 4. Example of matrix notes applied to key concepts in thermodynamics.

Process	1 st Law	Process & State Relations	Work	Heat
General Case	${}_1Q_2 - {}_1W_2 = U_2 - U_1$	$PV_1 = mRT_1; PV_2 = mRT_2$ $U_2 - U_1 = mc_v(T_2 - T_1)$ $H_2 - H_1 = mc_p(T_2 - T_1)$	$\int_1^2 PdV$	${}_1Q_2 = \int_1^2 PdV + mc_v(T_2 - T_1)$
Constant-P	${}_1Q_2 - {}_1W_2 = U_2 - U_1$	$P_1 = P_2; T_1/V_1 = T_2/V_2$ $U_2 - U_1 = mc_v(T_2 - T_1)$ $H_2 - H_1 = mc_p(T_2 - T_1)$	$P(V_2 - V_1)$	${}_1Q_2 = P(V_2 - V_1) + U_2 - U_1$ $= H_2 - H_1$ $= mc_p(T_2 - T_1)$
Constant-V	${}_1Q_2 - {}_1W_2 = U_2 - U_1$	$V_1 = V_2; T_1/P_1 = T_2/P_2$ $U_2 - U_1 = mc_v(T_2 - T_1)$ $H_2 - H_1 = mc_p(T_2 - T_1)$	0	${}_1Q_2 = mc_v(T_2 - T_1)$
Constant-T	${}_1Q_2 - {}_1W_2 = U_2 - U_1$	$T_1 = T_2; PV_1 = PV_2$ $U_2 - U_1 = mc_v(T_2 - T_1) = 0$ $H_2 - H_1 = mc_p(T_2 - T_1) = 0$	$PV_1 \ln(V_2/V_1)$	${}_1Q_2 = {}_1W_2$

Figure 5. Example of matrix notes applied to ideal-gas processes for a fixed-mass system.

volumes involving steady state and steady flow, the teacher can prompt the students to use self-explanation to complete the cell corresponding to the conservation of mass principle by asking them to ‘explain how the conservation of mass applies to these problems’. These student explanations should be monitored to ensure students are attending to and including the most important information.

Students can also be instructed to use self-explanation when completing worked examples. Here students can be assigned the task of writing verbal explanations for each step of the examples. Students who apply the self-explanation strategy in this manner learn more from worked examples than do students who do not use the strategy.⁴⁵

In addition to these uses, students can also be taught to use self-explanation during independent problem solving. Using the strategy in this context means that the student would explain each step of the problem as they are completing it, including explanations of how key principles are applied. Similar to how this strategy can be used with worked examples, instructors can encourage this strategy use by assigning the task of writing out these explanations. Thus, the homework students turn in would include not only their worked out problem solution but also their written explanations.

Conditional Knowledge

Whether acquiring declarative knowledge of principles, or procedural knowledge of skills and strategies, a student’s ability to independently use this knowledge is affected by his or her

understanding of when and where this knowledge should be applied.²⁸ Thus, the goals of building students' conditional knowledge should also be embedded within the goals for teaching declarative and procedural knowledge.

We believe this goal can best be achieved by applying the teaching methods of explicit explanation, cognitive modeling, and active student engagement to the recommended practices described thus far. An instructor who uses explicit explanation directly tells student what they are doing and why they are doing it. Thus, when the matrix notes are first introduced, for example, the teacher should clearly and directly tell the students about the purpose of the matrix and how it should be used. When guiding the students through completion of the matrix, the teacher should engage in cognitive modeling to make visible the thinking that underlies the decisions that are made. Throughout the semester, the instructor should continue to make explicit reference to the matrix, explain how it can be used during problem solving, and both prompt and model its use. Similar methods can be applied to both worked examples and the self-explanation strategy. That is, the teacher should tell students what they are doing and why, as well as engage in modeling to support students understanding. When prompting students to use the self-explanation strategy, it is also important that this strategy be labeled and explained. These explanations can also accompany the teacher's use of the strategy during cognitive modeling.

Students' conditional knowledge can also be fostered through activities that draw their attention to the relationships among principles and to the similarities and differences among problems from different classes.⁴ An activity that can achieve this goal is a form of card sorting. Card sorting is a method used in educational research to infer the organization of participants' knowledge. As an example of how this could be applied to the organization of principles within the thermodynamics course, consider giving students a stack of cards on which words are written to represent key concepts and principles. Students would be directed to sort these cards into piles so that they place together those cards that belong in the same grouping. In a large class, a pencil-and-paper exercise is more practicable. An example list of terms and an expert's sorting of those terms is shown in Fig. 6. This activity aids learning because it forces students to become aware of how they believe these concepts and principles are related. The instructor may also use the results of students' sortings to gain insight into how students understand those terms, much in the same way an educational researcher would use the results to infer the quality of the knowledge structure (cf. Shavelson & Stanton⁵¹). Students' organization of symbolic (mathematical) representations also can be developed and assessed using exercises such as that shown in Fig. 7.

Similar techniques can also be applied to whole problems.⁵² Students can be given problems, such as the two shown in Fig. 8, and told to sort the problems according to the deep structures that define their class membership. The results of these sortings can be used in the same manner as the sortings of terms – instructors can infer how students' understand these problems and their ability to detect the deep structure of the problems. The instructor should also engage students in discussion of these sortings to provide the type of training that can improve their ability to recognize the features that determine deep structure (cf. Quilici & Mayer⁵⁵).

Arrange the following items into several groups of related items. Use as many groups as you need to parse these items. You may also use items more than once. If one of the items in your groups can be considered a heading, underline that term.

Items	Expert Groupings
1. Pressure	<u>State / Properties</u>
2. State	Pressure
3. Process	Energy
4. Equilibrium	Enthalpy
5. Energy	Entropy
6. Work	Temperature
7. Heat	Equilibrium
8. Mass	
9. System	<u>Process</u>
10. Control volume	Adiabatic
11. Conservation principles	Isothermal
12. Enthalpy	Isentropic
13. Entropy	
14. Properties	<u>Conservation principles</u>
15. 1 st law of thermodynamics	Mass
16. 2 nd law of thermodynamics	Energy
17. Adiabatic	1 st law of thermodynamics
18. Isothermal	
19. Isentropic	<u>Energy</u>
20. Temperature	Heat
	Work
	System
	Control volume
	Mass
	<u>2nd law of thermodynamics</u>
	Entropy

Figure 6. Exercise for students to develop understanding of how thermodynamic concepts relate and for instructors to assess student understanding.

Conclusions

The suggestions for thermodynamic pedagogy provided in this paper are grounded in a theoretical framework of student cognition, and each has also been empirically validated in educational research. We believe that these recommendations also provide a pragmatic means of improving students' learning and problem solving in an introductory thermodynamics course. These activities can be assimilated with minimal disruption to an instructor's typical routine, and we are eager to put them to use and validate them in this context. We encourage instructors to consider how some of these activities, such as completing portions of the matrix or applying self-

Arrange the following equations into several groups of related equations. Use as many groups as you need to parse these items. You may also use items more than once. Add comments as needed to explain your selections.

Equations

$${}_1Q_2 - {}_1W_2 = \Delta E$$

$${}_1W_2 = \int_1^2 PdV$$

$$h = u + Pv$$

$$Pv = RT$$

$$\Delta h = c_p \Delta T$$

$$S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T} \right)_{rev}$$

$$q = c_p (T_2 - T_1)$$

$$q = c_v (T_2 - T_1)$$

$$\sum_{In} \dot{E} - \sum_{Out} \dot{E} = dE / dt$$

$$\dot{Q} - \dot{W} = \dot{m}(h_2 - h_1 + (V_2^2 - V_1^2) / 2 + g(z_2 - z_1))$$

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1} \right)^{\frac{\gamma}{\gamma-1}}$$

Expert Groupings

Energy Conservation

$$\sum_{In} \dot{E} - \sum_{Out} \dot{E} = dE / dt \quad \text{Most General - Can be used to derive all}$$

of the following:

$${}_1Q_2 - {}_1W_2 = \Delta E \quad \text{Fixed mass - Change of state}$$

$$\dot{Q} - \dot{W} = \dot{m}(h_2 - h_1 + (V_2^2 - V_1^2) / 2 + g(z_2 - z_1))$$

Control volume - one inlet/outlet, SSSF

$$q = c_p (T_2 - T_1) \quad \text{Special case: Constant-pressure heat addition, ideal gas, average or constant specific heat}$$

$$q = c_v (T_2 - T_1) \quad \text{Special case: Constant-volume heat addition, ideal gas, average or constant specific heat}$$

Key Definitions

$${}_1W_2 = \int_1^2 PdV$$

$$h = u + Pv$$

$$S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T} \right)_{rev}$$

State Relations

$$Pv = RT \quad \text{Ideal-gas EOS}$$

$$\Delta h = c_p \Delta T \quad \text{Ideal-gas calorific EOS, average or constant specific heat}$$

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1} \right)^{\frac{\gamma}{\gamma-1}} \quad \text{Ideal gas, isentropic process}$$

Figure 7. Exercise for students to develop and demonstrate connections among symbolic thermodynamic relationships.

PROBLEM 1

A balloon at sea level contains 2 kg of helium at 30°C and 1 atm. The balloon then rises to 1500 m above sea level. At this height, the helium temperature is 6°C. Determine the change in internal energy (kJ/kg) of the helium.

COMMENTS

We recognize immediately that we are dealing with a fixed-mass system and that the helium can be treated as an ideal gas. With this recognition, we know that the internal energy is only a function of temperature. Because the initial and final temperatures are given, the internal energy change can be found readily by using table lookups or approximated using the calorific equation of state: $\Delta u = c_v \Delta T$. We have no need to apply the 1st law to solve this problem.

PROBLEM 2

Initially, 1 kg of water (liquid and/or vapor) at 1.5 MPa is contained in a rigid 1.5-m³ tank. The tank is then heated to 300°C. Determine the final pressure (MPa) and the heat transfer (kJ).

COMMENTS

We recognize once again that we are dealing with a fixed-mass system; however, unlike in Problem 1, the fluid in the system cannot be treated as an ideal gas. We also suspect that we may be dealing with a liquid-vapor mixture, which will complicate the solution. We recognize that with a rigid tank, no work will be done on or by the system; hence, application of the 1st law will determine the heat transfer provided we can determine the initial and final internal energies of the system. We see that the initial state can be defined from the given information, and that the final state can be determined knowing that we have a constant-volume process ($v_2 = v_1$) and a given final temperature.

Figure 8. Two problems sharing some common features with different deep structures.

explanation to work through problem solving, could be incorporated into their routine of classroom instruction. Other activities, such as writing self-explanations for worked examples or completing a card sorting activity, could be assigned as homework.

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