2006-862: APPLYING THE 'CATCH ALL' GENERAL CONTROL VOLUME AND THE REYNOLDS TRANSPORT EQUATION TO IMPROVE THERMODYNAMICS INSTRUCTION

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Applying The ‘Catch All’ General Control Volume And The Reynolds Transport Equation To Improve Thermodynamics Instruction.

Abstract

In the instruction of Thermodynamics it is usual practice in most courses and textbooks ^{to} introduce applications of the First and Second Laws of Thermodynamics via closed systems. As students progress the introduction of open systems then follows. In the author’s experience this method often leads to a perceptual disconnect in the ‘flow’ of the material and can be a cause of considerable confusion among students.

This paper describes a teaching approach whereupon a ‘catch all’ general control volume is introduced as the primary tool of the subject from day one of the course. The accumulation or reduction of a generic property within the control volume is shown to occur as a result of three possible processes; direct transfer across a boundary, direct transfer in conjunction with some ‘carrier’ flow, or finally from spontaneous generation or destruction within the control volume itself. The generalized Reynolds Transport Equation is then formulized from this scenario.

First applications of the Reynolds Equation are then introduced. For the First Law of Thermodynamics the property of ‘energy’ must of course be considered. Heat, work and mass transfer across the boundary, as well as the possibility of work done by a moving boundary and internal sources of ‘energy generation’ must all be described and included within the derivation. The recognition that a closed system is a simplification of the more general control volume now makes the traditional requirement for such a distinction trivial.

It is stressed throughout the course that the value of the Reynolds Transport Equation and the general control volume are their generality and aid to the visualization of fundamental physical processes. This rather than the mathematical rigor of their application is the primary focus of class instruction.

Finally the paper gives some consideration to the conceptually more challenging property of entropy. By describing it as a measure of “energy degradation” it is then included as just another property in the Reynolds Transport Equation. Its subsequent passage through, and generation within, the control volume then also become rather uneventful academic challenges.

Ultimately this approach to the teaching of Thermodynamics has been found very beneficial. Personal observations and student feedback reinforces the belief that a typical student would rather learn one, more complicated, generally applicable model than learn several simpler models which become more elaborate and have more ‘additions’ as problems become more complex.

Introduction

Papers on new ways to teach thermodynamics are somewhat rare and tend to focus on either the implementation of a new software package, such as presented by Dixon [1], Chang et al [2] or on ways to present existing materials in more diverse and, dare one say, "fun" ways (e.g. Elliot [3]). Papers that address how the basic laws are taught and in what type of progression concepts are introduced are, however, almost non-existent. An observer might conjecture therefore, that the subject is so well documented that any deviation would result in inferior instruction and so should be avoided at all costs. Student feedback however does not support this and the subject is still continually seen as one of the most academically challenging. Improvements can and should, therefore, be sought out.

Thermodynamics at the US Coast Guard Academy, as in most undergraduate programs, is invariably taught in the junior year of the engineering program. Again, as is typical, the course is three credit hours and, unfortunately, does not have a closely linked laboratory session. The course normally has a recommended text which new instructors usually follow fairly rigorously. The general philosophy of such texts is an incremental development of the application of the main two thermodynamic laws. This application is invariably done in series with the introduction of pure substances and then, later in the course, gas-gas and gas-vapor mixtures. Along the way the mysterious property "Entropy" is introduced and quickly put to use. Some texts enhance the second law slavishly and bring in new and exotic terms such as "Exergy" and "Availability" to further the belief that the subject is really some kind of "Harry Potter" wizardry.

This "bottom up" development of thermodynamics is tried and tested and whilst arguably successful usually leads to the criticism that the course really only got interesting in the last two weeks. i.e. It was only after most of the material had been covered that students could see "the wood for the trees". Whilst this may be expected of previous generations of students, the modern undergraduate is a product of a "high speed, instant access, immediate results" society. As such there is ever more frustration with the typical student not seeing the point of yet another simplified, closed system, piston example, particularly, when what they want to do is design rocket engines after two classes.

With the above in mind, the approach attempted by the author is, by contrast, a "top down" approach which begins by introducing a 'catch all' general control volume and the Reynolds Transport Equation from the very first class. This is rarely, if ever, done in thermodynamics texts and generally relegated to mid to late chapters even in fluids texts. (e.g. Munson et al [4] and Shapiro [5] give comprehensive, if understandably fluids based coverage of the topic.) Once these concepts have been covered the students are generally re-assured to know that this really is about as complicated as it gets. The majority of what follows in the course is simply applications of this primary tool. Just like a skilled carpenter equipped with a few simple tools can make all types of furniture, so a skilled student equipped with a few general purpose tools can solve all types of thermodynamic problems. The "skill" is not in memory retention but rather in the continued application of the tools to ever more challenging "projects". It is worth noting that the carpenter analogy is continued into other parts of the course. e.g. Just like a carpenter needs to know his/her wood, so the thermodynamic student should understand his/her working fluids.

The General Control Volume Approach

With the introduction of the control volume at the very beginning of the class the instructor has to assume that students do not yet have a grasp of energy, work, mass flow heat etc. in a thermodynamics context. This can be a good thing as what is important, at this stage, is the generality of the general control volume approach. It will be used for various different properties throughout the course and to this end the "property" (B), of interest in our control volume analysis is initially chosen as something that students can readily associate with. The author has used money, beer, gold dust and diamonds to name but a few. The fundamental concept of the general control volume is that it describes the accumulation, or lack thereof, of our property B.

Using, for example, money as the property B of interest we can consider figure 1 as our control volume (e.g. An imaginary federal mint). We will now consider how money gets into and out of the control volume as a means to generate the Reynolds transport equation. Let us imagine that money gets into the building directly by being mailed in and out (B_{in} and B_{out}). Money can also be "convected" in by trucks (m) which carry an average amount of coins (b) each. Finally money is spontaneously created (minted) and destroyed (melted down) within our imaginary mint (B_{gen}).

With all of these processes in mind we can say that the net rate of accumulation of money within the mint is given by the following equation :-

$$\underbrace{\dot{B}_{in}-\dot{B}_{out}}_{\text{Money mailed in – money mailed out}}+\underbrace{(\dot{m}b)_{in}-(\dot{m}b)_{out}}_{\text{Money trucked in – money trucked out}}+\underbrace{\dot{B}_{gen}}_{\text{Net money generated in control volume}}=\underbrace{\dot{B}_{CV}}_{\text{Net accumulation of money in the control volume}}$$

Reynolds Transport Equation

Once the introduction of the equation has been made it is a good idea to use various other "familiar" properties to undertake some basic calculations.

e.g. A carrot factory runs at steady state (i.e. There is no accumulation of carrots in the factory). Two tons are shipped out per truck and trucks run at five an hour, twenty-four hours a day. Due to waste and damage carrots are destroyed at a rate of one half ton per hour. No carrots are grown in the factory.

What rate of carrots needs to be trucked in per hour ?

If trucks out ceased operating at what rate would carrots accumulate in the factory?

Etc….

The whole point of questions like this are to get students familiar with the Reynolds Transport Equation "tool" before introducing more unfamiliar properties such as momentum, energy and entropy.

The introduction of the First Law of Thermodynamics.

Instead of money or carrots we now introduce the concept of energy as simply another property. Internal energy, Kinetic Energy and Potential Energy should all be fairly familiar to undergraduate students having previously taken several physics classes. Heat transfer and work however need to be introduced simply as "energy in transit". It is also never too early, the author believes, to point out that the subtle difference between work and heat is that work can always be converted completely to heat but that only a fraction of work can ever be transferred to heat. For that reason alone, we can argue that of the two forms, work is the "higher value" form of energy transfer. More on this with the second law introduction later.

So taking B as Energy in our Reynolds Transport Equation we can write the First law of Thermodynamics so :-

$$\boxed{\underbrace{\dot{Q} - \dot{W}}_{\text{Direct Energy Transferred in (Net Heat in – Net Work out)}} + \underbrace{(\dot{m}e)_{in} - (\dot{m}e)_{out}}_{\text{Net Energy convected in on the back of mass flow)}} + \underbrace{\dot{E}_{gen}}_{\text{Net Energy created within control volume. (e.g. Nuclear Energy)}} = \underbrace{\dot{E}_{CV}}_{\text{Net Energy accumulated in the control volume}}}$$

First Law of Thermodynamics

Before use can be made of the First law some subtleties of the equation need to be elaborated on.

1. Heat transfer $\dot{Q}$ is basically one of the three forms, radiation, conduction and convection. For Thermodynamics classes this is usually a given quantity or the unknown which we are solving for. Detailed calculations of heat transfer are usually left for later heat transfer courses.
2. Work, $\dot{W}$ is usually somewhat more complicated, in that this can compose mechanical work extracted or inputted via a shaft /turbine/compressor, electrical work (Volts and Amps crossing a boundary) or boundary work (Piston-cylinder arrangements) to name a few. The important thing is to recognize that work involves the transfer of energy that can readily be recovered. E.g. By lifting a weight, we can easily then lower and recover that energy, by passing electricity into a capacitor we

can discharge and again recover that same energy. The same cannot be said from say, heating an iron block.)

3. One area of confusion frequently encountered, particularly with more academic students who do not simply accept definitions, is the so called flow work term. This is work done by the act of a fluid pushing its way into or out of a control volume. This is included as the product of pressure, P, and volume, V, and is conventionally included within the convected energy term $\dot{m}e$ (where V is replaced by specific volume, v). The confusion arises because this is arguably work and should therefore be contained within the $\dot{W}$ term. By including it within the convected term it introduces an extra energy term compared to the energy accumulation term on the right hand side of the Reynolds Equation. The confusion is then arguably compounded when, as frequently is the case, it is combined with the internal energy, u, to give rise to a whole new property, Enthalpy, h.

   In conclusion it is necessary that that students practice the subject in a consistent manner with their peers. As such it is necessary to teach the convention and the fact that students need to distinguish between the forms of energy convected into a control volume :-

$$e_{convected} = u + Pv + \frac{V^2}{2} + gh$$

Convected energy includes flow work term Pv.

and the final energy that is accumulated in a control volume

$$e_{cv} = u + \frac{V^2}{2} + gh$$

Accumulated control volume energy has no flow work term.

Once the above points have been emphasized the inclusion of energy, in all its various guises, as simply another property in the Reynolds Transport equation usually makes the application of the First Law relatively straightforward. Figure 2 showing the general catch all control volume allows application of the first law to either "open" or "closed systems" . Closed systems simply have the convected energy terms omitted and the distinction is now a non-issue.

Reynolds Transport Equation and Entropy

As previously alluded to, it is important to stress early to students that work is a more valuable form of energy in transit than heat. This can best be illustrated by introducing the Carnot engine simply as an energy conversion device. Using it to show that only a fraction of work can ever be extracted from a given amount of heat transfer. The actual fraction being a function of the temperature of the heat source, T_H and the sink, T_L .

$$W = \eta Q \qquad \text{where } \eta \le 1 - \frac{T_H}{T_L} \text{ and } \eta < 1.0$$

i.e. For a given quantity of heat transfer Q only a reduced amount of work W can ever be extracted. The fraction extracted is given by the efficiency, η.

$$Q_H - Q_L = W$$

And hence

$$Q_H - Q_L = \eta Q_H$$

$$Q_H - Q_L \ge \left(1 - \frac{T_L}{T_H}\right) Q_H$$

Which re-arranges to :-

$$\frac{Q_H}{T_H} - \frac{Q_L}{T_L} \ge 0$$

These $\frac{Q}{T}$ terms are given the status of a property, entitled entropy.

Entropy is mathematically defined as the amount of heat transferred divided by the temperature at which the transfer occurs. Physically it is an indicator of how much of the possible work available was actually extracted. If entropy increases then we have allowed energy to “degrade” in value.

In summary therefore, energy stored at a higher temperature has the potential to be converted into more work than energy stored at a low temperature. As such every time heat is transferred from a high temperature source to a low temperature sink, unless all of the “potential work” is extracted a net degradation in the quality of energy available will have occurred. (i.e. An entropy increase). Apart from heat transfer it as also possible that when work is transferred inefficiently (e.g. friction or resistance losses occur) some of that work is transferred into “lower value” heat and hence entropy is again “generated” where none existed before. With these concepts explained the following application of Reynolds transport equation is effectively the formulation of the Second Law of Thermodynamics :-

$$\underbrace{\dot{S}_{in} - \dot{S}_{out}}_{\text{Net Direct Entropy Transferred in (i.e. Via heat conduction)}} + \underbrace{(\dot{m}s)_{in} - (\dot{m}s)_{out}}_{\text{Net Entropy convected in on the back of mass flow)}} + \underbrace{\dot{S}_{gen}}_{\text{Net Entropy created within control volume. (e.g. Turbulence)}} = \underbrace{\dot{S}_{CV}}_{\text{Net Energy accumulated in the control volume}}$$

Second Law of Thermodynamics

Once again the consistency of presentation allows students to feel comfortable with the formulation and quickly gain confidence with its subsequent application.

Conclusions

The generality of the control volume and the Reynolds Transport Equation are stressed from class one of the introductory thermodynamics course. From then on the Reynolds Transport Equation is treated as the primary tool on which all of the ensuing thermodynamic concepts build. The subtleties of energy and entropy are then capable of being introduced with the main teaching focus being on the different physical characteristics and purposes of these various properties instead of how to formulate quantitative solutions. This “top down” approach is in marked contrast with most existing courses and texts which take an incremental approach and rarely if ever highlight the uniformity of the control volume equations.

In summary the author has found this approach popular and successful with both thermodynamics and fluid mechanics students. Students comfortable with the transport equation are generally more confident in tackling new problems and generate the governing equations directly from first principles rather than relying on texts or previous similar examples .The method releases the students form mathematical formulations to allow them to focus more on the underlying physics and ultimately on the very subtleties that make the subject the fascinating fundamental science that it is.

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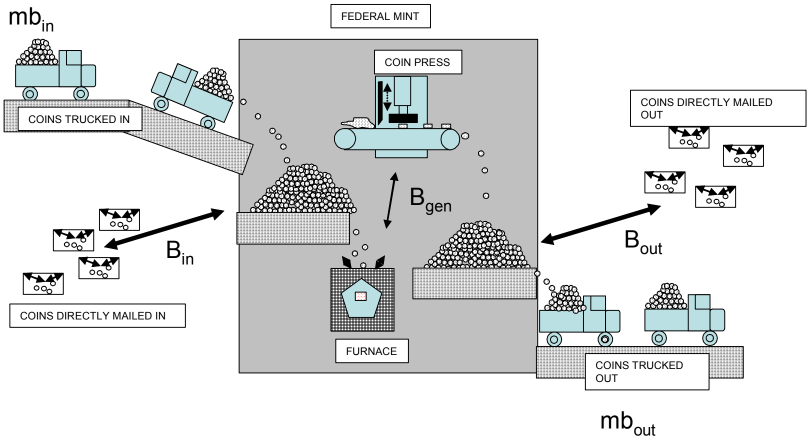

Figure 1. Money as a property in the derivation of the Reynolds Transport Equation

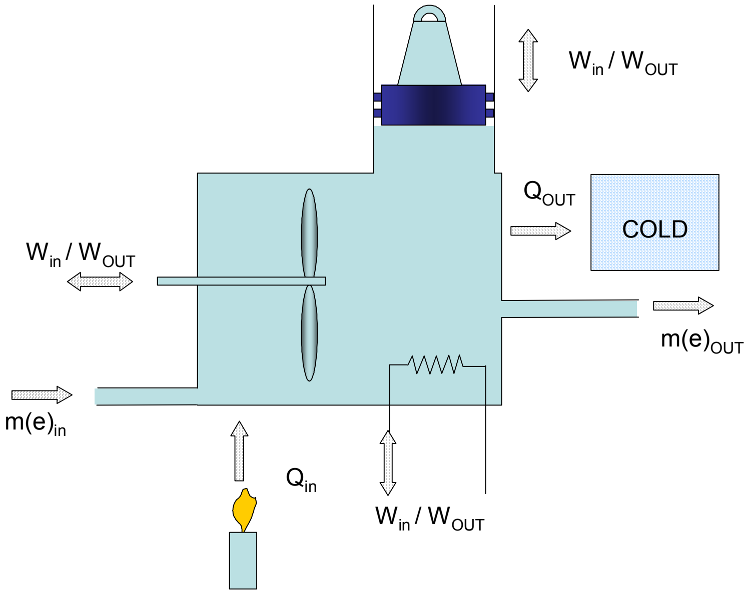

Figure 2. The general control volume for Energy.

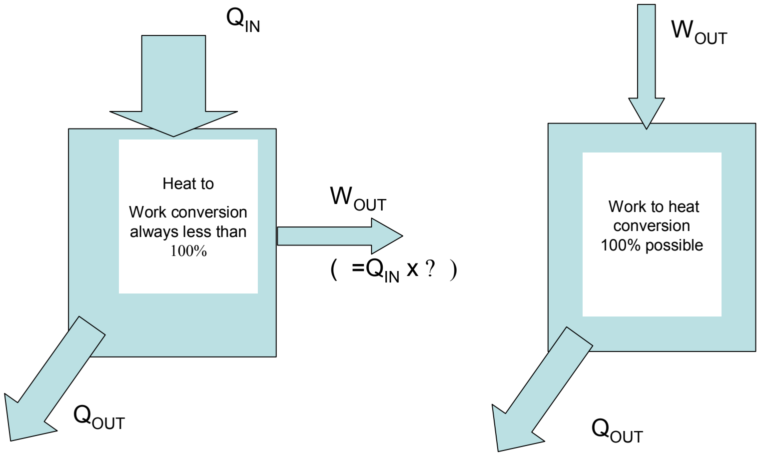

Figure 3. The Heat Engine as an "Energy" Conversion Device.