

2006-2448: ASSESSING CRITICAL THINKING IN MECHANICS IN ENGINEERING EDUCATION

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Assessing Critical Thinking in Mechanics in Engineering Education

Abstract. Typically, mechanics education in engineering schools focuses on communicating explicit content to students, but de-emphasizes the critical thought that underlies the discipline of mechanics. We give examples of the failure of students to apply basic principles of mechanics in solving problems. We develop assessment tools that measure critical thinking in student work, and how well mechanics textbooks engage students in critical analysis. Both tools focus on the treatment of three criteria that we judge to be fundamental, but that are commonly overlooked or undervalued: completeness of free body diagrams, consideration of physical dimension, and careful use of vectors, coordinates and sign conventions. Data collected from employing our assessment tools indicates that most of the time, students omit or misunderstand at least one critical idea when solving a problem, even when they obtain a correct answer. We also found that most of the textbooks surveyed exhibit at least one significant shortcoming pertaining to our criteria. Mechanics educators should vigorously emphasize fundamental aspects of mechanics, such as those that we suggest here, as a necessary (though insufficient) step to improve the ability of students to think critically and solve problems independently.

1. Introduction

Rooting Mechanics Education in Mechanics. Mechanics provides the educational foundation for nearly all branches of engineering, due to its dual role (1) to convey explicit content and subject matter (e.g. the behavior of mechanisms and structures), and (2) to develop analytical skills and rational thought (e.g. building equations, based on rational models, that describe physical phenomena). Our experience indicates that students and instructors in mechanics courses emphasize the explicit content, but often at the expense of developing analytical technique. This view echoes Schafersman, who, though not a mechanic, writes of the need to develop critical thinking in education:

Perhaps you can now see the problem. All education consists of transmitting to students two different things: (1) the subject matter or discipline content of the course ("what to think"), and (2) the correct way to understand and evaluate this subject matter ("how to think"). We do an excellent job of transmitting the content of our respective academic disciplines, but we often fail to teach students how to think effectively about this subject matter, that is, how to properly understand and evaluate it¹.

While in the short run the narrower focus on content enables students to (sometimes) get answers to some problems fairly quickly, students often lack even a *basic* working knowledge of how to consistently apply principles of mechanics to approach general problems – even problems that require only technique that they have already learned. We proffer that such shortcomings often result from the failure to carefully address fundamentals of mechanics in mechanics pedagogy [1]. Such fundamentals include the completeness of free body diagrams, the consideration of physical dimension, and the careful definition and use of coordinates and sign conventions.

Consider, for example, a student who incorrectly derives the equation of an oscillating mass as $m\ddot{x} - kx = 0$. The sign error in this equation may be, in the student's mind, simply due to a minor algebraic error that is of little consequence – “it's just a sign.” But it is likely that the source of this error lies not in careless algebra, but in misunderstanding, or not perceiving, the role of the coordinate x and the need to define it precisely with a sign convention. In this light, the error was arguably *conceptual*. Resolving this problem at its root – not just “fixing” the sign, but really establishing a proper coordinate – would likely lead to a deeper understanding that would be transferable to many other problems.

One approach to correcting this error is to identify its *context*. Fortunately, mechanics naturally lends itself to establishing well-defined categories that may be used to characterize various elements of a given problem. It is well accepted that a given mechanics problem comprises three basic elements: (1) Kinematics (geometrical properties), (2) Fundamental Laws of Mechanics (balance laws, such as Newton's Laws), and (3) Constitutive Laws (material properties). In their textbook *An Introduction to Statics and Dynamics*, Ruina and Pratap refer to these elements as the “Three Pillars of Mechanics”². They present the pillars as a fundamental concept in the introductory chapter, and repeatedly refer to them throughout the text [2]. The three pillars constitute a useful, consistent, and philosophically grounded framework with which to formulate and solve essentially all problems. We argue that all students in mechanics should learn to formulate and solve problems according to this framework.

Critical Thinking. By *critical thinking*, we mean a *systematic approach to problem solving, including complete and well-conceived problem formulation, generation of a solution, and careful assessment of the solution*. While this definition can be applied across a wide range of disciplines, it is somewhat narrow and operational. Many other definitions abound. Further discussion and ideas may be found in Schafersman¹, Gunnink and Bernhardt³, Bean⁴, Kanaoka⁵, and Paul and Elder⁶. Organizations that maintain related material are the Foundation for Critical Thinking⁶ and the Critical Thinking and Pedagogy group at National University of Singapore⁷. Also, The Scientific Reasoning Research Institute⁸ is a research organization at the University of Massachusetts-Amherst that has produced literature regarding critical thinking in physics and mathematics education.

Employing our definition, *critical thinking in mechanics* refers specifically to critical thinking applied to mechanics problems, using the framework of the Three Pillars. In this sense, was the student who wrote the equation $m\ddot{x} - kx = 0$ thinking critically? We provide a brief commentary below.

The equation is dimensionally balanced, indicating at least a partially correct application of Newton's 2nd Law, which is part of the 2nd Pillar. On the other hand, as was suggested previously, the sign error may indicate a conceptual error in establishing a coordinate, which is associated with the 1st Pillar. Even if this is the case, perhaps the student was thinking critically in the sense that he or she *applied* the Pillar of Kinematics, but did so erroneously. Finally, the student would be thinking critically if he or she examined the resulting exponential solutions, and realized that these solutions do not represent the expected oscillatory motion. The pinnacle of critical thought would be reached if the student used this realization to re-examine the entire problem solution, identify the error, and re-solve the problem correctly.

2. The Breakdown of Critical Thinking in Mechanics Education

In this section we examine evidence from situations in mechanics education in which students fail to employ critical thinking, and in which pedagogical materials fail to engage students in critical thinking. We believe that these examples are representative of typical situations encountered by students and instructors at many institutions, and that they provide a clear and accurate assessment of some fundamental issues that must be addressed.

Anecdotes from Student Questions. In the last two years or so, several of our former students have visited us to ask questions pertaining to their current course projects. Strikingly, although their questions varied in topic, all questions fit a disturbing pattern. In each case the students began by saying “we just have one question,” implying that only one ‘simple’ obstacle stood in the way of completing their project. After a few minutes of discussion, and discovering that their question was not so simple, and that it led to new questions, the students would concede that only a week remained in which to complete the project. Seeing that a week was not nearly enough time to adequately address their new realizations, the students would declare that their initial, though incorrect, assessment of the problem would be “good enough,” and that furthermore, they were sure that their instructor did not intend the problem to be as complicated as it now appeared to be. Below is an example that recounts one specific case, told from the point of view of the first author, paraphrased and slightly modified for brevity and simplicity.

Two former students came to ask me for some help with a class project from another instructor’s class. Their project was to analyze the stresses in a hook used to lift prefabricated walls upright. The hook was approximately J-shaped, with a lip. The hook grips one end of the wall and lifts, while the other end of the wall remains in contact with the ground.

The students said, “We just have one question. Can we assume that the hook is resisting all of the force?” I replied, “All of what force? Your question needs to be more precisely stated.” After pursuing this clarification for a few minutes, I was able to draw out from them that they had really meant to ask, “Can we assume that the hook supports the entire weight of the wall?” I said, “Draw a free body diagram, and you tell me. For simplicity, assume that the wall is flat on the ground and is just about to have one end lifted.”

After more prodding than should have been necessary for these students, who had completed Strength of Materials (in my class, no less!), one of them drew a simple FBD of the wall, and realized that it could be viewed as a uniform, simply-supported beam. The hook, therefore, would support *half* of the weight of the wall, $W/2$.

We were only just beginning. I then asked the students to draw a FBD of the hook itself, detailing how it carried the half-weight of the wall. The students proposed an upward force of the cable, equaling $W/2$, but they had some difficulty in seeing that, although the force of the wall on the hook was equivalent to a single downward force, the distributed contact resulted in a more complicated force distribution. I suggested that a simple model would be to assume that the wall contacted the hook at two places, on adjacent faces, without friction, rendering the hook a 3-force body, obliquely oriented (see Figure 1).

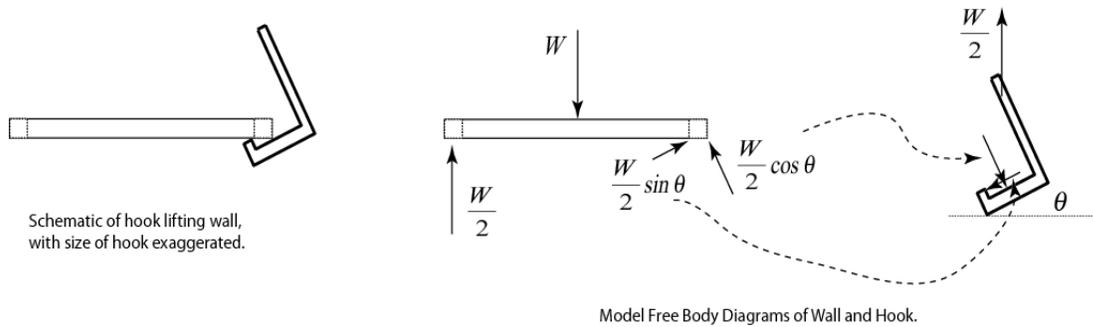


Figure 1. Sketches of hook lifting wall, and suggested Free Body Diagrams.

Beyond these issues, we discussed that the static analysis was only the first step to solve their problem. The students readily agreed that they needed to analyze the stresses, and suggested that they could apply ideas from Strength of Materials, such as the theories for axial bars and beams. I commended them for appealing to this line of reasoning, but I cautioned that these approaches were limited, and would be least useful precisely at the locations where the stresses may be most critical. I mentioned the finite element method, with which neither student had experience, and I also explained more about the importance of properly modeling the boundary conditions.

By this point, we had spent about an hour, and the students appeared somber. They told me that they thought that my suggestions were correct, but that they didn't have time to try tackling any of them, save for perhaps using the half-weight of the wall, instead of the full weight, in their originally intended analysis. They told me not to worry, because they were sure that what they were already doing would be sufficient to satisfy the expectations of their instructor. I have no idea how they actually solved their problem, and decided 'not to ask, not to tell.'

Regrettably, this example, which is representative of several other actual cases, reveals a serious lack of critical thinking on the part of our students. The imperative to address this problem lies well beyond academic perfectionism. The hook project was 'real-world,' and some of its essential analysis was amenable to techniques that the students surely knew. In this case, the static analysis of a simply-supported member was applicable. Had the students been given a simply-supported beam to analyze, they undoubtedly would have analyzed it correctly. But in a context in which the objects at hand were not so literally defined, the students could not apply, from scratch, a simple free body analysis; had they attempted this seriously, they would have at least discovered the answer to their immediate question.

What are the reasons for these lapses in critical thinking, and how can educators address this problem? Complete and definitive answers are likely to prove elusive, as a number of factors – for example, prior mechanics education, innate student ability or interest, demanding schedules and pressures, quality of instruction – are all influential, and are likely to vary significantly from case to case.

Nevertheless, we contend that some definite pedagogical improvements can be advanced, and that while limited in scope, they are necessary if we are to seriously engage our students in critical thinking. In the next two sub-sections we present tools to assess specific aspects of student work and assessments of some textbooks that illuminate some areas where effort should be placed.

Quantitative Measures of Student Performance. We identified three specific criteria against which to examine student homework: (1) completeness and correctness of Free Body Diagrams (FBD), (2) incorporating proper physical units (UNITS), and (3) proper use of vectors, coordinates, and sign conventions (VCS). While these criteria are not exhaustive, they constitute a useful set of criteria that can be reasonably quantified and measured, and their importance is certainly intuitive. As we demonstrate, the overall quality of student work is correlated to their performance in these critical areas. Therefore, we advocate pedagogy that emphasizes attention to these areas.

We developed a protocol to evaluate student performance on specific homework problems. For each problem, the instructor or grader would assign a sub-score for each criterion, from 0 – 3:

Free Body Diagrams (FBD)	0	1	2	3
Physical Units (UNITS)	0	1	2	3
Vectors, Coordinates, Sign Conventions (VCS)	0	1	2	3

where the sub-scores correspond to the following meanings:

- 0: serious error in final answer, and is attributed to poor application of criterion
- 1: serious error in final answer, but not attributed to application of criterion
- 2: final answer essentially correct, despite poor application of criterion
- 3: final answer essentially correct, and criterion was applied correctly

A detailed set of rules was established to determine each sub-score (see Appendix A).

The total score p for a given problem is the sum of the three sub-scores, i.e. $p = z_1 + z_2 + z_3$, where $z_i \in \{0, 1, 2, 3\}$. The resulting set of possible scores is $\{0, 1, 2, 3, 6, 7, 8, 9\}$. Scores of 4 and 5 are not possible, for if $p \in \{4, 5\}$, there must exist sub-scores z_i and z_j such that simultaneously, $z_i \in \{0, 1\}$, and $z_j \in \{2, 3\}$. However, this cannot occur, because if $z_i \in \{0, 1\}$ the final answer was incorrect; yet if $z_j \in \{2, 3\}$, the final answer was correct. Clearly, these two cases cannot occur simultaneously. Although this dichotomous scoring system rewards the attainment of a correct answer, students who obtain correct answers for correct reasons are further distinguished from those who get correct answers from incorrect reasons.

The dichotomy between sub-scores in $\{0, 1\}$ and sub-scores in $\{2, 3\}$ also implies that each score p arises from a *unique* triad of sub-scores, though the ordering of the sub-scores is not unique. For example, a score $p = 7$ can be realized as $(2 + 2 + 3)$ or $(2 + 3 + 2)$, but 7 cannot be realized as $(1 + 3 + 3)$; in other words, 7 can be realized only from two 2's and one 3. Thus, each score represents a *unique* level of total quality, but a given score does not uniquely indicate the level of quality derived from each individual criterion. As a result, this scoring system provides a *monotonic scale* against which to measure overall quality of work, but does not favor the importance of one criterion over another.

We employed this scoring system to assess five different homework problems from Dynamics and Strength of Materials classes. The average scores are reported in Table 1 (N is the number

of homework papers evaluated (*incidences*); the total N = 105 arises from five distinct assignments).

HW	N	FBD	UNITS	VCS	TOTAL
1	20	1.3	1.6	1.6	4.5
2	19	1.7	1.7	1.5	4.9
3	29	1.9	2.3	2.1	6.3
4	29	1.0	1.5	1.9	4.4
5	8	2.3	2.8	2.1	7.1
NET	105	1.6	2.0	1.8	5.4

Table 1. Results of Homework Assessments, by Criterion, and by Homework Assignment.

The results indicate, on average, a modest tendency for students to reach an acceptable final answer (the average score, 5.4, is greater than 4.5, which is the mean of the score set {0, 1, 2, 3, 6, 7, 8, 9}). However, the average tendency is also for students to neglect reasoning or make a significant error in each criterion (each sub-score average is a full point below 3, the score that requires the execution of the criterion without significant error). No single criterion emerges as an area of particular strength or weakness.

By consolidating the data across all criteria and all assignments, we reveal how often a correct (or incorrect) reason correlated to a correct (or incorrect) answer. An explanation for how the data was recompiled is in Appendix A, but roughly speaking, a “correct reason” correlates to the maximum sub-score for a given criterion. “Immeasurable Reason” refers to incorrect answers that we not directly attributed to any of the three basic criteria (e.g. a student who made no sensible progress). Also, the total number of total incidences here is 315, which is 3 times the number of problems studied (105), because each problem is scored against 3 distinct criteria.

N = 3 x 105 = 315	Correct Answer	Incorrect Answer	Total
Correct Reason	57 (18.1%)	7 (2.2%)	64 (21.3%)
Incorrect Reason	153 (48.6%)	29 (9.2%)	251 (79.7%)
Immeasurable Reason		69 (21.9%)	
Total	210 (66.7%)	105 (33.3%)	315 (100.0%)

Table 2. Correlation between Correct Reasons and Correct Answers.

Note that the tabulation generating Table 2 gives a generous impression of student work. For example, a student who arrived at a correct answer on the basis of 2 correct reasons (say FBD and VCS), but one incorrect reason (say UNITS), would contribute 2 tallies for “correct reason, correct answer,” and one tally for “incorrect reason, correct answer.” (A strict scoring system, requiring that *each* reason be correct for each correct answer, would identify this student entirely in the category of “incorrect reason, correct answer.”) According to this tabulation, most incidences (210, 66.7%), represent correct answers, but also, most incidences (251, 79.7%) also represent incorrect or immeasurable reasoning (immeasurable reasoning likely indicates incorrect reasoning not explicitly measured here, such as trigonometry errors.)

Analysis of Textbook Materials. If students are to be challenged to consider precise aspects of mechanics reasoning, such as the criteria identified here, it follows that textbooks and other educational materials bear the responsibility to present theory and problems with a commensurate level of detail. We therefore evaluated several major textbooks, critiquing them on the basis of the same three criteria as with our student assessment: (1) FBD, (2) UNITS, and (3) VCS. For simplicity, we restricted our evaluations to the standard first three chapters (or equivalent) of introductory Dynamics: Particle Kinematics, Particle Kinetics using Newton’s Law, and Particle Kinetics using Energy Methods. For each category we represented our general opinion of the book’s presentation of the criteria throughout these three chapters using comments of “consistent,” “inconsistent,” or “infrequent.”

In conducting the textbook evaluations, we did not exhaustively catalog every occurrence of each criterion. Rather, we formed judgments by finding two or three key examples in each text, and then browsing the rest of the selected pages to get a sense of how representative the examples were. An improved study would more exhaustively track each occurrence, and be evaluated by a panel of several people. Nevertheless, even if our conclusions are somewhat subjective, we submit that we have established a useful protocol for examining textbooks and that our results are reasonable.

Text	FBD	UNITS	VCS
Bedford/Fowler ⁹	consistent	inconsistent	inconsistent
Beer/Johnston/Clausen ¹⁰	mostly consistent	inconsistent	inconsistent
Boresi/Schmidt ¹¹	inconsistent	infrequent	inconsistent
Hibbeler ¹²	inconsistent	inconsistent	consistent
Meriam/Kraige ¹³	mostly consistent	infrequent	inconsistent
Ruina/Pratap ²	consistent	consistent	consistent
Tongue/Sheppard ¹⁴	consistent	consistent	consistent

Table 3. Textbook Assessments: Completeness of Free Body Diagrams (FBD); Inclusion of Physical Units (UNITS); Complete Use of Vectors, Coordinate Systems, and Sign Conventions (VCS).

Table 3 summarizes the results of the textbook evaluations. The texts that were selected were those that were readily available to us. Appendix B contains images of various selections that we examined, with further commentary.

According to our assessment, free body diagrams are generally used appropriately, but most textbooks have at least some instances of free body diagrams that excluded some forces. The most frequent misuse of free body diagrams seems to occur in problems solved using energy methods, in which some forces do no work, and thus do not enter into the calculations (see Appendix B, Figure B2). Even forces that do no work, or that otherwise may not enter into a calculation, impose real, physical effects, such as enforcing constraints. In some engineering situations, the examination of such ‘negligible’ forces is crucial; therefore, we advocate that *all* forces be included in solving problems, regardless of whether they appear in the calculations. Instruction that omits any force gives students an exit through which to escape the consideration and comprehension of the true physical reality of the problem at hand, and allows them to pursue lines of thinking according to their own, often flawed, reasoning [3]. Indeed, the absence of a free body diagram is the root cause of why the students who were trying to analyze the J-hook

(recounted above) had difficulty in even beginning the problem. To exclude a force, even one that appears to be irrelevant, is to surrender an important opportunity for the instructor to cultivate the complete understanding of the problem at hand.

Next, we found that nearly all textbooks frequently exclude units, especially in intermediate calculations, although usually they are attached to the final answer (see Appendix B, Figure B3). Repetitious inclusion of the units, accompanied by emphatic comments, provides a valuable opportunity for the educator to lead the students to realize that units reveal insights and special properties of the underlying mechanics. Students who develop the habit to consider and include units will be more disposed to critically assessing their own work, and ultimately, they will develop the habits of mind that will assist them in solving problems in more advanced subjects, such as fluid mechanics.

The texts had mixed evaluations on the use of coordinates and sign conventions. In many cases, careful establishment of coordinates is simply omitted (see Appendix B, Figure B1). Most texts do consistently define sign conventions for summing forces and moments (see Appendix B, Figure B3). However, the senses of the coordinates are often ambiguously sketched with double-headed or non-headed arrows. A well-defined coordinate should have a single-headed arrow pointing away from a reference point, defining a positive direction or orientation (a correct illustration is given in Appendix B, Figure B2). This might seem fussy, and agreeably, merely drawing proper arrowheads without explanation or emphasis is of little use. The point is that the careful establishment of a coordinate, including its sense, should be impressed upon the student as a fundamental part of the solution of any problem, and therefore, instruction should include the careful identification of coordinate directions. Such emphasis will prepare students for more advanced courses, such as Finite Element Analysis, in which a systematic establishment of coordinate definitions is required to formulate problems and interpret computed results.

3. Conclusions and Future Work

We have emphasized the need to train students in mechanics courses to think critically, grounding their problem-solving skills in the core ideas of mechanics itself. We have also defined reasonable and practical measures that can be used to assess both student work and educational materials. Our work shows that in general, students usually miss at least one critical element of a problem, even when they get the correct answer. *It is precisely this gap – between getting the right answer with faulty reasoning, and getting the right answer with correct reasoning – that must be filled if students are to become true problem solvers.* We have also demonstrated that textbooks often fall short in demanding critical thinking of their student readers.

We believe that the assessment tools that we present here are useful and innovative, but we also acknowledge their limitations. For example, our homework assessments might not be repeatable. For, would other instructors, using our same protocol, give the same analysis of what is acceptably correct? Indeed, the outcomes likely depend on the judgments of the evaluator. However, the assessment tool is likely to be effective for use by a given instructor.

We also recognize that our assessment tool may not categorize student errors uniquely. For example, is an incorrectly labeled force, say k vs. kx , an error in the free body diagram, or an

error in the use of physical units? Again, different evaluators might make different assessments, and perhaps further work could be done to catalogue and define various kinds of errors. Nevertheless, the overall assessment of how frequently students demonstrate proper reasoning will be reasonably invariant.

Allowing, then, that our assessment tools and procedures are reasonable, we hope that their underlying substance – demanding complete free body diagrams, including physical units, and requiring careful definition and use of coordinate systems and sign conventions – will inform pedagogy in mechanics, so that educators and educational materials will emphasize these concepts. In the future, we hope to use our assessment tools on a larger scale to determine, in fact, whether such pedagogical shifts would really improve students' understanding of mechanics, and their ability to independently solve problems.

Finally, while we have focused here on improving analytical technique and developing habits of mind, other teaching strategies, such as writing exercises, assignments with structured iterations for feedback and revision, and design projects, will also help students to cultivate their critical thinking abilities. We hope that our contribution complements these other activities.

In the end, merely presenting the fundamentals of mechanics, even if done with precision, will be of little use without engaged instruction. For what good will result, from, say, sketching coordinates with single-headed arrows, without an engaged discussion of *why* this is done? *Our real point is not simply to call for correcting details, but rather, to engender within mechanics pedagogy the well-conceived and planned articulation of the concepts that underpin these details.* Without such an approach, we will be left with our current situation, so keenly described by Hestenes et al.,

The implications could not be more serious. Since the students have evidently not learned the most basic Newtonian concepts, they must have failed to comprehend most of the material in the course. They have been forced to cope with the subject by rote memorization of isolated fragments and by carrying out meaningless tasks. No wonder so many are repelled! The few who are successful have done so by their own devices, the course and the teacher having supplied only the opportunity and perhaps inspiration¹⁷.

We educators can more positively influence the learning of our students if we recognize that sound education requires a mutually engaged relationship between the instructor and the student, in which the educator perseveres in challenging the student to understand the subject matter critically, and the student will embrace this as an opportunity for discovery, and not as a burden to be shed. If we fail to do our part, as engineering educators, to create such an engaged learning environment, we will have made no progress in attaining our goal of training students to think critically and become independent problem solvers.

Endnotes

[1] Perhaps one reason for this shortcoming in mechanics education is that in typical engineering programs, mechanics is taught as a *service* for degree-bearing disciplines, such as Mechanical Engineering and Civil Engineering. Few universities offer undergraduate degrees in the *discipline* of Mechanics.

[2] In introductory Statics and Dynamics, bodies of interest are often assumed rigid. In such cases, only pillars (1) and (2) are applicable, although pillar (3) is implicitly applied if one views rigidity as a limiting case of constitutive behavior.

[3] Physics educators have long perceived the tendency for students to follow their own intuition, rather than the actual dictates of the mechanics. Several researchers have investigated how students' preconceptions interfere with their ability to learn mechanics, notably Clement¹⁵ and McDermott¹⁶. As this research developed, the Force Concept Inventory (FCI) emerged as a tool to measure students' understandings or misunderstandings of how forces act on bodies (see Hestenes et al.¹⁷). Recently an ASEE group has been formed to collect FCI data (see Gray et al.¹⁸). We contend that our recommendation to construct *complete* Free Body Diagrams will help students to overcome many of their erroneous preconceptions.

Appendix A. Evaluation Form for Assessment of Student Homework

Use of CRITERION [FBD, Units, Coordinates, Vectors & Coordinates]		
CRITERION not used at all	Flag	Sub-score
serious error committed and is due to the absence of the FBD	1	0
serious error committed, but not due to the absence of the FBD	2	1
answer correctly obtained	3	2
CRITERION present but incomplete or incorrect		
serious error committed and is due to the poor FBD	4	0
serious error committed, but not due to the poor FBD	5	1
answer correctly obtained	6	2
CRITERION is present and essentially correct		
serious error committed	7	1
no serious errors and answer correctly obtained	8	3
correct reason, correct answer	corresponds to Flag #8	
incorrect reason, correct answer	corresponds to Flags #3 & #6	
correct reason, incorrect answer	corresponds to Flag #7	
incorrect reason, incorrect answer	corresponds to Flags #1 & #4	
incorrect answer, immeasurable reason	corresponds to Flags #2 & #5	

Appendix B. Excerpts from Textbooks

Example 15.2 Applying Work and Energy to a System

The two crates in Fig. 15.4 are released from rest. Their masses are $m_A = 40$ kg and $m_B = 30$ kg, and the kinetic coefficient of friction between crate A and the inclined surface is $\mu_k = 0.15$. What is the magnitude of the velocity of the crates when they have moved 400 mm?

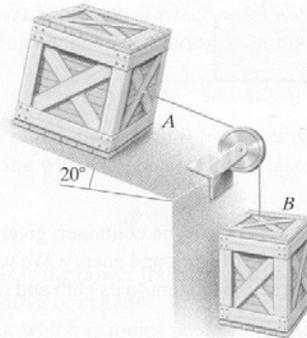


Figure 15.4

Strategy

We will determine the velocity in two ways.

First Method By drawing free-body diagrams of each of the crates and applying the principle of work and energy to them individually, we can obtain two equations in terms of the magnitude of the velocity and the tension in the cable.

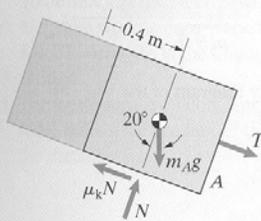
Second Method We can draw a single free-body diagram of the two crates, the cable, and the pulley, and apply the principle of work and energy to the entire system.

Solution

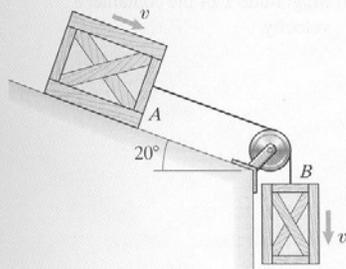
First Method We draw the free-body diagram of crate A in Fig. a. The forces that do work as the crate moves down the plane are the forces tangential to its path: the tension T ; the tangential component of the weight, $m_A g \sin 20^\circ$; and the friction force $\mu_k N$. Because the acceleration of the crate normal to the surface is zero, $N = m_A g \cos 20^\circ$. The magnitude v of the velocity at which A moves parallel to the surface equals the magnitude of the velocity at which B falls (Fig. b). Using Eq. (15.7) to determine the work, we equate the work done on A as it moves from $s_1 = 0$ to $s_2 = 0.4$ m to the change in the kinetic energy of A.

$$\int_{s_1}^{s_2} \Sigma F_t ds = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2;$$

$$\int_0^{0.4} [T + m_A g \sin 20^\circ - \mu_k (m_A g \cos 20^\circ)] ds = \frac{1}{2} m_A v_2^2 - 0. \quad (1)$$



(a) Free-body diagram of A.



(b) The magnitude of the velocity of each crate is the same.

Figure B1. In this excerpt of Example 15.2 from Bedford and Fowler⁹, the calculation of velocity depends on the use of a coordinate s . However, the sketches are absent of any mention of this coordinate, although the absolute distance of 0.4 m is labeled. Mechanics texts should emphasize careful establishment of coordinates, including their senses. The free body diagram is appropriate.

15.18 Swinging Pendulum

Problem Statement The pendulum bob shown in Fig. E15.18 weighs $W = 45 \text{ N}$. It is attached to the end of a light rigid rod whose mass is negligible. The pendulum is rotated from its initial vertical position through the angle θ .

- Determine the potential energy of the pendulum in this position.
- Let the pendulum be rotated until $\theta = 90^\circ$ and then released from rest. Determine the speed of the bob at the instant when $\theta = 0$. Neglect friction of the pin at A .

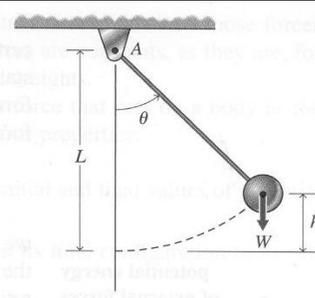


Figure E15.18

Solution Since friction is negligible, the system is conservative.

a. To move the pendulum to the position θ , we must raise the bob from its initial position (taken as the zero configuration, that is, the *configuration of zero potential energy*). Hence, we must raise the bob through a distance $h = L - L \cos \theta$. In doing so, we perform the work

$$U = -U_w = Wh = WL(1 - \cos \theta) \quad (a)$$

Equation (a) represents the negative of the work U_w of the gravity force W —that is, the work we perform against the force W . Therefore, the potential energy of the pendulum at the angle θ is equal to the work we do in raising the bob:

$$V = U = -U_w = WL(1 - \cos \theta) \quad (b)$$

Figure B2. In this excerpt of Example 15.18 from Borelli and Schmidt¹¹, a proper free body diagram of the bob, showing the force of the rod, is not provided. Even though the force of the rod does no work on the bob, and is thus not needed in the calculations, elementary mechanics texts should show all forces on bodies under consideration. The coordinate θ is properly sketched, with a single-headed arrow.

Sample Problem 3/1

A 75-kg man stands on a spring scale in an elevator. During the first 3 seconds of motion from rest, the tension T in the hoisting cable is 8300 N. Find the reading R of the scale in newtons during this interval and the upward velocity v of the elevator at the end of the 3 seconds. The total mass of the elevator, man, and scale is 750 kg.

Solution. The force registered by the scale and the velocity both depend on the acceleration of the elevator, which is constant during the interval for which the forces are constant. From the free-body diagram of the elevator, scale, and man taken together, the acceleration is found to be

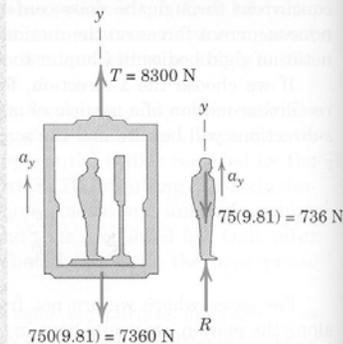
$$[\Sigma F_y = ma_y] \quad 8300 - 7360 = 750a_y \quad a_y = 1.257 \text{ m/s}^2$$

The scale reads the downward force exerted on it by the man's feet. The equal and opposite reaction R to this action is shown on the free-body diagram of the man alone together with his weight, and the equation of motion for him gives

$$[\Sigma F_y = ma_y] \quad R - 736 = 75(1.257) \quad R = 830 \text{ N} \quad \text{Ans.}$$

The velocity reached at the end of the 3 seconds is

$$[\Delta v = \int a \, dt] \quad v - 0 = \int_0^3 1.257 \, dt \quad v = 3.77 \text{ m/s} \quad \text{Ans.}$$



Helpful Hint

- If the scale is calibrated in kilograms it would read $830/9.81 = 84.6 \text{ kg}$ which, of course, is not his true mass since the measurement was made in a noninertial (accelerating) system. *Suggestion:* Rework this problem in U.S. customary units.

Figure B3. In Sample Problem 3/1 from Meriam and Kraige¹³, the calculations omit the inclusion of the physical units. Physical units are merely attached to the final answers. Elementary texts in mechanics should not only include, but should emphasize the presence of physical units in all calculations, as the physical dimension itself carries important meaning. The coordinates are well-defined.

EXAMPLE 3.16 CHANGE IN SPEED OF A MODEL PLANE

Figure 3.37 shows a 0.1 kg model airplane tethered to an operator at O by a massless control line. At the instant considered $\theta = 0$, $r = 6$ m, $v_t = 15$ m/s, and the thrust generated by the airplane is equal to 0.3 N. Use (3.52) to determine the angular speed of the airplane around O at $t = 3$ s.

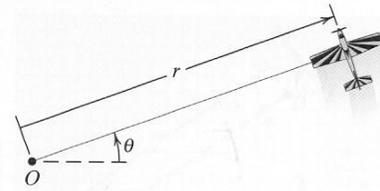


Figure 3.37 Tethered model airplane

Goal Determine the plane's angular speed around O after 3 s of applied thrust.

Given The airplane's mass and thrust and the distance from the airplane to its center of rotation.

Draw Figure 3.38 shows the forces acting on the airplane as well as a convenient set of cylindrical unit vectors.

Assume We will neglect the effect of gravity, assuming that the lift on the plane's wings counteracts it. Thus the only forces we need concern ourselves with are the thrust and the tension in the control wire.

Formulate Equations Equation (3.52) is directly applicable. The two forces that could produce moments about O are T_1 and T_2 , and their moment contribution is found from

$$\sum \mathbf{M}_O = \mathbf{r}_{r/O} \times (T_1 \mathbf{e}_\theta - T_2 \mathbf{e}_r) = r \mathbf{e}_r \times (T_1 \mathbf{e}_\theta - T_2 \mathbf{e}_r) = r T_1 \mathbf{k}$$

Thus

$$H_O(3) = H_O(0) + \int_0^3 r T_1 dt$$

The airplane's initial speed is 15 m/s, giving it an initial angular momentum of

$$H_O = (0.1 \text{ kg})(6 \text{ m})(15 \text{ m/s}) = 9.0 \text{ kg}\cdot\text{m}^2/\text{s}$$

Solve Putting the preceding equations together gives us

$$H_O(3) = 9.0 \text{ kg}\cdot\text{m}^2/\text{s} + (6 \text{ m})(0.3 \text{ N})(3 \text{ s}) = 14.4 \text{ kg}\cdot\text{m}^2/\text{s} \quad (3.53)$$

The magnitude of the angular momentum is equal to rmv_θ , which can be expressed in terms of $\dot{\theta}$ by utilizing $v_\theta = r\dot{\theta}$ to get $H = r^2 m \dot{\theta}$. Using the results of (3.53) gives us

$$14.4 \text{ kg}\cdot\text{m}^2/\text{s} = (6 \text{ m})^2 (0.1 \text{ kg}) \dot{\theta}$$

$$\dot{\theta} = \frac{14.4 \text{ kg}\cdot\text{m}^2/\text{s}}{(6 \text{ m})^2 (0.1 \text{ kg})} = 4 \text{ rad/s}$$

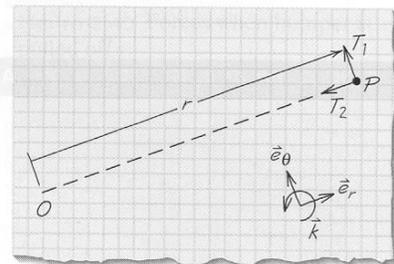


Figure 3.38 Force acting on model airplane

Figure B4. This excerpt of Example 3.16 from Tongue and Sheppard¹⁴ properly presents all key elements: the free body diagram includes all forces that act on the body, including one that does not appear in the calculations; coordinates are clearly drawn and properly defined; and physical units are included in all calculations, as appropriate.

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