AC 2011-662: ASSESSING ENGINEERING STUDENTS' ABILITIES AT GENERATING AND USING MATHEMATICAL MODELS IN CAPSTONE DESIGN

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Assessing engineering students' mathematical modeling abilities in capstone design

Abstract

In engineering capstone design, students need to use their previous knowledge to develop solutions to open-ended problems. A thorough solution to a capstone level problem often includes an appropriate computational or mathematical analysis. However, faculty are often disappointed in engineering students' ability to recognize when and how to apply mathematical analysis to their particular design solutions. This study assessed the capability of senior engineering students to apply mathematical modeling to design, and began the process of testing classroom interventions to rectify certain weaknesses.

This research was constructed around a framework that identifies 6 steps in mathematical modeling[1]. Students were given a scenario and asked to assist a hypothetical design team by creating a mathematical model that could be used in making decisions about the design of a phototherapy device to treat neonatal jaundice. The problem was posed in four iterations over the academic term, with each iteration requiring students to perform different steps in the modeling process.

In an earlier paper we explored how students interpreted the concept of "modeling," and how they decided what parameters were relevant. Most students had difficulty with these essential first steps of model creation, In subsequent iterations, students also demonstrated difficulty in representing a physical situation in equations, and in stating and justifying simplifications and assumptions. The last stages of modeling involve interpretation of the model, and here students proved to be better. They could, in general, relate graphical results from the mathematical model to experimental data obtained from a physical model. They were also able to use the model outputs to make design decisions, or explain why the existing model was inadequate for this purpose.

In the second year of the study, there was more instruction and review of students' performance after they worked on each stage of the problem. This improved performance. In the first year only 16% of students were able to generate equations (even incorrect ones), even though an equation for one element of the system had been given in class. In the second year this number increased to 29%. When students were asked to state assumptions they would use to simplify the system they planned to model, only 35% of students in the first year of the study stated assumptions that were relevant, but this number increased to 80% in the second year.

We conclude that even though students are exposed to certain aspects of modeling in earlier engineering courses, they may not recognize how to perform some of the required steps in an open-ended situation such as design. This prevents or constrains their use of modeling in this important context. Specific instruction in the steps of model creation can improve students' abilities. More work remains to optimize this instruction, and to determine whether the improvement resulting from instruction transfers from the scenario we created to the students' actual design projects.

Introduction

Mathematical modeling is essential to engineering practice and a valuable tool for engineering design. Engineers who generate mathematical models or use mathematical and conceptual knowledge to reason, interpret, and communicate solutions have some level of "quantitative literacy." Dossey[2] defines quantitative literacy as "the ability to interpret and apply these aspects of mathematics to fruitfully understand, predict, and control relevant factors in a variety of contexts." By "these aspects", Dossey means "data representation and interpretation, number and operation sense, measurements, variables and relations, geometric shapes and spatial visualization, and chance." The education of future engineers must prepare them to approach situations with quantitative literacy, at least with the tools in Dossey's list, and ideally with higher level tools including the ability to frame problems in terms of appropriate mathematical models and finding solutions to those models. Modeling can be used in the design process in many ways: to avoid expensive and time-consuming tests of physical prototypes, to guide the range of physical models that should be tested, to rule out seemingly reasonable designs that are destined to fail, to avoid overdesign of components, to explore the likely range of performance of a device, and to estimate failure rates of a device composed of many elements. Thus, there are many reasons why students should possess the capabilities to do modeling.

To further understand how to prepare students to have "quantitative literacy," we are investigating students' abilities at creating mathematical models in the context of design. In the first year of the project, we used several assignments related to a design scenario to determine where students struggle in moving from disciplinary knowledge and mathematical knowledge to the development of a mathematical model to solve a design problem. In the second year of the project, we attempted to provide instruction in modeling to help students overcome their difficulties.

Background Literature

Our study is exploring students' abilities at generating mathematical models to use during decision making in the engineering design process. We would like our capstone design students to understand the creation of mathematical models and be prepared to generate them in the workplace.

Bissell[3] argued that engineers do not usually start from scratch, or first principles in creating models, rather they "tend to select from standard well known models." Experienced modelers can "fine-tune existing models," having developed a modeling intuition, whereas novice modelers may need to have the creation of the model emphasized in instruction.

However, this study was also motivated by the opportunity to further understand where students may struggle in the building of a mathematical model, and how instruction might be used to improve those areas. Edwards[4] and Bissell[3] proposed flowcharts for the creation of mathematical models with the following general cyclical steps: 1. Identify the real-world problem, 2. Simplification, 3. Formulate the mathematical problem, 4. Obtain the mathematical solution of the model, 5. Interpret solution, 6. Compare with reality, 7. Return to step (1) or Present the results.

Gainsburg[1] drew on the ideas of several previous studies and identified six steps for what mathematical modeling should include:

1. Identify the real-world phenomenon
2. Simplify or idealize the phenomenon
3. Express the idealized phenomenon mathematically (i.e., "mathematize")
4. Perform the mathematical manipulations (i.e., "solve" the model)
5. Interpret the mathematical solution in real-world terms
6. Test the interpretation against reality

She studied the use of mathematical models in the workplace, answering the question "What does adult mathematical modeling look like?" Her study involved observing structural engineers at different levels of experience at an engineering firm solving a problem on supports and compression forces, and drew insight mainly from one extended and detailed observation of the interaction between a junior and a senior engineer. Our work used Gainsburg's six steps for the creation and use of mathematical models to evaluate students' abilities in different aspects of modeling. As implied by Gainsburg's framework, we consider a model to be more than a simple calculation. It needs to be a conception of a problem in a mathematical format that can yield different outputs depending on the values chosen for inputs, initial conditions, and/or model parameters.

Research Method

We used two sets of students taking the same Biomedical Engineering (BME) capstone design course. The first set of students took the course in 2009 (n=38), and the second set of students took the course the following year (2010, n=77). In this paper we shall refer to these two sets of students as BME09 and BME10.

Both sets of students were presented with a design scenario and asked to respond to questions designed to evaluate their abilities in the steps presented by Gainsburg. There were four stages to the scenario activity, each one focused on one or more of the steps, during the course of the academic quarter. We have used both quantitative and qualitative approaches to analyze student responses to document student abilities (and inabilities) in each stage.

Both sets of students were presented with the same scenario activity. The scenario was based on a project to design a phototherapy device to treat jaundice that is compatible with Kangaroo Mother Care (KMC) for premature infants in the developing world. Jaundice is a condition that is easily treatable with phototherapy, a process in which light is shone on the infant's skin. Phototherapy lights are used in conjunction with incubators in the developed world, but incubators are too expensive for use in the developing world. However, KMC is an alternative to incubators that utilizes a blanket-like device that wraps the baby and holds it close to the mother's chest, so that the skin-to-skin contact maintains the temperature of the baby, replacing the need for an incubator. A phototherapy device could be designed that is compatible with KMC.

The activities were presented as questions about the way in which phototherapy could be built into a KMC device. Students were asked to assist a “Phototherapy Design Team.” They were given enough information in the scenario that they would not need to do additional literature searches. Students were also given the three main design requirements for the device as established by the American Association of Pediatrics (AAP)[5]: 1) blue light with a wavelength between 430 and 490nm is the optimum light for phototherapy, 2) the average surface area of the baby that should be illuminated is $0.24m^2$, or approximately 75% of the baby, and 3) the spectral irradiance, or strength of the light source, should be above 30 $\mu W/cm^2/nm$. Students were told that phototherapy could be provided with blue LEDs that could be attached to a flexible substrate that could be put next to the baby inside the KMC, and the device would be run by battery power. The scenario then instructed the students that the next step was to model the device.

The phototherapy problem was tractable but required many decisions and could be approached in different ways in order to approximate a realistic design. However, because it was open-ended, we felt that analysis of students’ capabilities would be more feasible if we evaluated them on different aspects of modeling in different stages of the activity. For this reason, questions designed to address the six steps of the modeling process presented in Gainsburg’s framework were presented in four activities (iterations), each of which was completed by the students in a different week.

Any solution to this problem requires that the designer decide how many LEDs would be required, how they would be distributed, and whether they would be touching the baby or positioned at some distance away, and this in turn requires some understanding of the light distribution provided by one LED. These are elements that lend themselves to mathematical modeling.

Students had all taken a physics course covering light, waves and optics, but few if any had used this type of information in an engineering context. They had all had freshman design courses in which they had learned the design process and worked in teams on various types of designs. Few students had taken any additional design courses before their capstone course. In the capstone, they were learning further aspects of the design process, and were organized into teams to work on different problems posed by actual clients. None of the teams were working on KMC phototherapy. Students were not informed about our conception of the stages of modeling. They performed the activities for our study as regular activities for the course, for which they received credit. They were informed that their work on these activities would be helpful to their own design projects and that they would also be analyzed as part of a research project.

The difference between the two sets of students is that BME10 received planned interventions, or lecture/discussions, in between the four activities. These lectures provided instruction on what mathematical modeling is, why it is important, and major mistakes that had been made in the previous stage. The intent of these lectures was to bring all students back to the same starting place for the next stage and get them to reflect on how their work differed from what an experienced modeler might have done. The students in BME09 did not receive these lectures. However, the BME09 students did receive one short lecture between activities 2 and 3 to clear up their misconceptions about light and to provide a mathematical description for the light distribution on a surface coming from a single LED. A modified version of this lecture was included in BME10 as well. This was the only case in which the lecture provided information directly useful for the next activity. The following table summarizes the activity schedule for the two sets of students, and the steps of Gainsburg's framework addressed in each iteration.

	BME09	BME10	Gainsburg steps
Stage 1 (week 1)		Class 0: Survey – Conceptions of Modeling	
	Class 1: Activity - Iteration 1	Class 1: Activity - Iteration 1	1, 2
		Class 2: Lecture 1	
Stage 2 (week 2)	Class 2: Activity - Iteration 2	Class 3: Activity - Iteration 2	1, 2
	Class 3: Correction lecture	Class 4: Lecture 2	
Stage 3 (week 3)	Homework: Activity - Iteration 3	Homework: Activity - Iteration 3	3, 4
	Class 4: Activity - Iteration 3	Class 5: Activity - Iteration 3	3, 4
		Class 6: Lecture 3	
Stage 4 (week 4)	Class 5: Activity - Iteration 4	Class 7: Activity - Iteration 4	5, 6
Stage 5 (end of term)		Class 8: Survey – Conceptions of Modeling	

Table 1 – Schedule of activities and lectures in BME09 and BME10

Implementation Strategy

In this section we lay out the strategy we used to present Gainsburg's modeling steps. This strategy is presented chronologically with indications on which sets of students participated in each part.

<u>Survey - Conceptions of Modeling</u>: BME10

Students in BME 10 completed a survey prior to the start of the activity. This survey asked students for their conceptions of modeling and when they had encountered modeling in previous courses. These were general questions and the students were not presented with the scenario. This survey was administered again after the completion of the academic quarter. These survey results are described by Carberry[6], and are not reported here.

Activity - Iteration 1**:** BME09 and BME10

This iteration assessed students' ideas of what constitutes modeling, and covered the first two steps in Gainsburg's framework, 1. Identify the real-world phenomenon, and 2. Simplify or idealize the phenomenon. We planned this iteration to be very open-ended. Students were asked to "tell the Phototherapy Design Team what you think should be modeled, how you would approach the modeling, and how you expect the model to eventually be helpful in the design." Ideally, we hoped that students would provide their conceptions of what modeling is, and not just list of the steps in the overall design process. Note that we never said "mathematical model" or anything comparable.

This activity was completed in class in order to collect students' individual responses. Students had 45 min to complete the task. Student responses were collected and were analyzed in terms of the first two steps in Gainsburg's framework using a rubric reported previously[7].

Lecture 1: BME10

This lecture was delivered in the class period following iteration 1. Students' responses to iteration 1 were given a first-pass review by instructors to identify major mistakes for discussion in the lecture. This lecture was designed to inform students of the importance of modeling. Students' own responses from iteration 1 were discussed and compared to our desired approach of using mathematical models. Steps one and two of Gainsburg's framework were not discussed in this lecture, since iteration 2 of the activity would also be covering these steps.

The intent of this lecture was to ensure that students were clear about what a mathematical model is and the benefits of using mathematical modeling. Students were also advised that a mathematical model was needed to help design the phototherapy device.

Activity - Iteration 2: BME09 and BME10

Just prior to this iteration, the instructor discussed the difference between sterations 1 and 2 – that mathematical models were desired for the activity, not physical models or experiments. The instructor and the student worksheet itself indicated that what the Phototherapy Team needed was a *mathematical model* to help them design the device, given certain specifications/design requirements.

This iteration addressed steps one and two of Gainsburg's framework again, but with a guided approach. Specifically, in addition to highlighting "mathematical," in the instructions students were asked to sketch the system they planned to model, list relevant parameters and variables, give reasons why those parameters are important to the creation of the model, note any relationships between parameters or variables, note any judgments that would have to be made about the model components, describe possible geometries they may consider for the device, and propose possible mathematical approaches to the problem.

This strategy allowed us to return everyone to a common starting point, regardless of their performance on iteration 1 and it allowed us to probe deeper into their capabilities on these two steps. Again, this activity was completed in class in order to collect students' individual responses. Student responses were analyzed with the same rubric as for iteration 1 so that responses could be compared. Student responses were compared to each other and to their own responses from iteration 1.

Correction Lecture: BME09

When BME09 students' responses to iteration 2 were reviewed by instructors, it became clear that there were some misconceptions about the behavior of light and some confusion about making assumptions. BME09 students were given a short lecture on the cosine law for light distribution from a point source. We expected students to use this cosine law equation for one LED to create equations for a phototherapy device consisting of an LED array.

Lecture 2: BME10

As in BME09, students were also given a short lecture over the cosine law for light distribution from a point source. However, BME10 students were given an expanded lecture that discussed identifying relevant parameters and variables, distinguishing parameters to be modeled from design requirements, creating relevant sketches, and making assumptions. (These were problematic areas in BME09.) The lecture gave an example solution covering parameters and assumptions for iteration 2, while staying general about the geometries. We did not want to provide an example geometry for the array because we did not want to constrain how students would perform in iteration 3.

Activity - Iteration 3**:** BME09 and BME10

Because both BME09 and BME10 had lectures between iterations 2 and 3, the students were again brought to a common point of departure for iteration 3. The activity associated with this iteration was designed to address steps three and four of Gainsburg's framework: 3. Express the idealized phenomenon mathematically (i.e., "mathematize"), and 4. Perform mathematical manipulations.

Students were asked to find the equations and list the assumptions that would be useful to create the model. Then students were asked to manipulate the equations. To begin to address step four of Gainsburg's framework, students were asked to calculate irradiance as a function of position on the skin. Because students would need additional time outside of class to look up or develop the mathematical equations, this part of iteration 3 was given as homework. The rubric for scoring the assumptions students listed in the creation of their model is shown below in Figure 1.

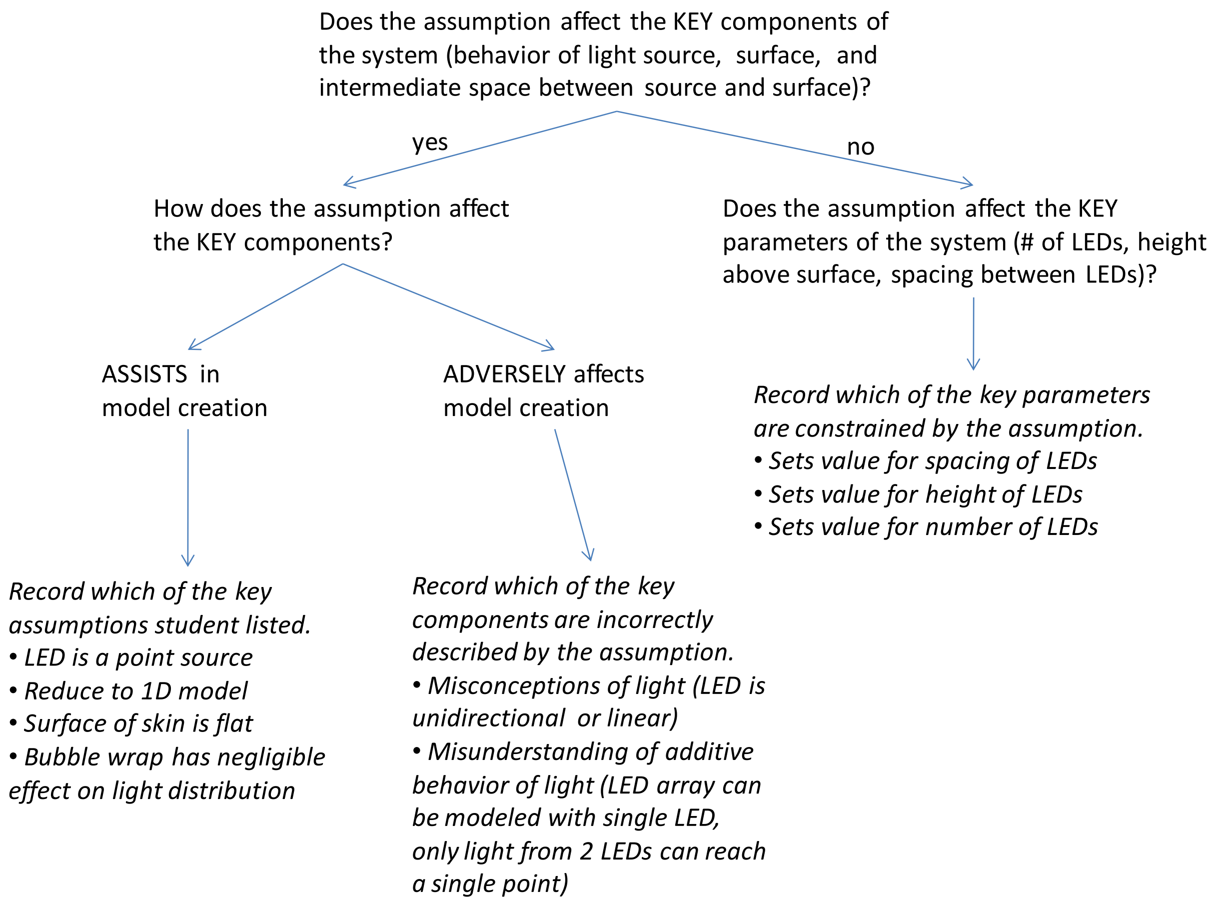


Figure 1 – Rubric for categorizing students' assumptions in iteration 3.

In the following class, students were asked to submit comments on some assumptions that could be made in their models. Students were asked about the error associated with the assumptions and whether they thought the model's predictions would be accurate for a real situation.

For the purposes of simplifying the analysis of students' abilities at "mathematizing" the phenomena, we gave the students a few key simplifications in the hopes that their equations would be similar (i.e. a baby wrapped in a thin blanket could be approximated as two concentric cylinders, in which the diameter of the inner cylinder approached the diameter of the outer cylinder leading to essentially parallel plates when looking at a small patch of the circumference. This indicated that spherical coordinates were not necessary).

<u>Lecture 3</u>: BME10

The third lecture took place after the homework and in-class portions of iteration 3 were completed and reviewed by instructors. This lecture began with a reminder of why mathematical modeling is important in design, then reviewed the steps of Gainsburg's framework that had been completed (1-4) and briefly named the last two steps (5-6). No lecture material was presented on the final two steps; these steps were merely listed to complete the overall picture of model creation. An example solution to previous iterations was presented, including parameters, assumptions, sketches, and equations. Again, an effort was made to correct misconceptions about the behavior of light and the assumptions that were made.

<u>Activity - Iteration 4</u>**:** BME09 and BME10

In the last iteration, students were provided with a model consisting of equations for a linear array of LEDs and graphical outputs from that model, as well as experimental data generated by a design team that had worked on the phototherapy device in 2008. This allowed us to start the students with common information again, and attempt to isolate their capabilities in the last two steps of Gainsburg's framework from their performance in earlier steps. Thus, this iteration evaluated: 5. Interpret the mathematical solution in real-world terms, and 6. Test the interpretation against reality.

Students were asked to explain the model outputs in their own words, to comment on whether the experimental data verified the model, and to provide a recommendation for the design of the Phototherapy device based on the model outputs. Due to time constraints, this activity was completed as a homework assignment in BME09, and in-class in BME10.

Results

Identifying parameters and variables

Our previous paper[7] reported in some detail on iterations 1 and 2 in BME09. For BME10, no difference in the use of mathematics was expected on iteration 1 and none was found. In Figure 2 below we can see that mathematical approaches (both mathematical modeling and the use of physical models or experiments accompanied with predictive equations) were considered by very few students. Only 6 of the74 students in BME10 students associated modeling with any type of mathematical approach. Iteration 1 set the stage for the instructor to discuss mathematical modeling and we expected this discussion to improve performance on iteration 2. As shown in Figure 3, students in both BME09 and BME10 increased the number of mathematical approaches considered (the first two bars in Figure 3 sum to a total of 18% and 29% respectively) relative to their performance in iteration 1, but the increase in BME10 was much greater after the lecture.

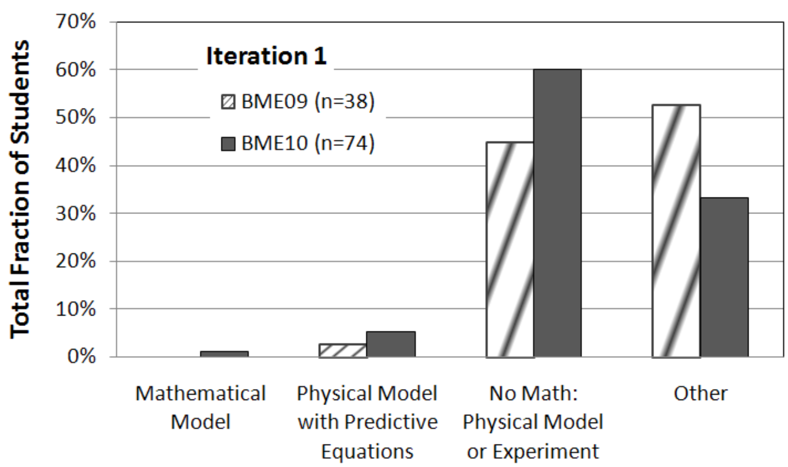


Figure 2 – Performance on iteration 1 for students from both BME09 and BME10 when asked how they would model the phototherapy design problem.

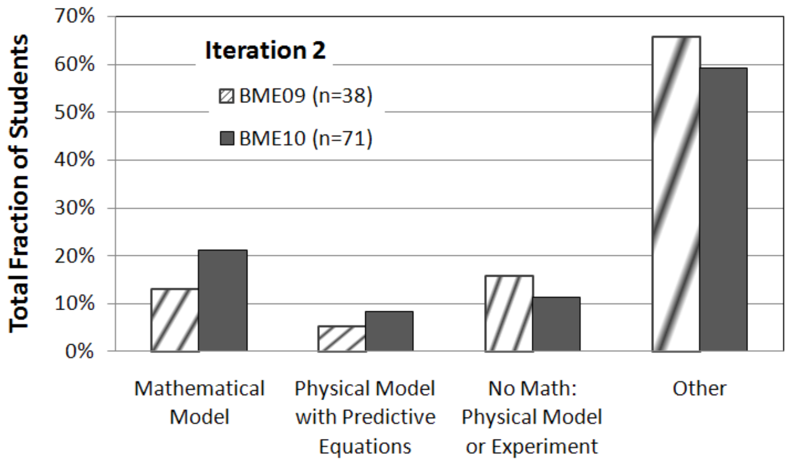


Figure 3 – Performance on iteration 2 for BME09 and BME10 students.

Making assumptions

Iteration 3 covered steps 3 and 4 of Gainsburg's framework: "Express the idealized phenomenon mathematically (i.e., 'mathematize')" and "Perform the mathematical manipulations (i.e., 'solve' the model)." Students were asked to write the equations to be used in the model and list any assumptions they needed to make. Students were then asked to manipulate the equations in order to calculate irradiance as a function of position on the skin.

Assumption	BME09 (n=31)	BME10 (n=69)
Assumptions that would assist in creating the model		
LED modeled as a point source	5 (16%)	23 (33%)
Bubblewrap has a negligible effect	3 (10%)	39 (57%)
Simplify to a one-dimensional model	1 (3%)	9 (13%)
Surface is flat	7 (23%)	23 (33%)
Assumptions that constrain the parameters		
Set value for height of LEDs above surface	3 (10%)	9 (13%)
Set value for spacing of LEDs in array	1 (3%)	4 (6%)
Assumptions that are not exactly right		
Baby and mother are modeled as cylinders	6 (19%)	0
Assumptions that adversely affect the model		
Light is unidirectional or linear	2 (6%)	2 (3%)
Single LED model	1 (3%)	2 (3%)
Only light from 2 LEDs reach any given point	0	8 (12%)
No assumptions	15 (48%)	0

Table 2 – List of assumptions students used in order to create their model (number indicates the frequency with which students mentioned this assumption)

Table 2 shows the frequency with which students made particular assumptions in the creation of their mathematical model. The number of students completing this part of iteration 3 was 31 for BME09 and 69 for BME10. Roughly half of the students in BME09 did not list any assumptions for their model, even though they were asked to do so. The request for assumptions was part of the question asking for the equations necessary to build the model. We felt that students may have skipped over the request for assumptions while reading this question, so an editorial change was made to the BME10 student worksheet to display our request for assumptions more prominently. All students in BME10 provided at least one assumption.

In addition, we noted an improvement in the types of assumptions students in BME10 were making. In particular, we tracked the number of times students listed the key assumptions that would assist in creating the model. Of those key assumptions, no student in either BME09 or BME10 stated all four in their responses. Table 3 below shows the percentage of students listing multiple key assumptions.

	Students listing the key assumptions that would assist in creating the model				
	all 4	***3***	***2***	***1***	***none***
BME09	0%	0%	10%	26%	65%
BME10	0%	16%	22%	45%	19%

Table 3 – Percentage of students from BME09 and BME10 that stated one or more of the key assumptions that would assist in the creation of the model.

Overall we see that students in BME10 were more likely to list more of the key assumptions. In fact, 16% of the BME10 students listed 3 of the key assumptions. In addition to an increase in the frequency that a key assumption was listed, an overall increase in assumptions, including those that would adversely affect the model, was seen in BME10.

The four assumptions listed under the top category "Assumptions that would assist in creating the model" were utilized in *all* of the students' models, even if the students did not explicitly state them. The problem statement even explicitly stated that the LEDs should be modeled as point sources. And while all of the students' models included this assumption, only five of 31 students in BME09 and 23 of 69 students in BME10 stated that this assumption was needed for their model. This indicated that they either felt that it was unnecessary to restate what was in the problem statement, or they did not realize they were making the assumption. The latter explanation is more likely in the case of the assumption that the bubblewrap used as a spacer between the LED array and the infant's skin had a negligible effect on the light distribution. No student included any equations in their model to account for the scattering of light that could occur through the bubblewrap, yet only three of 31 students in BME09 and 39 of 69 students in BME10 stated that they used that assumption.

Some students made assumptions that were not wrong, but placed an unnecessary constraint on one of the parameters in their model. Ideally the students would have used their models to investigate how changes in height of the LED from the skin and spacing of the LEDs in the array would impact spectral irradiance on the infant's skin. Yet, some students made assumptions that gave height or spacing a set value. They reasoned that this would simplify their model, and evidently did not see that these assumptions would limit the parameter space.

Six students in BME09 stated the assumption that the mother and baby were to be modeled as cylinders. This assumption likely stems from the correction lecture held prior to this iteration.

In the lecture we discussed assumptions, and explained how we came to the assumption that the baby could be modeled as a flat surface. We began by simplifying the shape of the baby as a cylinder, and that the blanket could surround it, similar to a concentric cylinder. Because the diameter of the baby and the diameter of the surrounding blanket were very similar in size, and the curvature was small relative to the distance between the blanket and the skin, for very small regions the two could be viewed as parallel plates. This led to the assumption that for short distances the baby would appear as a flat plate. The students making the simplification that baby and mother were cylinders probably attended the lecture and only remembered part of the discussion. In BME10, the students had a more detailed lecture, and more students stated the assumption of the flat plate, and no students stated the assumption about the cylinders.

Several students made assumptions that adversely affected their model. These mainly fell in two categories: misconceptions of the behavior of light or simplifying too far. A few students believed that light shines in only one direction from a point source, like a laser beam. Others thought that light had a more conical shape, and that only the two closest LEDs would shine on any given point on the skin's surface. A couple of students stated that the equations for a single LED would be adequate to model the LED array. These assumptions prevented the model from including the additive effect of the surrounding LEDs on the total light falling at a point on the skin.

Generating model equations and performing mathematical manipulations

In addition to stating assumptions, students were asked to provide the equations that would constitute their model for the light emitted from an LED array. Table 4 lists the types of equations students provided. The largest fraction of students, 39% in BME09 and 58% in BME10, provided an equation for light emitted from one LED as their model. This equation was given by the instructor in the lectures preceding this iteration. Many students provided the equation exactly as it had been written on the board, and others modified it slightly by changing letters for variables or (incorrectly) dividing the entire equation by wavelength in order to match the units of spectral irradiance, which have wavelength in the denominator. Six students in BME09 and nine in BME10 provided irrelevant or incorrect equations. Some of these equations were found in phototherapy literature and did not apply to the problem at hand, and other equations were simply formed because the units of the equation variables matched the units of spectral irradiance.

Final Equation	**BME09 (n=31)**	**BME10 (n=69)**
One-LED equation	12 (39%)	40 (58%)
Incorrect or Irrelevant equation	6 (19%)	9 (13%)
Containing a sum:		
Correct summation equation	0	6 (9%)
Summation of light from 2 to 5 LEDs	4 (13%)	10 (14%)
Incorrect equation containing a sum	2 (6%)	4 (6%)
No final formula	7 (23%)	0

Table 4 – Final equation generated for students' models

Six of 31 BME09 students (19%) and 20 of 69 BME10 students (29%) provided an equation with a sum of light from multiple LEDs, so there was some improvement following the lecture. However none of the BME09 students and only six of the BME10 students were able to manipulate the equations to provide the correct summation equation for a line of LEDs. Four students in BME09 and ten students in BME10 came close by summing light from the nearest two to five LEDs in the array, but did not arrive at the general equation that would include light from LEDs outside that range. This meant that the students tacitly made the assumption that the light from further LEDs would be negligible. An example of a student's calculations leading up to a summed equation is shown in Figure 4.

Two BME09 and four BME10 students had an incorrect equation with a sum. Seven students in BME09 were unable to finish the mathematical manipulations, even though this portion of the iteration was given as a homework assignment and time was not limited. These students did not provide a final formula.

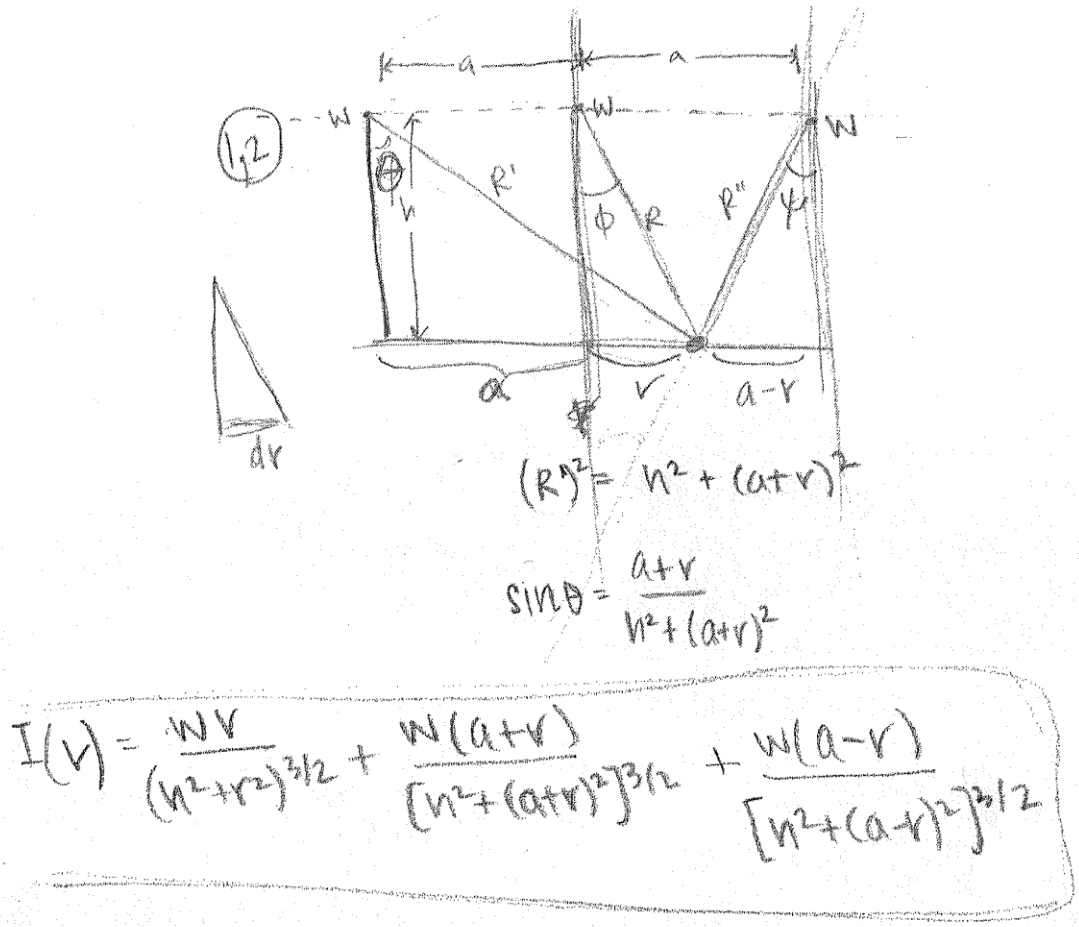


Figure 4 – Example of a student's sketch and mathematical manipulations used to generate model equations (BME09 student). The equation was considered correct because the additive behavior of light is correctly modeled, however it neglects the light from neighboring LEDs beyond a distance of *a*, without stating this as an assumption.

Evaluating model and assumptions

In the class period following submission of their homework on iteration 3, students were given the remaining portion of this iteration, in which they were asked to discuss the assumptions made in the model. The first assumption (that the LED behaves as a point source) approximates the light source as a mathematical point, emitting light radially. In the first question, students were asked to discuss whether the LED was well-modeled as a point source. Students could either agree or disagree with this assumption. Table 5 summarizes the students' responses to the question of whether the LEDs are well modeled as point sources. It must be noted that when we graded the student worksheets we did not mark students' opinions of the assumptions as right or wrong, but graded the justifications for their choice.

	BME09 (n=32)	**BME10 (n=75)**
Provided a response with reasonable justification	18 (56%)	48 (64%)
Provided a response with INCORRECT justification	6 (19%)	10 (13%)
Provided a response WITHOUT justification	2 (6%)	8 (11%)
No response (or no definitive response	6 (19%)	9 (12%)

Table 5 – Student assessment of the assumption that an LED is well modeled as a point source

Fifty-six percent of BME09 students and 64% of BME10 students provided a yes or no response to the question of whether the LED was well-modeled as a point source along with a reasonable justification. Typical justifications for "yes" responses were: 1) that the LEDs were small, or were small in relation to their surroundings, 2) light from LEDs is emitted radially, and 3) the point source assumption reduces model complexity. Typical justifications for "no" responses were: 1) that the focusing of LED lenses and 2) the larger size of the LED bulb may affect the light distribution leaving the LED. A few students, six in BME09 and ten in BME10, persisted in their misconceptions about light, despite addressing this issue in the lecture prior to iteration 3. These students listed incorrect justifications for choosing to whether to use the point source assumption, including: 1) light from an LED is unidirectional and 2) only light that is normal to the surface is absorbed by the infant's skin. Two students in BME09 and eight students in BME10 did not provide a justification for their response. The remaining students did not provide a definitive answer on whether the assumption was acceptable. These students often presented arguments for particular cases in which the assumption would be valid or invalid.

It was encouraging to note that the fraction of students able to provide a reasonable justification for their choice increased after students were exposed to the lecture (from 56% in BME09 to 64% in BME10), and the fraction of students with misconceptions about light decreased (from 19% in BME09 to 13% in BME10). However it was discouraging to see that the fraction of students unable to provide a justification increased (from 6% in BME09 to 11% in BME10).

When asked how well the model they built, including the assumptions, represented a real device, students again presented their argument for whether the model was a good or bad representation of a real device and provided a justification for their choice. Again, we must note that students' responses were not graded for whether their choice was right or wrong, but whether they provided reasonable justification for their choice. These results are shown in Table 6.

	BME09 (n=32)	BME10 (n=75)
Provided a response along with discussion on direction of error	***2 (6%)***	***27 (36%)***
Reasonable justification for choice	2 (6%)	26 (35%)
INCORRECT justification for choice (misconceptions of light)	0	1 (1%)
Provided a response without any discussion on direction of error	***27 (84%)***	***37 (49%)***
No definitive answer	***3 (9%)***	***11 (15%)***

Table 6 – Student assessment of whether the mathematical model is a good representation of a real device.

The majority of students in both BME09 and BME10 provided an opinion on whether the model was a good or bad representation. However, only six percent of students in BME09 discussed the direction of error associated with the model assumptions (whether the model over- or underestimates the real phenomena). This fraction increased to 36% in BME10. Some example justifications that students provided were: 1) the model would overestimate the actual irradiance produced by the device because the model did not include the scattering of light by the bubblewrap used as a spacer between the LEDs and the infant's skin, 2) the model would underestimate the irradiance produced by the device because an actual device would have a two dimensional LED array, which would produce higher values of spectral irradiance than the one dimensional line of LEDs in the mathematical model, and 3) the model would overestimate the actual irradiance produced by the device because of the uneven nature of the infant's skin.

In BME09, 84% of the students provided an opinion on whether the model was a good representation of a real device, but failed to discuss the direction of the error associated with the model assumptions. This number dropped to 49% in BME10, indicating that students in BME10 were analyzing their models more deeply than students in BME09. This deeper analysis is a skill that is vital to seeing the relation between the model and the physical situation and then evaluating whether a model is a decent approximation of that physical situation.

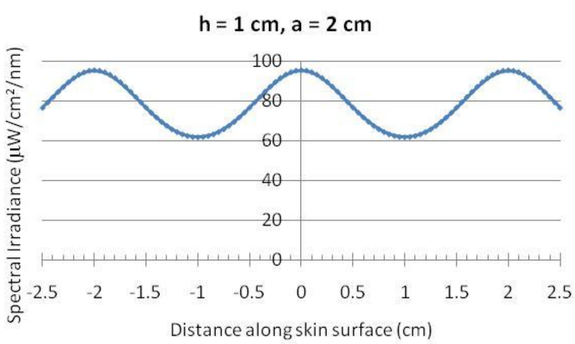
Of the students who did not provide a discussion on the direction of the error associated with the model assumptions, twenty students in both BME09 and BME10 stated instead how the model could be improved. Students stated that some assumptions (point source, negligible influence of bubblewrap, and 1D simplification) may not be accurate. Students also wanted to include properties of the baby, such as dimensions and curvature; assumptions that would make the model more complex. Two students each year thought that the model was for only one LED when it was shown in the problem statement for a one-dimensional line of LEDs.

Three students in BME09 and eleven students in BME10 were unable to provide an opinion on whether the model represented a real device. These students presented arguments for both how the model output could and could not match the output of a real device, if certain conditions were met. It is interesting to note that in BME10, four students stated that it would be preferable to perform experiments or create physical models to test before developing the mathematical model.

Interpreting data

In the fourth iteration of the activity, students were given graphs of model outputs for spectral irradiance for five cases. The plots provided had heights of the LEDs above the infant's skin ranging from 0.5cm to 2cm and the spacing of the LEDs in the one-dimensional line ranging from 2cm to 4cm. Students were also provided with a set of experimental data for spectral irradiance for a single line of LEDs with height set at 1cm and 2cm and spacing set at 2cm and 4cm. Figure 5 shows an example of the model output along with the experimental data that was provided to students.

a)



b)

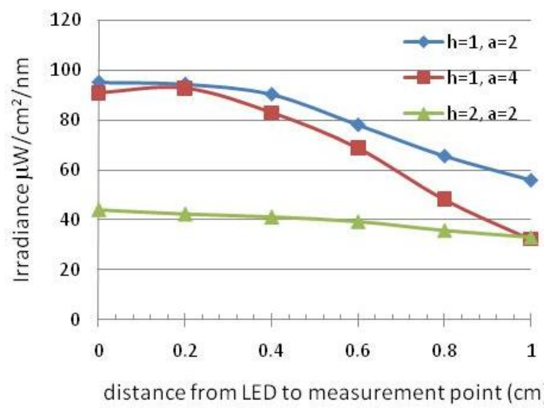


Figure 5 – a) An example of model output and b) an example of experimental data, both provided in the student worksheet.

Students were then asked whether the experimental data validated the mathematical model. Ideally, we expected students to say either yes or no, and then provide some justification for their answer. Table 7 summarizes the students' responses.

	BME09 (n=32)	BME10 (n=66)
Provides a correctly justified response	***30 (94%)***	***61 (92%)***
Yes, model matches experimental data	25 (78%)	46 (70%)
Yes, it matches, but more data is needed	2 (6%)	7 (11%)
No, experimental data is not close enough	2 (6%)	4 (6%)
No, too few data points	2 (6%)	4 (6%)
Provides an incorrectly justified response	***1 (3%)***	***3 (5%)***
Provides no response	***1 (3%)***	***2 (3%)***

Table 7 – Student assessment of whether the experimental data validate the model

The majority of students from both BME09 and BME10 could provide a justified opinion on whether the experimental data validated the mathematical model. Typical comments that were made by students about the experimental data were: 1) yes, the experimental data match the model output closely, 2) yes, the experimental data appear match, but additional data points would be needed, 3) no, the experimental data are not close enough in all cases, and 4) no, there are too few data points to make a proper validation. Additional concerns voiced by instructors, but not mentioned by students, were: 1) the absence of multiple experimental runs and 2) the physical model (single row of LEDs) used for the experiments does not completely describe the actual physical device to be built (a two-dimensional grid of LEDs). Yet, we can see in Table 6 that most students, when given model output and experimental data, are able to interpret the data, form an opinion on its validity, and justify that opinion, and this level of ability did not change between BME09 and BME10.

At the end of the worksheet students were asked to make a recommendation on the design of the actual device, based on the model output and experimental data. We looked for whether students were able to interpret model outputs and experimental data, and use that data to make an informed design decision on ideal values for key parameters (specifically height of LEDs above skin's surface and spacing of LEDs within the array). This is summarized in Table 8. Thirty-seven percent of BME09 students and 53% of BME10 students provided a recommendation on a preferred height above the skin and spacing between LEDs in the array. Of these recommendations, some followed the size constraints placed upon the design (the height constraint was placed on the design during the first iteration and was available in the background information during other iterations). It was encouraging to see that more students in BME10 were able to use model output and experimental data to make a design decision. It was only a small concern that some of the students forgot the design constraints when making their recommendation.

	BME09 (n=32)	**BME10 (n=64)**
Used model and experimental data to make a height and spacing recommendation that considers constraints on the device size	*2 (6%)*	*16 (25%)*
Used model and experimental data to make a recommendation that DOES NOT consider constraints on the device	*10 (31%)*	*18 (28%)*
Unable to make a recommendation on height and spacing	*20 (63%)*	*30 (47%)*
• Wants a less simplified model	6%	10%
• Wants to use more complex experiments	10%	7%
• Describes the process for selecting height and spacing, yet does not make a decision	47%	30%

Table 8 – Student recommendation for design team based on model output and experimental data. (Spacing of LEDs in array, height of LEDs above infant's skin).

Sixteen percent of BME09 and seventeen percent of BME10 students could not provide a recommendation on height and spacing, and instead recommended changes to the model or experiments that would increase their complexity, such as revising the model to include a two dimensional LED array and bubblewrap. Forty-seven percent of BME09 and 30% of BME10 students could not make a recommendation, rather stating that the design team should use the model to find the best values for spacing (*a*) and height (*h*) that would provide a uniform light distribution and spectral irradiance above the required 30μW/cm^2/nm dosage. In these cases we are unable to determine whether students are able to correctly interpret model outputs, because they did not complete their analysis. However, it was encouraging to see a decrease in incomplete analyses in BME10.

Discussion

What causes the greatest difficulty in modeling for students?

This work was motivated by the importance of modeling in design, coupled with the faculty's frustration at the failure of prior students in biomedical engineering capstone design to make use of mathematical modeling in their design projects. However, modeling is a multistep effort as Gainsburg and others have shown, and it was unclear before our study what step or steps caused students' failure to incorporate mathematical modeling into design. Therefore, we sought first to identify the steps causing trouble. Our approach to this was assisted by a framework in which specific elements in modeling can be identified. The strategy was to have all students develop and interpret a mathematical model for a particular design scenario. We chose a problem that would be moderately difficult for the students, but one that we expected them to have the knowledge and resources to tackle. The mathematical model required aspects of geometry and algebra rather than solutions of differential equations. This should have been simpler for them than identifying boundary conditions and solution methods for differential equations.

While some aspects of the study require further analysis, we can draw some conclusions about students' performance on the six steps of Gainsburg's framework.

1. Identify the real-world phenomenon
2. Simplify or idealize the phenomenon

In an earlier paper[7] we discussed students' abilities in BME09 on these aspects of modeling. Students were instructed to "tell the Phototherapy Design Team what you think should be modeled, how you would approach the modeling, and how you expect the model to eventually be helpful in the design." Students hardly ever identified *mathematical* modeling as an important aspect of this, and, once prompted specifically about mathematical models in iteration 2, they had difficulty identifying the parameters of the physical situation that would be adjusted in their model, as opposed to those parameters that were design requirements and were therefore not adjustable. We found that many students did not represent the physical situation with useful sketches, a point that is important to the next stages of modeling as well.

3. Express the idealized phenomenon mathematically (i.e., "mathematize")
4. Perform the mathematical manipulations (i.e., "solve" the model)

We asked students to do these steps in iteration 3. Students were asked to give the equations that would form the basis of the model. In BME09, few students could do this (Table 4), even though a starting point for the model was discussed in class before this iteration. In BME09 no students were able to create a correct general mathematical model for a one-dimensional line of LEDs. Only six students were able to propose a mathematical equation for summing a discrete number of LEDs in a linear array, and two of those final equations were incorrect. Finding the appropriate equations also requires identifying and justifying the assumptions underlying those equations, but is a distinct activity. Students were also generally weak in this phase (Table 2). Finally, in the homework phase of iteration 3 we also asked students to do step 4, phrased as: "calculate irradiance as a function of position on the skin." We have not fully analyzed this yet, but because few students could do step 3, they did not make it to step 4, and we did not learn as much from this as we had hoped. A separate iteration, which could not be incorporated in the class because of time constraints, would have been necessary to address this more fully, by giving the equations and asking for outputs. However, because the relevant equation could be easily entered into Matlab or Excel, we tentatively assume that if we had given students the equation, they could have generated appropriate outputs in terms of graphs or maximum and minimum values of irradiance on the skin. Step 4 is one in which students receive extensive practice in engineering courses, so we were less concerned with their capabilities in that step.

5. Interpret the mathematical solution in real-world terms
6. Test the interpretation against reality

Iteration 4 was concerned with these steps. Deciding whether a model is adequate to represent a real world situation is a critical element in doing mathematical modeling. In general, students performed somewhat better in these phases, but gave answers that were incomplete.

When we asked whether one of the assumptions (to model the LEDs as point sources of light) was appropriate, 56% of BME09 students gave a response with a reasonable justification (Table 5). When we asked whether experimental data we provided justified the use of the model, 94% of students in BME09 drew reasonable conclusions and justified them (Table 7). In response to the more complex question of whether the model represented the actual situation, which is different than the experiments, which were themselves only a physical model of the true situation of the baby receiving phototherapy, most students were able to draw conclusions (Table 6). However, very few said whether the model would likely underestimate or overestimate the actual light at the baby's skin, a critical piece of knowledge in one's confidence in making design decisions based on a mathematical model. Overall, the stronger performance on this stage of the activity, however, indicates that once data from a mathematical model was provided, students were relatively proficient in data interpretation, but a bit weaker in their critical analysis of the data.

Does it help to teach specific aspects of modeling?

In going through the process of generating the scenario and analyzing student responses, the investigators and instructors realized in a deeper way just how complex mathematical modeling is. We recognized that while students had seen models of physical behavior in many courses and of many types – control system diagrams, circuit equations, mechanical models of muscle and joints, pharmacokinetic models, action potential models, etc – they had probably never had to start from a physical situation before and generate the models, considering the assumptions, simplifications, and relation back to the physical situation. They may not have even understood that the equations in earlier courses were in fact models. Thus, in BME10, we investigated whether some instruction about modeling would help. We did not provide answers to the questions posed by the iteration that was coming up, but after each iteration we discussed what students had not done well in the previous iteration, and why it was important. In general, we attempted to alert them to the purposes of modeling and the importance of defining the right parameters, making simplifications, sketching, choosing equations, and relating the model to reality. This allowed them to perform better in some of the specific tasks we asked them to do, and in which they had been weak previously. They stated assumptions better (Table 2 and Table 3), created appropriate equations more frequently (Table 4), and more often identified the direction of the mismatch between model outputs and the physical situation (Table 6). They did not improve substantially in other areas of model interpretation, but this was where performance was best initially.

How can we improve computational skills in design further?

By the end of BME10, students were aware of at least some of the complexities and value of modeling as reflected in a post-course survey[6]. The results obtained with the BME09 cohort show that instructors cannot assume that students know what is meant by modeling or how to do it, and the BME10 results show that direct instruction in the purposes and process of modeling is likely to be valuable. We need to be aware that the scenario work may underestimate the students' true capabilities. Our iterations were done in class or as homework, and these may be less engaging activities than the students' own projects, in which they are more invested. More information about modeling could be incorporated in the next offering of the course, but we also need to analyze final reports from the BME09 and BME10 classes, to look for improvements in modeling in that context, and gain a better understanding of their performance when they are presumably performing more optimally. This will also help us learn whether the scenario work transferred, or whether it was confined only to their performance in that context. Of course analyzing real design reports has the disadvantage that we learn only about teams, or the strongest member of a team in modeling, rather than all the individuals.

Another next step is to investigate whether engineering students in different majors look like the biomedical engineering students we studied. In some majors, including electrical and mechanical engineering, design largely involves producing an artifact as it often does in biomedical engineering, whereas in chemical engineering design is often about taking a chemical process and putting it on an industrial scale. We believe that most of the same issues should apply in either case, but this remains to be investigated. It is also possible that the curriculum in certain majors promotes modeling better.

We believe that additional practice, which students will receive if they pursue careers having an element of design, is valuable for the development of expertise in particular, and of modeling specifically. It is difficult to provide that extra practice in full design challenges in the crowded undergraduate curriculum, but we would argue that courses prior to capstone design should touch more often on model creation and interpretation as well as model solution. A possible difference between engineering courses and the real world is that in most design arenas, engineers might not have to be so flexible that they would in courses, and would not have to start from scratch as we asked our students to do. This should make modeling easier in the real world. However, Gainsburg emphasizes that even if one is doing a constrained type of modeling, such as structural modeling using principles of mechanics and off-the-shelf modeling packages, many decisions have to be made related to assumptions and interpretation, and a great deal of flexibility and insight about the modeling process is required. Thus, educating students in the many aspects of modeling is well worth time in the curriculum.

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