AC 2010-2387: ASSESSMENT OF BOUSSINESQ APPROXIMATION IN A FLUID MECHANICS COURSE

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DR. MYSORE NARAYANAN obtained his Ph.D. from the University of Liverpool, England in the area of Electrical and Electronic Engineering. He joined Miami University in 1980 and teaches a wide variety of electrical, electronic and mechanical engineering courses. He has been invited to contribute articles to several encyclopedias and has published and presented dozens of papers at local, regional, national and international conferences. He has also designed, developed, organized and chaired several conferences for Miami University and conference sessions for a variety of organizations. He is a senior member of IEEE and is a member of ASME, SIAM, ASEE and AGU. He is actively involved in CELT activities and regularly participates and presents at the Lilly Conference. He has been the recipient of several Faculty Learning Community awards. He is also very active in assessment activities and has presented more than thirty five papers at various conferences and Assessment Institutes. His posters in the areas of Assessment, Bloom's Taxonomy and Socratic Inquisition have received widespread acclaim from several scholars in the area of Cognitive Science and Educational Methodologies. He has received the Assessment of Critical Thinking Award twice and is currently working towards incorporating writing assignments that enhance students' critical thinking capabilities.

Assessment of Boussinesq Approximation in a Fluid Mechanics Course

Abstract

There is an absolute need for an in-depth coverage of certain important topics in an undergraduate engineering curriculum especially in the area of Thermodynamics and Fluid Mechanics. This need arises basically from the feedback received from the alumni and also from some members of the Industrial Advisory Board. A small group of employers has also indicated that there is a need for increasing the academic rigor in certain courses. The author is of the opinion that all undergraduate engineering students must know, in addition to various other topics, the five theorems that are normally encountered while treating the subject matter of Thermodynamics and Fluid Mechanics. The five theorems are Green's Theorem, Gauss' Theorem, Stokes' Theorem, Buckingham-Pi Theorem and Boussinesq approximation. The author considers this 'set' as a part of accommodating academic rigor. The author has tried to meet these needs while he was teaching courses in Thermodynamics and Fluid Mechanics. In this presentation, the author describes how he has tried to incorporate the principles of Boussinesq approximation in a junior level fluid mechanics course. He has also outlined methods to assess students' knowledge in certain specific areas.

Introduction

Boussinesq approximation is named after the French physicist and mathematician Joseph Valentin Boussinesq for his invaluable contributions in the area of hydraulics and fluid mechanics. Boussinesq was the professor of mechanics at the Faculty of Sciences of Paris, before retiring in 1918.

There are several mathematical models to describe Boussinesq approximation and Boussinesq equations. Boussinesq approximation is normally encountered in three general areas.

1. Buoyancy: Assuming small differences in density of the fluid, one can utilize Boussinesq approximation for determining buoyancy-driven flow calculations.
2. Waves: Assuming gravitational actions, one can utilize Boussinesq approximation for analyzing the propagation of long water waves on the surface of fluid layer.
3. Viscosity: Eddy Viscosity to model Reynold's Stresses is another area where Boussinesq approximation has helped in Turbulence modeling.

It is essential that students have an adequate background of partial differenital equations before they take a course in Thermodynamics and Fluid Mechanics. In this paper, the author presents some ideas pertaining to the utilization of those mathematical techniques in a Fluid Mechanics course. Boussinesq approximation for water waves is an important topic that majority of engineering students must know and understand. The approximation is valid for non-linear waves and fairly long waves. The students should also understand that fluid being studied is incompressible and is an assumption that is used in the theory of convection. Some instructors are of the opinion that Boussinesq Equations are normally not taught in an undergraduate curriculum. The author is in partial agreement with this idea. Regardless, the author wants to introduce the importance of Boussinesq approximation in an undergraduate curriculum, because many students may not choose to go to a graduate program in fluid mechanics.

Boussinesq Equations

It is important to observe that buoyancy-driven flow experiments, data collection and calculations have been carried out on horizontal turbulent buoyant jet with small density variations only. Validity of Boussinesq approximation has been verified in these instances. However, if one tries to analyze horizontal turbulent buoyant jet with relatively larger density variations, one finds inadequate data in the literature. Perhaps, there is enough experimental data, availabile only to certain specific researchers.

It is possible to obtain an accurate description of wave evolution in coastal regions utilizing Boussinesq-type equations. Boussinesq models gained prominence during the past few decades because of the availability of high-speed computers and simulation software. The original equations were derived by Boussinesq in 1871, using a 'depth-averaged' model. However major subsequent contributions to the subject matter took place during the 1960's, wherein 'variable-depth' models were introduced.

Let us consider for example, the potential flow over a horizontal bed.

Let us consider a three-dimensional space with co-ordinates, x, y, z.

However, for this example let us consider only the two dimensional plane x and z.

If h is the mean water depth, and z is the vertical coordinate, then $z = -h$.

One can arrive at a Taylor Expansion of the velocity potential $\phi\,(x,z,t)$ around the bed level, $z = -h$.

Assume that

u is the horizontal flow velocity component,

w is the vertical flow velocity component,

g is the acceleration due to gravity and

η is free surface elevation.

Then, horizontal and vertical flow velocities can be accounted for while deriving the partial differential equations.

$$\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}[(h+\eta)u_b] = \frac{1}{6}h^3 \frac{\partial^3 u_b}{\partial x^3}$$

$$\frac{\partial u_b}{\partial t} + u_b \frac{\partial u_b}{\partial x} + g\frac{\partial \eta}{\partial x} = \frac{1}{2}h^2 \frac{\partial^3 u_b}{\partial t \partial^2 x}$$

It is normal practice to solve a system of conservation equations of an integral model using a fourth order Runge-Kutta technique. The ultimate objective is to obtain a set of numerical solutions to the problem in question.

Buoyancy Motion

If one assumes that buoyancy can drive the motion even when the temperature variations and the density variations are very small, one can understand the basis for Boussinesq Approximation. Boussinesq Equations can be derived based on these fundamental assumptions.

Supposing one considers determining the flow under certain specified conditions.

Let ρ_b and T_b represent the density and temperature of a 'bottom' layer.

If the temperature difference between the top layer and the bottom layer is very small and if

α is the coefficient of expansion, one can write:

$$\rho = \rho_b \{1 - \alpha (T - T_b)\}$$

From which one can obtain the buoyancy term: $g(\rho - \rho_b)$

In other words, when one considers buoyancy-driven flow, density differences are small enough to be ignored. However, density differences cannot be considered negligible, when they appear in conjunction with acceleration due to gravity, g.

Another viewpoint would be while considering hot and cold-water flow.

Let ρ_h ρ_c represent the densities of hot and cold water respectively.

The difference $\delta\rho = (\rho_h - \rho_c)$ is very small and can be ignored.

However, if gravity were to be considered, then:

$$g' = g\{(\rho_h - \rho_c)/\rho\}$$

In the above equation: $\rho = \rho_h$ **OR** ρ_c

The important governing term that defines buoyancy is:

$$g\,\alpha T$$

Boussinesq Approximation can also be illustrated using the temperature control of an office. Consider an office room with an open window. During the summer, the room is cool inside and the outside temperature is hotter than that of the room. There is a density difference in the air, which drives the hot air from outside to enter through the open window and rise upwards towards the ceiling of the room. If the same situation were to be observed during the winter, the room is warmer inside than the outside. Again, the density difference directs and drives the air in to the room, however, this time it moves downwards towards the floor. Observe that in both cases, the direction of airflow is always from outside to inside. Assuming symmetry and treating the airflow as Boussinesq, the room can be modeled as warm room outside out or as a cold room outside in.

Anup A. Shirgaonkar and Sanjiva K. Lele of Stanford University have examined the validity of the Boussinesq approximation for shallow flows with stratification where the atmosphere can be incompressible or compressible, with possible existence of strong base flow motion and externally imposed small initial density perturbations (Shirgaonkar & Lele, 2006).

Implementation

The author has not listed out the complete mechanics of obtaining the partial differential equtions. It can be found in standard textbooks. The author has primarily focussed on the importance of introducing Boussinesq Approximation in an undergraduate curriculum. The author proposes to implement this in one single lecture of 50 minutes duration. The students will be provided a brief review of how partial differential equations are treated before Boussinesq Approximation is introduced. A short homework assignment is also planned wherein the students are required to read, research, and report their findings in a 400-word essay that includes a historical perspective as well. At present, the author does not have plans to include a question pertaining to Boussinesq Approximation in a quiz or test or an examination. The author plans to utilize a rubric that is similar to Washington State University's Critical Thinking Rubric for purposes of assessment. The author tried to assess the level of student understanding for the exposure provided in lecture classes of fifty minutes duration. Furthermore, one should be recognize that each topic will be different and the difference may be huge and significant, particularly from the learner's point of view. In addition, each instructor's delivery style is different, one may even arrive at two different sets of data for the same subject, and topic when two different instructors are involved (Narayanan, 2007, 2009).

Assessment and Conclusions

The methodology for conducting assessment is shown in Appendix A.

Appendix B shows how a detailed assessment matrix can be generated using the principles outlined by Washington State University Rubrics.

Using EXCEL spreadsheet mode values are determined and a bar chart has been generated which is shown in Appendix C.

Analysis of data collected has been recorded in Appendix D.

After a brief introduction, review exercise it appears that the students have a good understanding and appear to be comfortable with this mathematical technique of partial differential equations. This shows an excellent score of **5** on Likert scale.

Students have shown to possess a good grasp of the basic principles of fluid mechanics, such as the concepts of Buoyancy & Metacenter. They also understand when to use Bernoulli's equation and are capable of handling the necessary mathematical calculations. This again, shows an excellent score of **5** on Likert scale.

A strong course in Hydraulics and Fluid Mechanics has provided the students an opportunity to acquire an adequate understanding of flow rate, viscosity and Reynolds number. This shows an acceptable score of **4** on Likert scale.

Students have difficulty in understanding the principles of Buckingham Pi Theorem. In this case, it appears as though they need a refresher course in calculus and differential equations. This records a score of **3** on Likert scale. We need to move towards **4.**

Some students find it difficult to grasp the concept of surface integral and volume integral. This can be seen while the students are learning about Stokes' Theorem. It has recorded a score of **3** on Likert scale. This must be improved to record at least **4.**

It seems that the students have a good grasp of Green's Theorem and Gauss' Theorem. They both show a respectable score of **4** on Likert scale.

Finally, we arrive at the Boussinesq Approximation Assignment. One can see that the concepts are tough and students have to put in extra effort to appreciate the need for this topic. Regardless, they have shown, interest, and have secured an adequate level of 3 on Likert scale.

Furthermore, when a homework assignment is given to them, they seem to fair better. Given the freedom of a take-home assignment, the students have shown that they can read a topic, reflect on it and report their findings in an excellent manner. This shows an excellent score of **5** on Likert scale.

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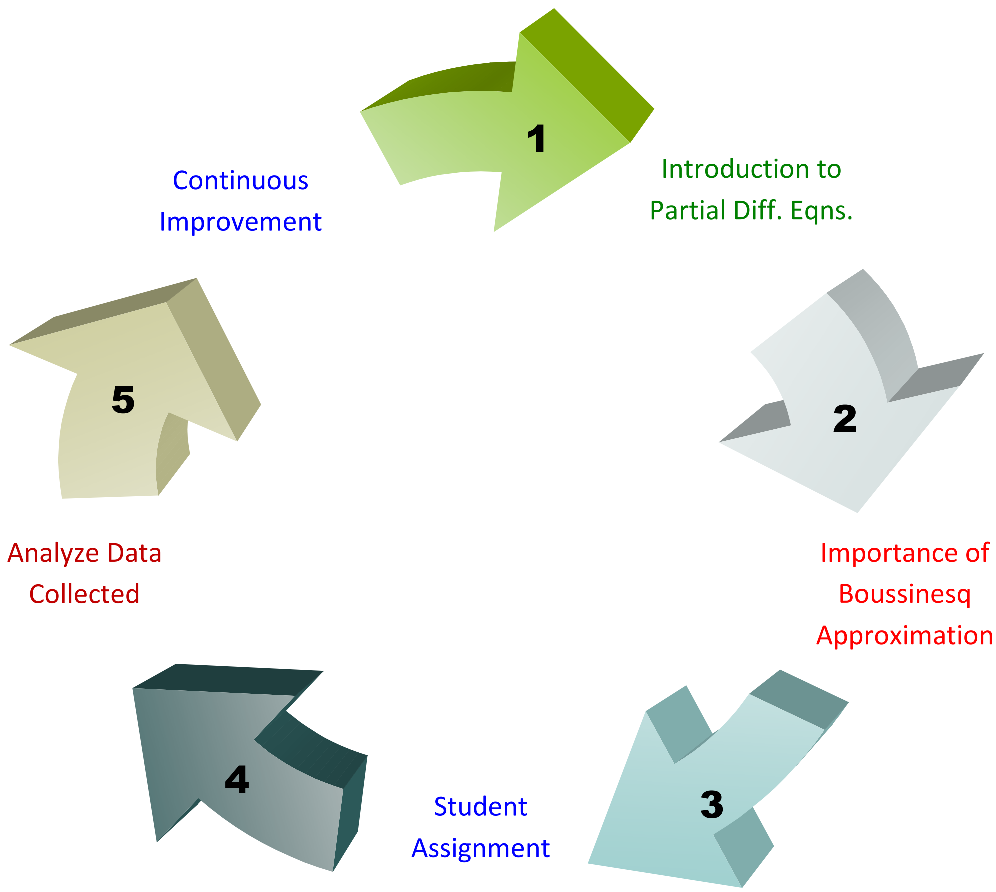
APPENDIX A: Methodology used by the author.



The author has previously used this approach in other research and other ASEE publications.

APPENDIX B : Matrix Generated using *Washington State University* Rubrics

Boussinesq Approximation Assessment

#	Course:Fluid Mechanics TOTAL xx STUDENTS #	A	B	C	.	.	.	.	X	Y	Z	MEDIAN	MODE	AVG.
	Washington State University RUBRICS LIKERT SCALE WEIGHT DISTRIBUTION: **(1 : Strongly Disagree; 5 : Strongly Agree)**													
1	Introduction to Partial Differential Equations	3	5	5	.	.	.	.	5	4	4		5	
2	Buoyancy, Metacenter, Bernoulli	3	5	5	.	.	.	.	5	4	5		5	
3	Waves, Flow, Viscosity, Reynolds Number	4	4	3	.	.	.	.	3	4	3		4	
4	Buckingham Pi Theorem	4	4	3	.	.	.	.	4	5	4		3	
5	Stokes' Theorem	5	4	3	.	.	.	.	3	4	3		3	
6	Green's Theorem	3	3	5	.	.	.	.	4	3	4		4	
7	Gauss' Theorem	3	3	5	.	.	.	.	5	4	4		4	
8	Boussinesq Approximation Assignment	4	4	3	.	.	.	.	3	4	3		3	
9	Homework Assignment and 400 Word-Essay	4	5	4	.	.	.	.	5	4	5		5	

Example of how data can be collected.

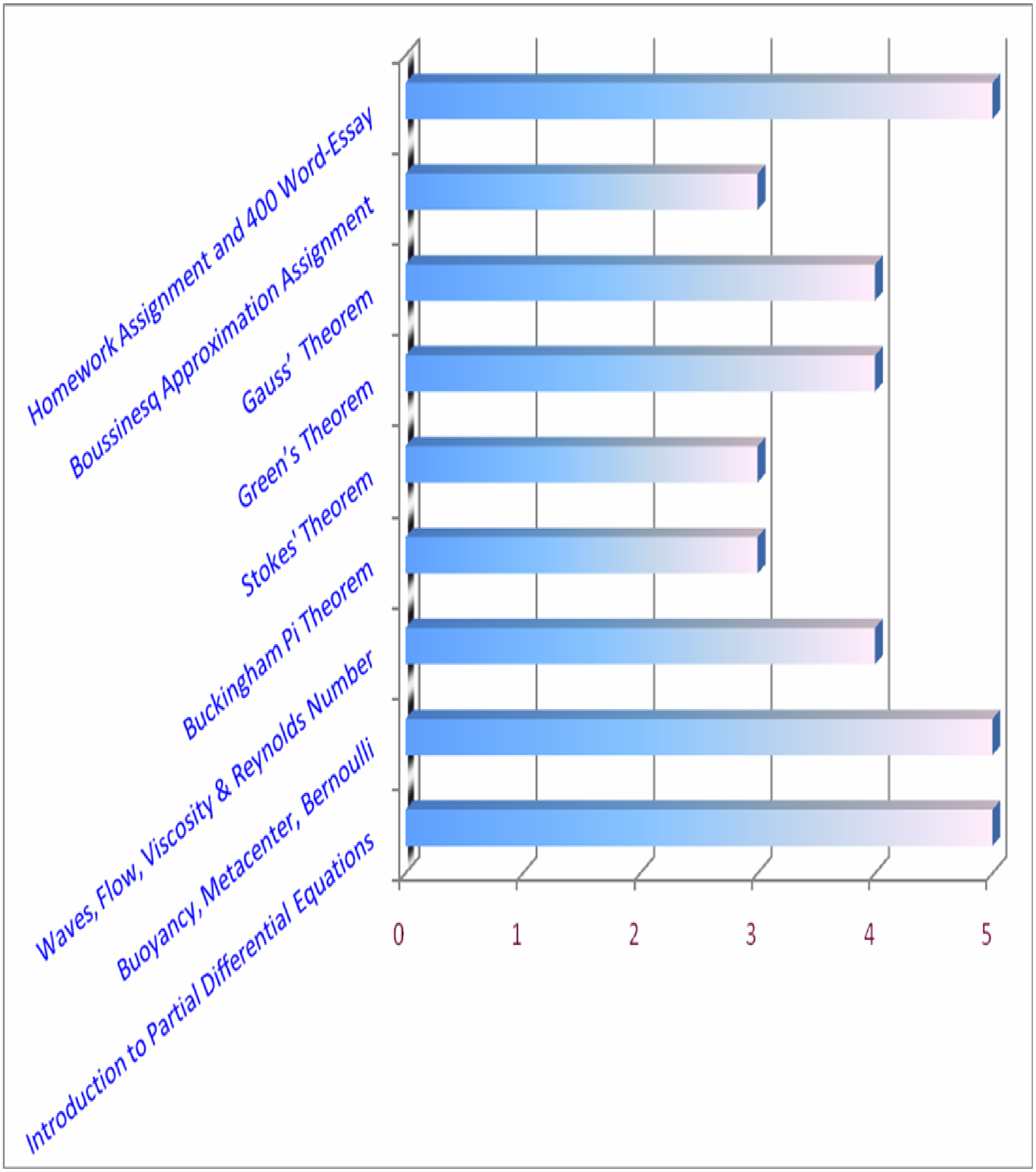
Partial list of topics that can be observed.

Data Collected by Dr. Mysore Narayanan.

APPENDIX C : Bar Chart of Data Collected

Rubrics courtesy of Washington State University, Pullman, WA.

Subject Studied: Fluid Mechanics



APPENDIX D : Analysis of the Bar Chart and Conclusions

Partial list of topics observed

1. **Introduction to Partial Differential Equations:** After a brief introduction, review exercise it appears that the students have a good understanding and are fairly comfortable with this mathematical technique. This shows an excellent score of **5** on Likert scale.

2. **Buoyancy, Metacenter, Bernoulli:** Students have shown to possess a good grasp of the basic principles of fluid mechanics. They understand Bernoulli's equation and are capable of handling the necessary mathematical calculations. This again, shows an excellent score of **5** on Likert scale.

3. **Waves, Flow, Viscosity and Reynold's Number:** With a detailed course in Hydraulics and Fluid Mechanics, the students have acquired an adequate understanding of all these topics. This again, shows an excellent score of **5** on Likert scale.

4. **Buckingham Pi Theorem:** Students have difficulty in understanding the principles in this case. It appears as though they need a refresher course in calculus and differential equations. This records a score of **3** on Likert scale. We need to move towards **4.**

5. **Stokes' Theorem:** Some students find it difficult to grasp the concept of surface integral and volume integral. This records score of **3** on Likert scale. Attempts must be made to move this to **4** atleast.

6. **Green's Theorem:** It seems that the students have a good grasp of this topic. This shows a respectable score of **4** on Likert scale.

7. **Gauss' Theorem:** Some students have learnt this in College Physics. Therefore, this again, shows a respectable score of **4** on Likert scale. This indicates that the students have a reasonable understanding of the topic.

8. **Boussinesq Approximation Assignment:** The concepts are tough and students have to put in extra effort to appreciate the need for this topic. Regardless, they have shown and interest and have secured an adequate level of **3** on Likert scale.

9. **Homework Assignment and 400 Word-Essay:** Given the freedom of a take-home assignment, the students have shown that they can read a topic, reflect on it and report their findings in an excellent manner. This shows an excellent score of **5** on Likert scale.