

Basic Vibration Design to Which Young Engineers Can Relate: The Washing Machine

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Abstract

A first course in vibration engineering is typically a content based, engineer science offering with limited time and resources for engineering design. This paper offers one example of an early design project in vibration engineering with strong instructional content that enhances the learning environment. It is crafted in a manner that is within the student's capability to complete, yet offers a taste of the interesting applications that lie ahead in their engineering education.

I. Introduction

Typical early undergraduate vibration courses focus on background theory that is used in later, more senior-level design projects and course work in the engineering curriculum. Our experience is that students are motivated at all levels of learning by real world problems that demonstrate relevance of the material. The challenge is to craft design projects at this early stage in the engineering student's career that are within their capabilities to complete, yet offer a taste of the interesting engineering applications that lie ahead.

A first course in vibration engineering is historically a content based, engineer science offering with limited time and resources for engineering design. Our goal is to introduce the design process in a manner that compliments the course material. Our students take this course prior to a formal offering in design morphology. Despite this possible shortcoming, we find that problem formulation and analysis, search for alternative solutions, decision making, and documentation with specification of results can all be accomplished without formal design training.¹

II. Design in Engineering Science Courses

Not all of the Accreditation Board for Engineering and Technology (ABET) defined fundamental elements of design are included in this process. Often, time constraints in the engineering science requirements can preclude some areas of full construction, testing, and evaluation. Despite the observation, we find it extremely beneficial to include as many design components as possible. Our desire is to promote student creativity in a meaningful design experience that allows optimal mentoring opportunities.

As with ABET, we recognize that design can not be taught in one course and that it is "an experience that must grow with the student's development."²

We find this introductory type of design reinforces subsequent course work in the formal design process. Students are later able to better grasp the design actions of establishing needs, understanding the problem and generating potential solutions, evaluating those solutions, and documenting their work.³ In the absence of a freshman-level design course in our curriculum, we find early design experiences in typical engineering science courses indispensable.

This paper offers one example of an early design project in vibration engineering with strong instructional content that enhances the learning environment. The students engage in reflective engineering problem definition and solution procedures, work as part of a team, communicate their engineering ideas, and achieve high performance. Our target audience is students in the second semester of their engineering curriculum. The design is a culminating event for the first half of the course.

III. Background of the Problem

The appliance industry is continually moving toward optimizing washing machine design and reducing costs. Current trends are toward more lightweight and composite materials. These lightweight designs increase the possibility that oscillatory walk will occur during the spin cycles of the machines. The problem of walk is receiving increased interest by appliance manufacturers.⁴

Literature on the topic of washing machine oscillatory walk is limited at best. The competitive and proprietary nature of the appliance industry contributes to this observation. This fact makes the problem even more intriguing for students as they analyze and design a practical application that has not been solved in common engineering texts on the subject.

IV. Design Problem Statement

Virtually every campus has laundry facilities for students. Most students are therefore familiar with the unwanted vibrations that occur when an unbalance of clothes accumulates during the spin cycle. Our design groups are told that they have been hired as consultants to investigate the problem and make recommendations concerning the design.

The students are required to develop a physical model that is limited to a single-degree-of-freedom (SDOF) system as a first-cut engineering analysis. Parameters of the model (e.g., masses, dimensions, rotational speeds, etc.) are estimated by physically observing the washers. The students are told that budget and other constraints preclude them from disassembling the machines, although we are currently acquiring an experimental washing machine so that these measurements could be taken directly in future semesters.

Student design groups are required to meet with their senior design consultant (their professor) to review and approve their physical model and methodology to complete the design prior to continuing beyond this point. This in-progress review allows the instructor quality control and a programmed interaction with the students that enhances the learning experience.

The remainder of the problem statement is left quite open-ended. Students are told to submit a design that limits the unwanted vibration and generates transmitted forces at a level that preclude oscillatory walk of the machines.

V. Design Approach

Most design groups approach modeling the inner housing and rotating drum assembly of the machine as a simple SDOF with a rotating imbalance. The assembly is connected to the remainder of washing machine sitting statically on the floor (See figure 1).

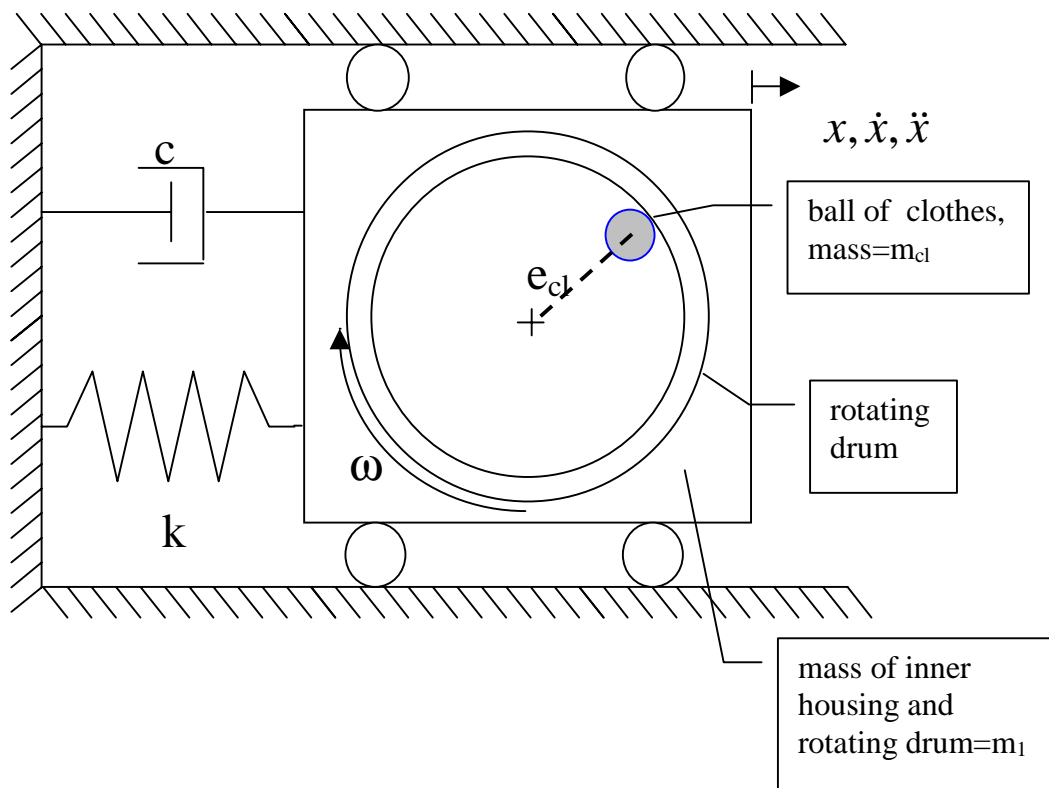


Figure 1. SDOF Model: Top View

To ensure that the system does not experience oscillatory walk, most students use a second SDOF model to analyze the entire machine sitting on the floor subject to transmitted forces through the suspension system (See figure 2). If the transmitted forces are greater than the opposing Coulomb dry friction forces, the washing machine experiences walk.

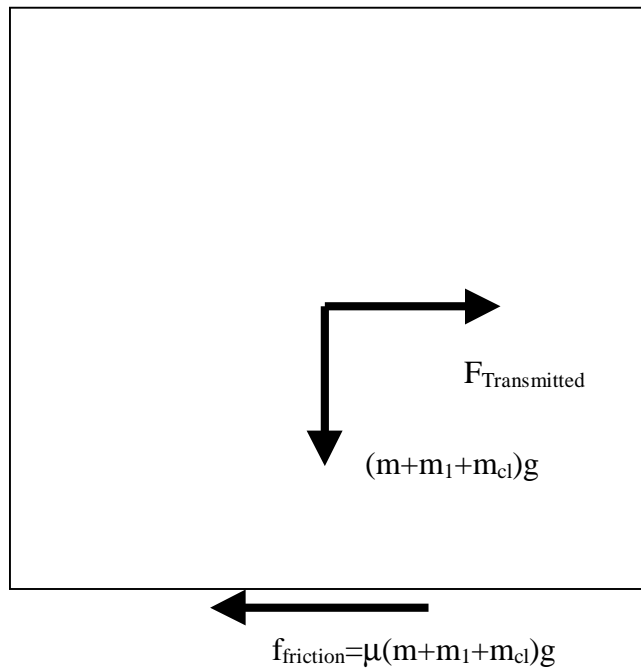


Figure 2. SDOF Model: Side View

The students are required to use their engineering judgment in estimating the parameters of the model. Table 1 lists typical values that students choose.

Table 1. Baseline Parameters

Parameters	Symbol	Value
Mass of the Entire Machine	m	$\frac{300lb}{32.2 \frac{ft}{sec^2}}$
Mass of Inner Housing and Rotating Drum	m_1	$\frac{50lb}{32.2 \frac{ft}{sec^2}}$
Mass of Unbalanced Clothes	m_{cl}	$\frac{5lb}{32.2 \frac{ft}{sec^2}}$
Coefficient of Static Friction with Floor	μ	0.5
Radial Distance to Unbalanced Clothes	e_{cl}	10 in
Spin Speed	ω	650 rpm
Suspension Spring Constant	k	varies
Suspension Damping Ratio	ζ	varies

Although any of the values listed in Table 1 could be altered in the design, students typically vary the suspension spring and damping characteristics to meet the objective of limiting unwanted vibration and generating transmitted forces at a level that precludes

oscillatory walk. The amplitude of the displacement of the inner housing and rotating drum is given by⁵:

$$\bar{x} = \frac{e_{cl}\beta\omega^2}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega\omega_n)^2}}$$

where

$$\beta = \text{mass ratio} = \frac{m_{cl}}{m_1 + m_{cl}}$$

$$\omega_n = \text{natural frequency of the inner housing and drum} = \sqrt{\frac{k}{m_1 + m_{cl}}}$$

$$\zeta = \text{damping factor} = \frac{c}{2(m_1 + m_{cl})\omega_n}$$

Figure 3 shows a plot for the baseline parameters. Suspension parameters of $\zeta=0.4$ and $k=35$ lb/ft were used to start the analysis. Note that the system is operating well above resonance. A rather reasonable displacement amplitude value, \bar{x} , of 0.912 inches is determined at the current spin cycle operating condition with an unbalance of 5 lb.

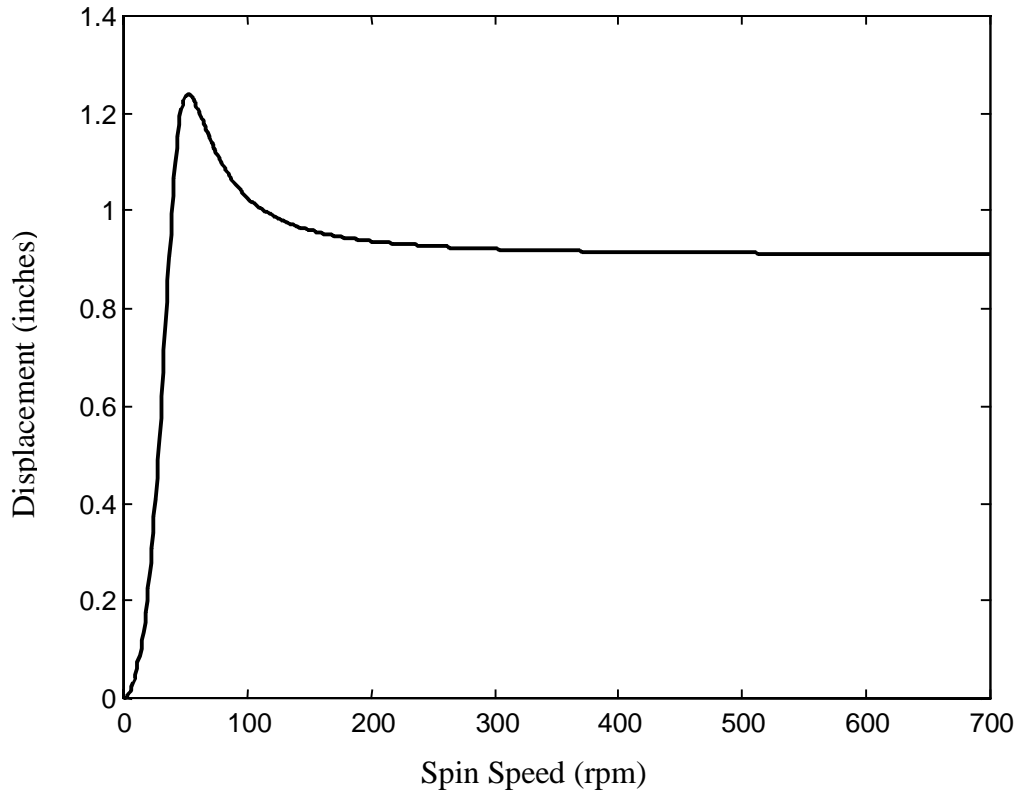


Figure 3. Displacement Amplitude vs. Spin Speed for Baseline Parameters

It is often important from a teaching perspective to express the value of the amplitude of displacement in dimensional units (i.e., inches). This gives the student an intuitive feel whether the results make sense from an engineering perspective. Another teaching point can be made by encouraging the development of a non-dimensional expression for displacement as:

$$\frac{\bar{x}}{e_{cl}\beta} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

where

$$r = \text{frequency ratio} = \frac{\omega}{\omega_n}$$

This form of the expression reduces the number of parameters in the design and allows the analysis of characteristic curves regardless of specific parameter values.

In continuing with the design, while holding the damping ratio constant at $\zeta=0.4$, the spring constant characteristics of the suspension can be varied. Figure 4 is a plot of the results for $k=35$ lb/ft, $k_1=5$ lb/ft, and $k_2=100$ lb/ft.

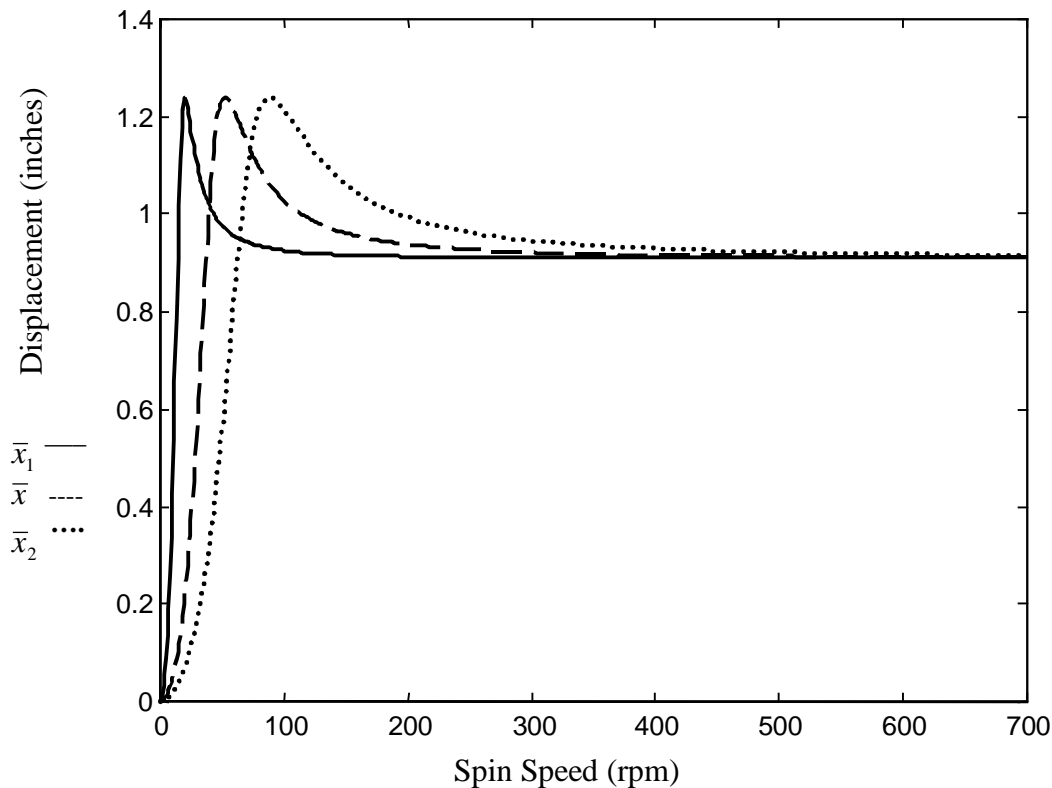


Figure 4. Displacement Amplitude vs. Spin Speed; \bar{x} corresponds to $k=35$ lb/ft; \bar{x}_1 corresponds to $k_1=5$ lb/ft; \bar{x}_2 corresponds to $k_2=100$ lb/ft.

Students should note that by changing the stiffness, the damping ratio normally changes. In order to hold the damping ratio at a value of $\zeta=0.4$, the damping constant, c , must be changed proportionally with the stiffness. The relationships between k , ζ , and c are additional important teaching concepts for the student. By holding ζ constant and varying k , the occurrence of the resonant peak shifts.

Similarly, Figure 5 is a plot in which the spring constant, k , is held constant at the baseline parameter of 35 lb/ft and the damping factor, ζ , is varied for three values of $\zeta=0.4$, $\zeta_a=0.2$, and $\zeta_b=1.0$. Students should note the assumption of linear viscous damping in the model and subsequently this graph. Typical optimized suspension models assume linear viscous damping while standard washing machines generally use Coulomb friction damping. These friction dampers are inexpensive, but inherently nonlinear and difficult to hold at precise levels.⁶

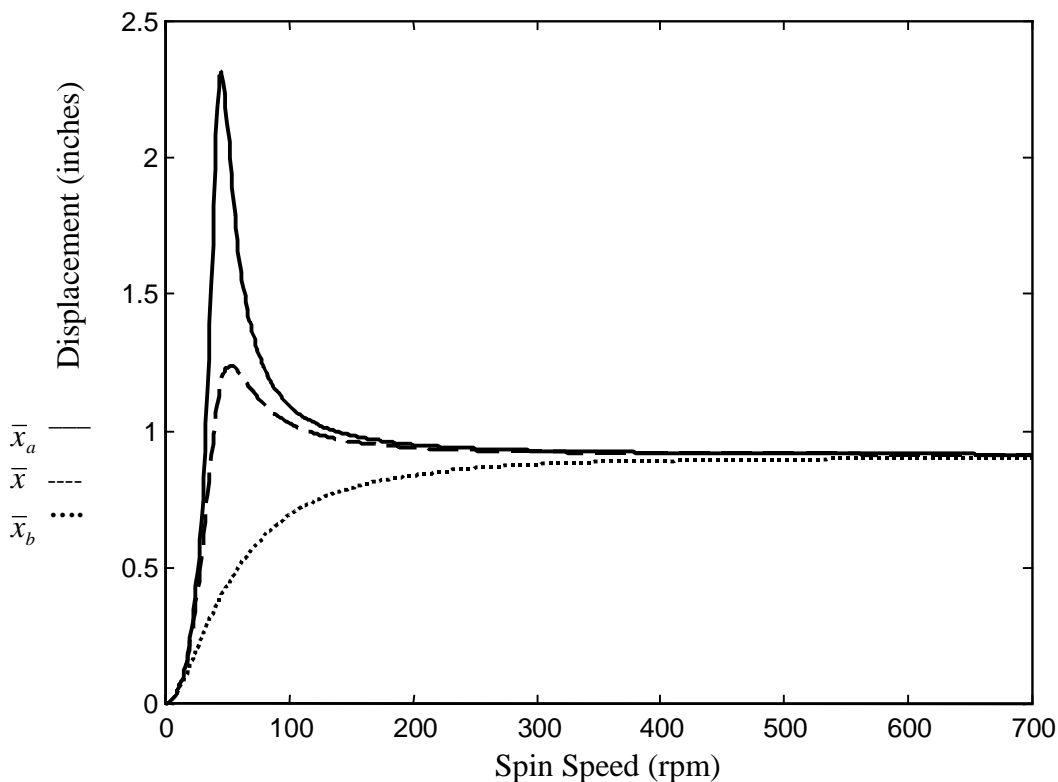


Figure 5. Displacement Amplitude vs. Spin Speed; \bar{x} corresponds to $\zeta=0.4$; \bar{x}_a corresponds to $\zeta_a=0.2$; \bar{x}_b corresponds to $\zeta_b=1.0$.

We note that increased damping decreases the amplitude of displacement, particularly around resonance. While operating well above resonance, in the region of a spin speed of 650 rpm, this effect is minor. In this region the graph asymptotically approaches a value of $e_{cl}\beta$. Thus in this region, the major controlling factors are the eccentricity of the load and the mass of the unbalanced clothes.

Next, the students examine the transmitted forces to the remainder of the machine and determine the conditions that potentially cause oscillatory walk. The amplitude of the transmitted force can be expressed as⁵:

$$F_T = \frac{e_{cl} m_{cl} \omega^2 \sqrt{(\omega_n^2)^2 + (2\zeta \omega \omega_n)^2}}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta \omega \omega_n)^2}}$$

As discussed earlier, additional learning points can be made by examining the non-dimensional form of the transmitted force as well:

$$\frac{F_T}{e_{cl} \beta k} = \frac{r^2 \sqrt{1 + (2\zeta r^2)^2}}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}}$$

Figure 6 is a plot of the dimensional transmitted forces for the spring constant, k , held constant at the baseline parameter of 35 lb/ft and the damping factor, ζ , varied for three values of $\zeta=0.4$ (baseline), $\zeta_1=0.2$, and $\zeta_2=1.0$.

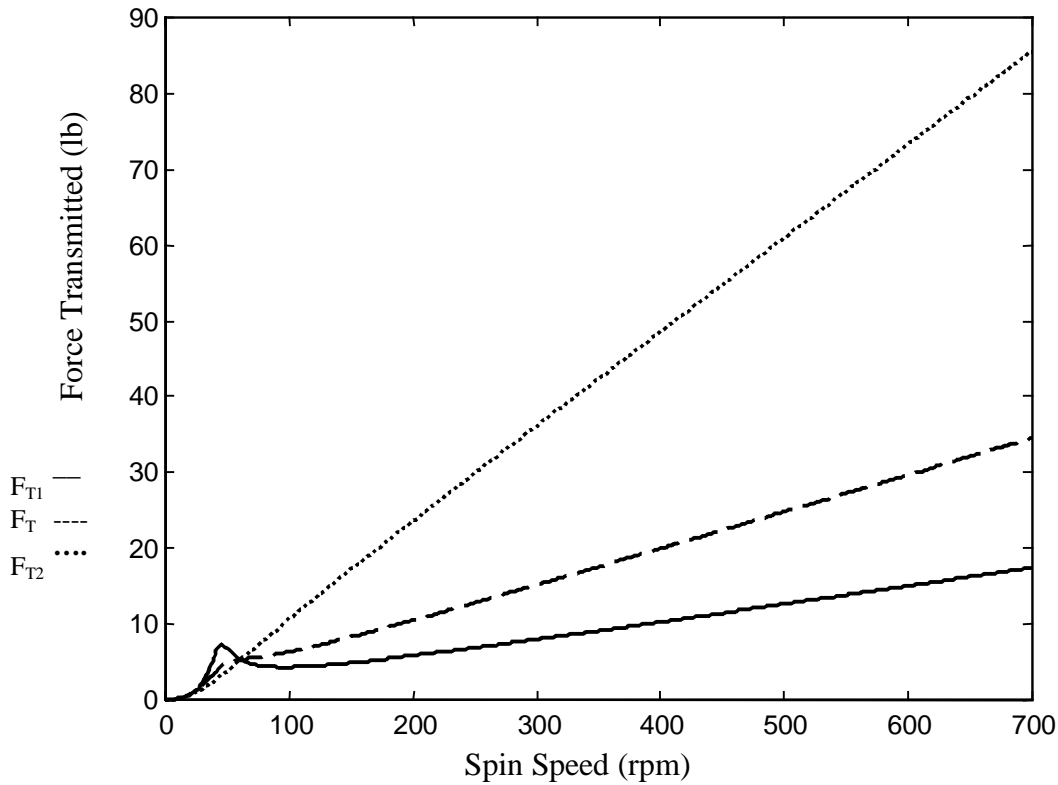


Figure 6. Transmitted Force vs. Spin Speed; F_T corresponds to $\zeta=0.4$; F_{T1} corresponds to $\zeta_1=0.2$; F_{T2} corresponds to $\zeta_2=1.0$.

This graph illustrates interesting results that the student must analyze. As with the displacement amplitude, the transmitted forces decrease near resonance with increased damping. Above resonance, however, the transmitted forces actually increase with increased damping. Therefore, there is a tradeoff that the student should make note of between controlling the displacement amplitude and the transmitted forces simultaneously.

Washing machine tipping is considered a safety hazard in the United States and it is not allowed as a failure mode. The oscillatory walk failure mode, therefore, is designed as a form of slip rather than tip.⁴ Assuming as shown in Table 1 that the total weight of the machine is 300 lb and the coefficient of static friction with the floor is 0.5, the transmitted force required to overcome the static friction force is μN or 150 lb. We see from figure 6 that this is not a problem at the parameters currently specified. The student can easily vary the baseline parameters to predict when oscillatory walk will occur.

VI. Other Design Factors

The students may comment on other design factors in the problem that may help meet the objective of reducing unwanted vibration and transmitted forces that cause oscillatory walk. Traditionally, to counteract these forces, additional mass may be added to the system. Research shows that this mass can be as much as 60% of the washing machine's original mass. Problems occur though, in that the addition of this large quantity of mass can result in other structural failures or an overdesign of the remaining structural system due to the static requirements of the additional mass.⁴

Other discussions may center on bolting the machines to the foundation or floor. This has actually been tried in the past to control walk due to excessive unbalanced forces.⁷ A special stand has been patented in Germany that locks the machine's feet in place.⁸ Students should discover that this can lead to excessive forces being transmitted to the floor, however, and could become a customer complaint because of unacceptable levels of floor vibration.⁴

VII. Conclusions

The design project introduced in this paper can be integrated into a typical vibration engineering science course. Its content deals directly with material covered in this type of course and relates topics from the classroom to real world engineering applications. The project emphasizes the importance of the theoretical background and enhances the learning environment by showing relevancy of the material. These objectives are accomplished at a level that is consistent with the education of undergraduate vibration students at this point in their engineering curriculum.

VIII. Acknowledgments

The views expressed herein are those of the authors and do not purport to reflect the position of the U.S. Military Academy, the Department of the Army, or the Department of Defense.

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