Bernoulli Balance Experiments Using a Venturi

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Abstract

Two simple and inexpensive venturi experiments were developed for use in either the laboratory or classroom. The purpose of this paper is to present the equipment, procedures and experimental results for these experiments, as used in a junior level fluids laboratory. In the first experiment, a shop fabricated venturi was employed to determine the experimental minor loss coefficient, K, in an unsteady-state system. The throat velocity determined by the Bernoulli balance was about 16% higher than the measured experimental velocity. The minor loss coefficient was 0.86, which indicates only about 15% of the energy contained in the throat stream was recovered. This value is consistent with results reported in the literature. In the second experiment, a venturi was fashioned from small, inexpensive funnels and once again used to determine the experimental minor loss coefficient, this time in a steady state system. The average calculated minor loss coefficient was 0.28 and deviated about 3.6% among the three experimental runs. The experimental throat pressures ranged from 26.2-28.5 in Hg absolute, and the theoretical and experimental pressures agreed within 5%.

Introduction

One of the main objectives of engineering education is to effectively transfer subject information to the engineering students. A number of methods have been developed for enhancing student learning including multimedia developments,[1,2] active, problem-based learning,[3] collaborative learning,[4,5] and participation in cooperative education.[6] Several papers have specifically addressed methods for improving or supplementing the teaching of engineering including the use of spreadsheets to solve two-dimensional heat transfer problems,[7] the use of a transport approach in teaching turbulent thermal convection,[8] the use of computers to evaluate view factors in thermal radiation,[9] implementation of a computational method for teaching free convection,[10] and the use of an integrated experimental/analytical/numerical approach that brings the excitement of discovery to the classroom.[11] Supplemental experiments for use in the laboratory or classroom have also been presented, including rather novel experiments such as the drying of a towel[12] and the cooking of French fry-shaped potatoes.[13] Suggestions for the integration of engineering material into the laboratory and classroom have been described by Penney and Clausen,[14-20] who presented a number of simple hands-on experiments that can be constructed from materials present in most engineering departments. This cross-course integration of course material has been shown to be a very effective learning tool that causes students to think beyond the content of each individual course.[21]

Many different types of meters are used industrially to measure the rate at which a fluid is flowing through a pipe or channel. The selection of a meter is based on the applicability of the

instrument to the specific problem, its installed costs and operating costs, the desired range of flow rates and the inherent accuracy of the flow meter.[22] In some situations, only a rough indication of the flow rate is required; at other times, highly accurate measurements are necessary to properly control or monitor a process.

The venturi meter is one of the more common full-bore meters, i.e. meters that operate by sensing all of the fluid in the pipe or channel. In the operation of a venturi meter, upstream and downstream pressure taps are connected to a manometer or differential pressure transmitter (see Figure 1). This pressure differential is then used to determine the flow rate by applying a Bernoulli balance from the entry of the throat of the meter. The angle of the discharge cone is typically set between 5° and 15° to prevent boundary layer separation and to minimize friction. Typically, 90% of the pressure loss in the upstream cone is recovered, making the venturi very useful for measuring very large flow rates, where power losses can become economically significant. Thus, the higher installed costs of a venturi (over an orifice) are offset by reduced operating costs.[22, 23]
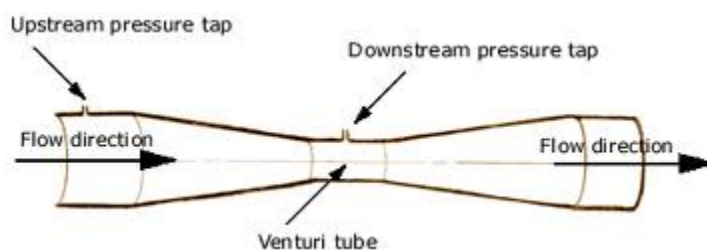
Figure 1. Venturi Meter Operation

The purpose of this paper is to demonstrate two simple and inexpensive laboratory experiments for analyzing the performance of a venturi. These experiments may be performed as laboratory exercises or as classroom demonstrations. In the first experiment, a shop fabricated venturi was employed to determine the experimental minor loss coefficient, K, in an unsteady-state system. In the second experiment, a venturi was fashioned from small, inexpensive funnels and once again used to determine experimental minor loss coefficient, this time in a steady state system. Junior level Chemical Engineering students at the University of Arkansas helped to develop and perform these experiments as part of the requirements for CHEG 3232, Chemical Engineering Laboratory II.

Commercial Venturi Experiment

The first experiment employed a shop fabricated acrylic venturi (see Figure 2), with a throat diameter of 4.4 mm ($\frac{3}{16}$ in) and a pressure tap in the throat. The venturi was assembled in the flow system of Figure 3, consisting of a feed reservoir, an exit reservoir, connecting piping and plastic tubing which served as simple water-filled manometers. Tap water served as the test fluid.

Figure 2. Photograph of Shop Fabricated
Venturi

Figure 3. Flow System with Venturi
Installed

Experimental Procedures

To ready the system for operation, the top reservoir was filled to its top mark, and the bottom
reservoir was completely filled, such that the exit pipe was submerged as shown in Figure 4. All
air was cleared from the system by suctioning. At time zero, the stopper was removed from the
bottom of the piping system, and then the time required for the upper reservoir to drain was
recorded as the liquid level passed graduated marks. Several additional measurements were
made:

- The distance from the top of the feed reservoir to the water level
- The distance from the top of the feed reservoir to the venturi throat
- The distance from the top of the feed reservoir to the water level in the bottom reservoir
- The diameter of the feed reservoir

Figure 4. System Exit in Bottom Reservoir

Equipment List

- Shop fabricated venturi, 1 in NPT fittings, equipped with pressure tap at its throat (45° inlet angle, 30° outlet angle, 4 mm ($\frac{3}{16}$ in) throat)
- Water reservoirs (buckets), 2
- Copper tubing, 19 mm ($\frac{3}{4}$ in) od, with fittings
- Copper tubing, 6 mm ($\frac{1}{4}$ in) od
- Rubber stopper
- Tygon tubing, 12 mm ($\frac{1}{2}$ in) od
- Stand for apparatus, as available
- Garden hose
- Stopwatch

Results and Discussion

Table 1 presents experimental data and Figure 5 shows a plot of the height measurements with time as the feed reservoir drained. As is noted in Figure 5, the height in the reservoir decreased linearly with time.

Table 1. Measured Heights

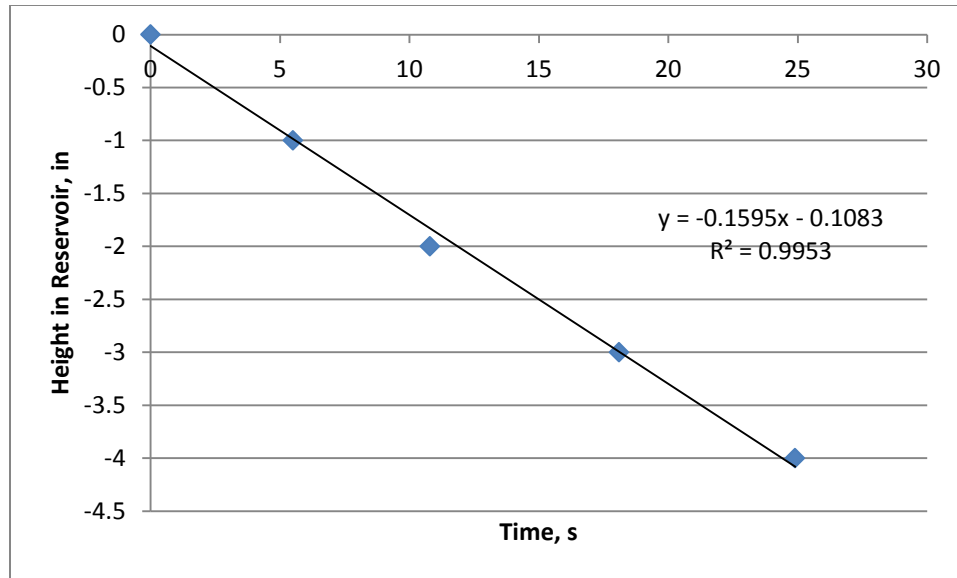| Distance from Feed Reservoir to . . .	Measured Distance, m
Water level	-1.589
Throat	-0.513
Water level in bottom reservoir	-1.019
Feed Reservoir Diameter	15.2 cm (6 in)

Figure 5. Height Measurements with Time as the Feed Reservoir Drained

Data Reduction

Experimental Velocity through the Throat

The velocity through the throat was calculated by dividing the volumetric flow rate from the feed reservoir by the cross-sectional area of the reservoir. The areas of the feed reservoir and venturi throat were determined as follows:

$$A_{fr} = \frac{\pi D_{fr}^2}{4} \tag{1}$$

$$A_t = \frac{\pi D_t^2}{4} \tag{2}$$

Since the height in the feed reservoir decreased linearly with time, the rate of change may be calculated by the equation

$$\frac{dH}{dt} = \frac{\Delta H}{\Delta t} \tag{3}$$

The volumetric flow rate from the feed reservoir is then calculated as

$$Q = \frac{dH}{dt} A_{fr} \tag{4}$$

Finally, the throat velocity is obtained by

$$v_{vc} = \frac{Q}{A_t} \tag{5}$$

Throat Velocity from the Bernoulli Balance

McCabe *et al.*[22] show the Bernoulli balance (with α terms set to zero) as

$$\frac{P_1}{\rho} + gZ_1 + \frac{v_1^2}{2} + W = \frac{P_2}{\rho} + gZ_2 + \frac{v_2^2}{2} + h_f \tag{6}$$

In applying the Bernoulli balance across the manometer, the pressure at the throat is obtained in terms of the elevations, with W and h_f both equal to zero.

$$\frac{P_m}{\rho} + gH_m = \frac{P_t}{\rho} + gH_t \tag{7}$$

But $P_m = 0$, because the reference pressure is atmospheric. Thus,

$$\frac{P_t}{\rho} = g(H_m - H_t) \tag{8}$$

The Bernoulli balance is now applied from the liquid level in the feed reservoir to the throat, with $W = 0$ and friction neglected between the feed reservoir and the throat.

$$\frac{P_{fr}}{\rho} + gH_{fr} = \frac{P_t}{\rho} + gH_t + \frac{v_t^2}{2} \tag{9}$$

Making use of Equation (8), with $P_{fr} = 0$, Equation (9) becomes

$$gH_{fr} = \frac{P_t}{\rho} + gH_t + \frac{v_t^2}{2} = g(H_m - H_t) + gH_t + \frac{v_t^2}{2} \tag{10}$$

But the height in the feed reservoir is the reference point. Thus,

$$v_t = (2gH_m)^{1/2} \tag{11}$$

Permanent Friction Loss in the Venturi

In applying the Bernoulli balance between the liquid level in the feed reservoir and the liquid level in the discharge reservoir, the following relationship is obtained:

$$h_f = gH_p \tag{12}$$

The permanent friction loss is normally correlated in terms of the velocity in the throat, i.e.,

$$h_f = K\frac{v_t^2}{2} \tag{13}$$

where K is the minor loss coefficient for the venturi. Equating Equations (12) and (13) yields

$$K = \frac{2gH_p}{v_t^2} \tag{14}$$

Discussion of Results

Throat Velocity

The experimental velocity from Equation (5) is 4.82 m/s and the velocity from the application of the Bernoulli balance (Equation (11)) is 5.58 m/s. The experimental velocity should be less than the Bernoulli balance velocity because the velocity near the venturi walls is lower than the centerline velocity. The experimental velocity is lower than the Bernoulli balance velocity by about 16%.

Venturi Minor Loss Coefficient

The experimental minor loss coefficient from Equation (14) is 0.86, which indicates that only about 15% of the energy content of the vena contracta stream is recovered. This is a low efficiency, which is a result of the large 30° included angle of the diffuser. Duggins[24] found that the pressure recovery efficiency for a 30° included angle conical diffuser was about 0.25 for an area ratio of 3.265 (a diameter ratio of 1.8). The area ratio for the current diffuser was about $\left(\frac{0.75in}{\frac{3}{16}in}\right)^2 = 16$. With the flow separation which occurs in a 30° included angle diffuser, the shop fabricated venturi is too long for optimum pressure recovery.

Conclusions

1. The throat velocity determined by the Bernoulli balance was about 16% higher than the measured experimental velocity, probably caused by
   a. the boundary layer effects in the small $\left(\frac{3}{16}in\right)$ throat, and
   b. the effect of the jet vena contracta occurring somewhat downstream of the throat of the venturi
2. The minor loss coefficient for the venturi was about 0.86, which indicates only about 14 % of the energy contained in the vena contracta stream was recovered. This value is consistent with results reported in the literature.

Funnel Venturi Experiment

A venturi experiment was also performed with a "homemade" venturi, fashioned from 3 in diameter plastic funnels. Figure 6 shows a photograph of the supplies used to construct the venturi. Epoxy was used to securely fasten the top funnel into a 3 in hole at the bottom of a bucket. The second funnel was then attached to the first funnel using epoxy with the aid of a 10 cc syringe. Tygon tubing was split and then attached to the ends of each funnel with putty. Hose clamps were also placed on the ends to further secure the tubing. The split in the tube was repaired by using clear packaging tape. A 9.5 mm (0.375 in) tube was placed through the tygon tubing at the throat of the venturi to measure the pressure with a vacuum gauge. A PVC pipe splint and putty were used to support the tube and keep it straight.

Figure 6. Supplies Used in Preparing a Funnel Venturi

Experimental Procedures

The experimental set-up for the funnel venturi system was slightly different from the procedures used with the commercial venturi (see Figure 7), primarily in the use of a vacuum gauge to measure pressure in the throat and the use of a constant level feed reservoir (operation at steady state). Figure 8 shows a close-up of the pressure tap at the throat.

To begin the experiment, the feed and bottom reservoirs (buckets) were filled to the top with tap water. The water flow from the tap was adjusted to keep the height of the water in the feed reservoir constant. Once the system was operating at steady state, the distance from the water level in the feed reservoir to the water level in the bottom reservoir was measured, and the pressure at the throat of the venturi was recorded. The flow rate of water was determined by diverting the feed hose into a reservoir (bucket) for a measured time period. This procedure was repeated at two additional flow rates.

Equipment List

- Funnels, 2, 76.2 mm (3 in) top diameter, 10 mm (0.3855 in) nozzle diameter
- Water reservoirs (buckets), 3
- Balance
- Epoxy
- Plumber's putty
- Duct tape
- Packaging tape
- Electrical tape
- Syringe, 10 cc, to disperse epoxy

- 1 ½ in PVC pipe splint
- Hose clamps
- Tygon tubing, 35 mm (1 $\frac{3}{8}$ in) od
- Copper tubing, 6 mm ($\frac{1}{4}$ in)
- Syphon
- Stand for apparatus, as available
- Vacuum gauge, 0-30 in Hg
- Garden hose
- Stopwatch

Figure 7. Flow System with Venturi   Figure 8. Close-up of Venturi Tap at the Throat

Experimental Data

The data from the three experimental runs are presented in Table 2. The units are mixed (SI, English units) because they represent the actual experimental data taken in the laboratory. As expected, the vacuum in the vena contract increased as the the flow rate of water increased.

Table 2. Experimental Data from the Funnel Venturi Experiment

Run	P_t, in Hg (gauge)	H_p, m	Water Collected, lb_m	Time for Collection, s
1	3.7	-0.560	44.5	39.79
2	3.0	-0.495	45.5	49.24
3	1.4	-0.307	43.0	55.16

Data Reduction

Experimental Velocity through the Throat

The velocity in the throat is once again found using Equation (5)

$$v_t = \frac{Q}{A_t} \tag{5}$$

However, in these steady state experiments, the volumetric flow rate is found by the equation

$$Q = \frac{m\rho}{t} \tag{15}$$

Permanent Friction Loss in the Venturi

The minor loss coefficient for the venturi is once again found by the equation

$$K = \frac{2gH_p}{v_t^2} \tag{14}$$

Bernoulli Balance to Find Throat Pressure

A Bernoulli balance may be written for the system between the top of the feed reservoir and the throat of the venturi:

$$\frac{P_{fr}}{\rho} + gH_{fr} = \frac{Pt}{\rho} + gH_t + \frac{V_t^2}{2} \tag{16}$$

After rearrangement, the theoretical pressure in the throat can be calculated using the equation

$$p_t = \rho \left[\frac{P_{fr}}{\rho} + g(H_{fr} - H_t) - \frac{V_t^2}{2} \right] \tag{17}$$

The experimental absolute pressure in the throat can be found by subtracting the vacuum gauge pressure from atmospheric pressure. Comparisons can then be made between the theoretical and experimental pressures.

Discussion of Results

Table 3 shows results for the velocity in the throat vena contracta, the minor loss coefficient, and experimental and theoretical throat pressures for the three experimental runs. The calculated velocities were 6.72, 5.56 and 4.69 m/s, for the three experimental runs. The calculated minor loss coefficients were 0.25, 0.31 and 0.27. Although the coefficient was not constant for the three runs as expected, the percent deviation for these values was only 3.5%, an acceptable experimental error. The experimentally obtained throat pressures for each flow rate were 26.22, 26.92 and 28.52 in Hg, and the theoretical throat pressures calculated from experimental velocities were 25.02, 26.93 and 27.69 in Hg. The theoretical and experimental pressures for each data point agreed within 5%, again an acceptable experimental error.

Table 2. Reduced Results from the Experiments

Run	v_t (m/s)	K	P_t, in Hg abs	
			Experimental	Theoretical
1	6.72	0.25	26.22	25.02
2	5.56	0.31	26.92	26.93
3	4.69	0.27	28.52	27.69

Conclusions

1. The calculated minor loss coefficients were 0.25, 0.31 and 0.27, a scatter of 3.6%.
2. The experimentally obtained throat pressures for each flow rate were 26.22, 26.92 and 28.52 in Hg, while the theoretical throat pressures based on experimental velocities were 25.02, 26.93 and 27.69 in Hg. The theoretical and experimental pressures agreed within 5%.

Nomenclature

A_{fr}	Area of feed reservoir, m^2
A_t	Area of throat, m^2
D_{fr}	Diameter of feed reservoir, m
D_t	Diameter of throat, m
H	Height, m
H_{fr}	Height of liquid in the feed reservoir, m
H_m	Distance from the top of the feed reservoir to the water, m
H_p	Distance from the top of the feed reservoir to the water in the bottom reservoir, m
H_t	Distance from the top of the feed reservoir to the throat
K	Minor loss coefficient for the venture, dimensionless
P_1	Pressure at position 1, atm
P_2	Pressure at position 2, atm
P_{fr}, P_m	Pressure at manometer entrance, ($P_{fr} = P_m$ = atmospheric pressure)
P_t	Pressure in throat, atm
Q	Volumetric flow rate from feed reservoir, m^3/s
v_1	Velocity at position 1, m/s
v_2	Velocity at position 2, m/s
v_t	Velocity in throat, m/s
W	Work done on/by the system, m^2/s^2
Z_1	Height at position 1, m
Z_2	Height at position 2, m
$\frac{dH}{dt}$	Height change in the feed reservoir with time, m/s
g	Acceleration of gravity, m/s^2
h_f	friction in the system, m^2/s^2
m	Mass of collected water, kg
t	Time, s
Δ	Change in . . .

ρ Fluid (water) density, kg/m^3

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