



Retrieval Practice and Spacing: Effects on Long-Term Learning among Engineering Precalculus Students

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Abstract

Mathematical competency is vital to success in engineering. Competency requires not only short-term mastery of mathematical concepts, but also long-term retention. Research in cognitive psychology shows that the more times information is retrieved from memory, the more likely it is to be remembered over the long-term. Research also shows that increasing the amount of time between retrievals increases long-term retention. Which of these interventions—increasing the amount or the temporal spacing of retrieval practice—has the greater impact on long-term retention? Supported by NSF’s Improving Undergraduate STEM Education (IUSE) program, we answered this question by independently manipulating the amount and the spacing of retrieval practice in a precalculus course for engineering students. This was done by varying the number of quiz questions concerning key precalculus objectives (amount of practice) and varying whether those questions were massed on a single quiz or distributed across quizzes in multiple weeks (spacing of practice). Long-term retention was assessed in a test of precalculus knowledge administered one month after the end of the precalculus course. We found that students were significantly more likely to retain precalculus objectives when quiz questions had been spaced versus massed. Increasing the number of quiz questions did not significantly affect retention. These findings suggest that educators wishing to increase students’ long-term retention of mathematics knowledge should increase the spacing, rather than the amount, of retrieval practice in their courses.

Introduction

College courses require students to learn large amounts of information, but students can rapidly lose the ability to recall information from previous courses or semesters [1 - 4]. This loss of information is especially harmful when success in upper-level courses depends on students’ ability to remember foundational information learned in lower-level courses. Engineering students are a prime example of those who rely heavily on retention of lower-level course content. Currently, one-third of students who start an engineering program fail to complete it [5], possibly due in part to failure to retain foundational knowledge.

Research in cognitive psychology suggests that simple changes could be implemented in classrooms in order to increase retention of information. Students report choosing to reread (or reread) material over any other study technique [6], but this is not an optimal strategy for maximizing long-term retention. Instead, once students understand a concept or procedure, they should begin actively retrieving relevant information from memory—a technique known as retrieval practice. Retrieval practice increases long-term retention of information to a greater extent than does reread (e.g., [7]). Effects of retrieval practice have been extensively studied in the laboratory, typically using verbal materials. For example, Karpicke and Roediger [8] had

participants learn Swahili-English word pairs (e.g., *mashua*-boat). After participants reached the criterion of being able to produce the correct English translation a single time, they either repeatedly restudied the word pair, repeatedly retrieved the English translation, or did not encounter the word pair again. In the latter condition, when items were neither retrieved nor restudied following initial learning, recall of their English translations was only 33% after a one-week delay. Restudy increased recall only minimally and non-significantly to 36%, but repeated retrieval increased recall to 80%. Classroom studies have also supported the notion that retrieval practice enhances memory (e.g., [9]).

Because retrieval practice is a potent memory-enhancing technique, and because many students fail to spontaneously engage in it, cognitive psychologists have called on educators to incorporate more retrieval practice into their pedagogical practices (e.g., [10, 11]). Instead of relying on students to practice retrieval during their own study time, educators should increase the frequency with which students retrieve information as part of required coursework (e.g., on quizzes or homework assignments). This should have the effect of increasing the information's long-term retention.

Simultaneously, cognitive psychologists have also called on educators to change the temporal distribution of retrieval attempts. Some college courses, including many mathematics courses, already involve some amount of retrieval practice (e.g., on problem sets). While this is preferable to the absence of retrieval practice, it is not necessarily optimal because all the practice may occur in a short temporal window. For example, after learning how to perform a particular mathematical operation, students may complete all related problems in the span of a few hours. Cognitive psychologists refer to this as (temporally) massed practice. Although massed practice is commonplace in educational settings, it yields poor long-term retention when compared to practice that is distributed or spaced over a longer period of time (e.g., [12, 13], [14]). An example of spaced practice would be completing multiple problems requiring the same mathematical operation over the course of several days or even weeks, with considerable intervals of time between each instance of retrieval. The finding that spaced practice (versus massed) leads to superior long-term retention is known as the spacing effect. Cognitive psychologists have encouraged educators to make use of the spacing effect by requiring students to retrieve critical information in spaced fashion (e.g., [10, 11, 15]).

In sum, cognitive psychologists have offered two strong recommendations to educators who wish to increase students' long-term retention of course content: Increase the amount of retrieval practice required of students and increase the spacing of retrieval practice. While these recommendations are sensible given large research literatures generally supporting the existence of retrieval practice and spacing effects, questions arise when educators are faced with putting the recommendations into practice. One question is whether the two proposed interventions will be effective in an educator's particular instructional field. Retrieval practice and spacing effects have most often been studied in laboratory settings using verbal materials. Relatively little

research has been conducted in actual college classrooms and even less has focused on retention of mathematics knowledge. Consequently, educators who teach mathematics courses for engineering students might well wonder whether retrieval practice and spacing effects can be leveraged to increase long-term retention in their classes. To our knowledge, only two prior studies are germane. Hopkins and colleagues [16] showed, in a precalculus course for engineers, that spacing the administration of quiz questions across several weeks, versus massing them on a single quiz, led to increased retention of key course objectives (e.g., *Simplifying Exponential Expressions Involving Rational Exponents*) on the final exam in the course. Students who received spaced quiz questions versus massed ones also performed better on exams in a calculus course the following semester, suggesting that spacing increased retention, not only within a single semester, but across semesters. This study did not examine the effect of increasing the *amount* of retrieval practice, but the study we describe next did.

In a follow-up to Hopkins et al. [16], Bego and colleagues [17] again examined the effect of increasing the spacing of retrieval practice in a precalculus course for engineers while simultaneously examining the effect of increasing the amount of retrieval practice. They replicated the finding that spacing increased retention of key course objectives on the final exam in the course and found that increasing the amount of retrieval practice had a roughly comparable effect (although there was some evidence that the effect of increasing spacing was more robust than the effect of increase the amount of practice). Hence, both interventions were effective for increasing retention within a semester. However, the study did not examine the impact on retention in a subsequent calculus course.

While Hopkins et al. [16] provided evidence that spacing increases retention across semesters, it is unknown whether increasing the amount of retrieval practice does the same. Within a semester, increasing the amount of retrieval practice had an effect comparable to increasing the spacing, but it is an empirical question whether this equivalence is preserved as time passes. What is effective for enhancing memory in the short-term need not be effective for the long-term. The retention of information across semesters (the long-term in this context) is important to examine, given the cumulative nature of developing expertise in mathematics.

The study presented here is an extension of Bego et al. [17]. The same students who took the precalculus course were tracked if they continued into a calculus course the following semester. These students were tested on the knowledge of key course objectives from precalculus at the beginning of the calculus course. Both the amount and the spacing of retrieval practice had been manipulated in the precalculus course on an objective-by-objective basis, permitting a test of how these factors influenced across-semester retention.

Method

Design

This study had a 2 (amount of retrieval practice: three vs. six) X 2 (timing of retrieval practice: massed vs. spaced) within-subjects design.

Participants

Participants were students enrolled in *Engineering Analysis I* in Spring 2017 who had previously taken *Introductory Calculus for Engineers* in Fall 2016. To be included in the analyses reported here, students must have completed all quizzes and the final exam in *Introductory Calculus for Engineers* and also completed the diagnostic readiness test in *Engineering Analysis I*. A total of 51 students met inclusion criteria.

Course Format and Materials

Introductory Calculus for Engineers was based on *Precalculus: A Right Triangle Approach* by Kirk Trigsted. The course format included weekly class meetings, practice problems, individualized study plans, weekly quizzes, unit exams, and a final exam. Based on the NCAT emporium model (<http://www.thencat.org/R2R/AcadPrac/CM/MathEmpFAQ.htm>), weekly class meetings consisted of group activities led by instructors and teaching assistants. There were no traditional lectures. Practice problems, study plans, and quizzes were assigned using Pearson's MyMathLab® online software. At the end of each unit, students were given an in-class exam, and at the end of the semester, students were given a cumulative final exam.

Procedure

We manipulated the number and distribution of weekly quiz questions that covered specific course objectives. Four objectives (e.g., *Simplifying Exponential Expressions Involving Rational Exponents*) from each of the first eight weeks were selected from a larger pool of objectives. The selected objectives (32 in total) were designated *target objectives* for the purposes of this research. For each target objective, six questions were chosen from the MyMathLab® question bank. Questions were assigned to four different conditions: *less practice, massed* (three questions, all on a single quiz), *less practice, spaced* (three questions, across three quizzes), *more practice, massed* (six questions, all on a single quiz), and *more practice, spaced* (six questions, across three quizzes). In the spaced conditions, we used the same spacing schedule as in Hopkins et al. [16]: the question(s) covering a target objective appeared on i) the quiz in the week that the objective was taught, ii) the quiz two weeks after the objective was taught, and iii) the quiz four weeks after the objective was taught. For each student, one objective was quizzed in each of the four conditions. Table 1 depicts how objectives would be quizzed in each condition.

Table 1: Implementation details for the four treatment conditions.

Condition	Weekly Quiz			
	1	2	3	4
Less practice, massed	Questions 1-3	-	-	-
Less practice, spaced	Question 1	Question 2	-	Question 3
More practice, massed	Questions 1-6	-	-	-
More practice, spaced	Questions 1,2	Question 3,4	-	Question 5,6

Critically, assignment of objective to quizzing condition was counterbalanced. This means that each objective appeared in each condition for an equal number of students in *Introductory Calculus for Engineers*. Counterbalancing ensures that any differential retention of course objectives as a function of quizzing condition cannot be attributed to the difficulty of the objectives assigned to a particular condition (i.e., item effects). Rather, differential retention must be due to manipulations of amount and/or spacing of retrieval practice.

Students from *Introductory Calculus for Engineers* who continued into *Engineering Analysis I* the following semester were tracked for the purpose of assessing across-semester retention of target objectives from *Introductory Calculus for Engineers*. The measure of retention was described to students as a diagnostic readiness test for *Engineering Analysis I*. It was administered one month after the cumulative final exam in *Introductory Calculus for Engineers*. Students were informed that the test was required for the course, but that performance would not affect their grade. The diagnostic readiness test contained questions covering all the target objectives from *Introductory Calculus for Engineers* (one to three questions per objective). The order of questions was randomized for each student.

Results

To assess long-term retention of target objectives, we calculated proportion correct for questions covering each objective on the diagnostic readiness test in *Engineering Analysis I*. Proportion correct was submitted to a 2 (amount of retrieval practice: three vs. six) X 2 (timing of retrieval practice: massed or spaced) within-subjects ANOVA. Our initial analysis included data from all participants. Only the main effect of timing was significant, $F(1, 50) = 7.87, p = .007, \eta^2 = .136$. Students answered more questions correctly if corresponding quiz questions had been spaced in the previous semester ($M = .59$) than if they had been massed ($M = .54$). There was no

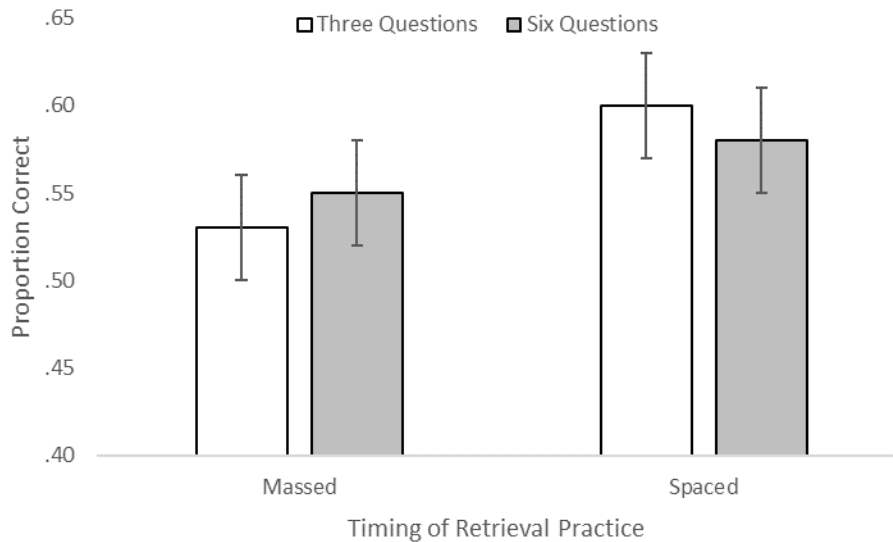


Figure 1: Proportion correct on the long-term retention test in the four conditions: less practice, massed; less practice, spaced; more practice, massed; and more practice, spaced.

suggestion of a main effect of amount of retrieval, $F < 1$, or of an interaction between amount and spacing of retrieval, $F < 1$. Figure 1 clearly depicts the main effect of timing, without any main or interactive effect of amount of practice.

It bears remembering that assignment of course objective to condition was counterbalanced in *Introductory Calculus for Engineers* such that every objective was quizzed in every condition for the same number of students. However, because we could not control which students met inclusion criteria for this study (i.e., completion of all critical measures in *Introductory Calculus for Engineers* and advancement to *Engineering Analysis I*), objectives were not perfectly counterbalanced in the analysis reported above. The numbers of students in the four counterbalancing cells, although equal at the start of *Introductory Calculus for Engineers*, were not equal in the analysis reported above. Specifically, the numbers in the four cells were 14, 13, 10, and 14. To ensure that lack of balance was not responsible for the results of our previous analysis (which included all students), we randomly selected 10 students in each of the counterbalancing cells that initially exceeded 10 and reran our analysis on a perfectly balanced subset of 40 students. We repeated this process 100 times. Despite the lack of power inherent in reducing our sample size by nearly 22%, a significant main effect of timing ($p < .05$) was obtained in 71% of cases. The median p value for the effect was .0355 for these 100 random subsets. This finding strongly suggests that the significant main effect of timing in our initial analysis was not due to item effects arising from unbalanced assignment of items to conditions. When we analyzed random subsets of perfectly counterbalanced data, we observed in a clear majority of cases that memory was significantly better for objectives whose quiz questions had been spaced in the preceding semester versus massed.

Discussion

Having previously found that within-semester retention of precalculus knowledge benefits equally from increasing the amount of retrieval practice and increasing the spacing of retrieval practice [17], the present study examined across-semester retention as a function of the same two interventions. We found that, one month after the completion of a precalculus course, memory was significantly better for precalculus objectives that had been the target of spaced quizzing versus massed quizzing. This finding conceptually replicates Hopkins et al. [16], wherein across-semester retention was measured in a less direct way than done here. In the present research, the average benefit associated with spacing was 5%, which is equivalent to half a letter grade.

In striking contrast to the benefit associated with spacing, we obtained no evidence that across-semester retention was bolstered by increasing the amount of retrieval practice in the precalculus course. In other words, it did not matter (for across-semester retention) whether students answered six quiz questions about a given course objective in precalculus or only three. This manipulation had no significant effect on students' ability to answer questions about those objectives one month later. We reiterate that a larger amount of retrieval practice *was* associated with increased retention within the precalculus course itself. It appears, however, that a mnemonic benefit associated with increased retrieval practice does not endure after one month. Future research will be necessary to explain why increasing the spacing of retrieval practice has a lasting effect on retention (at least out to one month) but increasing the amount of practice does not.

The present findings provide valuable information for instructors whose aim is to increase retention of foundational mathematics knowledge. Although cognitive psychologists have recommended increasing both the amount and the spacing of retrieval practice, it may be sufficient to do only the latter. This means that students need not endure longer homework assignments resulting from an increased number of retrieval attempts and instructors need not face the prospect of scoring those additional attempts. While spacing questions across quizzes requires foresight and strategic planning by teachers, it is nonetheless a fairly easy and no-cost way to increase retention.

By increasing retention of foundational mathematics knowledge, spacing retrieval practice could increase the number of students who complete engineering programs. Future research could explore whether spacing also has the potential to enhance long-lasting memory in other STEM fields wherein success depends on the cumulative acquisition of knowledge.

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