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Board 132: Notes on Design of Keyed Joints

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Notes on Design of Keyed Joints

Abstract
The purpose of this paper is to make it easier for beginners to design keyed joints, primarily in machine design courses. End-milled profile keyseats are apparently simple features. Even so, in teaching this topic, some points are challenging; these include how to assume a reasonable keyseat fillet radius, how to account for notch sensitivity, and whether keys fail due to shear or compression. Moreover, because keyed joints involve disparate topics—static failure, fatigue failure, stress concentration, engineering graphics, and manufacturing—beginners benefit from having these topics addressed together.

This paper offers the following main recommendations:

- Account for shaft curvature when dimensioning a keyseat.
- Specify fillet radii based on sizes of bull nose end mills. Chamfer keys.
- Neglect notch sensitivity in shaft fatigue calculations.
- Size keys based on compression failure formulas, despite keys typically failing in shear.

These recommendations may differ from guidance in textbooks and standards—but they are better grounded in the evidence and offer the clarity needed for design calculations.

In addition to literature review, this paper reports novel finite element analysis of key failure, showing that large fillet radii do not compromise key strength, which is complex and dominated by shear. Because tight fillet radii weaken the shaft, larger fillet radii are recommended. Those points would lead to shafts that are more economical to produce and less likely to undergo fatigue failure, without compromising key strength.

Introduction
This project arose due to difficulties with teaching and learning design of keyed joints in the context of a project-centered machine design course; that context is worth describing here. In this course, students do a project in which they find a need that a person has that could be solved with a machine; they then develop a paper design for this machine, culminating in a bill of materials and engineering drawings of stepped shafts. The major steps of the project are conceptual design, design of an "effector" (typically converting rotary to linear motion), selection of spur gears, design of keyed joints, selection of rolling-element bearings, design of shafts, and design documentation. The project is scaffolded in detail, with 33 discrete steps, several of which are assigned in each weekly homework assignment. The course content follows this sequence, as well. Because power transmissions are ubiquitous in machines, this course scope enables a

balance between open-ended design and the authentic technical guidance needed to iteratively develop a paper design.

Students had particular struggles with design of keyed joints. Although textbooks provide essential guidance on this topic, insufficient detail is offered to draw and tolerance keyseat. Moreover, some traditional guidance apparently includes some errors, arising from applying notch sensitivity data to geometries different from those tested, and from an oversimplified model of key failure. These errors are not conservative. Taken together, larger fillet radii are more beneficial to the shaft and less detrimental to the key than was previously recognized.

This paper focuses specifically on keyed joints; other types of joints have their benefits but the analysis presented here is not relevant to them. Also, this paper focuses on joints with profile keys and end-milled keyseats because they are better understood than other types of keyed joints. The purpose of this paper is to provide improved technical guidance on design of keyed joints, rather than to show that one instructional approach is superior to another; we assume that having clearer guidance would make the topic easier to learn but we did not measure this.

It is worth asking why this problem was not previously recognized. Two explanations are both likely. First, working engineers design shafts based more on prior experience than on mathematical analysis, with redesigns using larger diameters at sites of failure. Second, two mistakes may cancel each other out: although keyseats often are drawn as having internal square angles, machinists may intentionally add fillets to prevent shaft cracking in production, or wear on square end mills may result in acceptable fillet radii.

Thus, although shaft and key failure are not ubiquitous, these issues impair student learning because their resolutions depend on tacit knowledge and realistic factors in part fabrication, while students require explicit and cohesive guidance to develop paper designs. Thus, the primary purpose of this paper is to ease student learning of design of keyed joints by providing clear guidance. Additionally, the recommendation to use larger fillet radii is beneficial for practicing machine designers.

The following sections describe key and keyseat dimensions, keyseat failure, and key failure.

Dimensions

Keyseat Dimensions

Fig. 1 shows a keyseat for a shaft diameter D, key width W, key height H, and fillet radius r. For a square key, $H = W$. Due to the curvature of the shaft, the depth of cut is greater than $H/2$, by a chordal height $Y = \frac{1}{2}(D - \sqrt{D^2 - W^2})$. Chordal height increases the depth of cut by approximately 15%. The distance from the bottom of the keyseat to the bottom of the shaft is thus $S = D - H/2 - Y$. Because S can be measured with calipers, a keyseat drawing should show D, r, W, and S. These dimensions follow from the depth control formulas in ANSI ASME B17.1-1967 [1].

Typical Key Sizes

Typical key and keyseat width and depth are given by ANSI ASME B17.1-1967, Table 1 [1]. Key sizes should be based on what is commercially available; three suppliers are considered here. McMaster-Carr keystock [2], Huyett [3], and Daemar [4] all offer keystock that correspond to

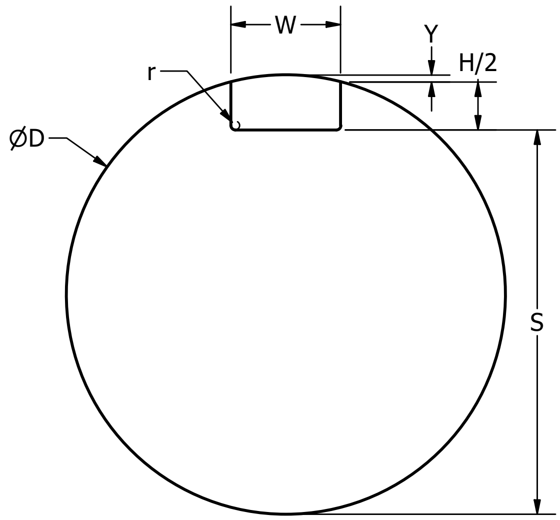

Figure 1: Schematic of keyseat dimensions, redrawn from part of [1].

Table 1: Common bull-nose end mill radii

Mill diameter, in	Corner Cut Radius, in
1/8	0.015, 0.02
3/16	0.015, 0.02, 0.03
1/4	0.015, 0.02, 0.03, 0.045
5/16	0.015, 0.02, 0.03, 0.045
3/8	0.015, 0.02, 0.03, 0.045
1/2	0.015, 0.03, 0.045, 0.06

B17.1 Table 1; McMaster-Carr's selection is more limited and on the small end. Other suppliers offer selection that goes beyond the standard, as well. Huyett's catalog is especially helpful in showing various types of stock.

Keyseat Fillets and Key Chamfers

Keyseats are not typically filleted to a controlled radius, and keys are not typically chamfered [1]. A fillet radius must be assumed to calculate stress concentration; if this dimension is too small, the stress will be higher than designed for. Fillet radius can be controlled with cutting tools such as bull nose end mills, which are also known as rounded edge or corner radius end mills. Table 1 shows corner radii for bull nose end mills available from McMaster-Carr [5]. Other suppliers, such as Carbide End Mill Store [6] and MSC Direct [7], have greater selection. The corner radii for these tools are reasonable to specify as fillet radii.

It seems that keyseats are often cut with nominally square end mills, which is problematic for a few reasons. A perfectly square keyseat would have infinite stress concentration. Cutting a square keyseat renders the shaft prone to cracking during production. In practice, the corners of square end mills are prone to wear. Thus, keyseats cut with square end mills probably have fillets with a small and uncontrolled radius. Because the fillet radius determines the stress concentration, this dimension should be controlled.

Keystock should be chamfered to accommodate filleted keyseats. The chamfer should be a little larger than the fillet radius; a chamfer of 5/4 the fillet radius is roughly consistent with Ref. [1].

ASME B17.1 Table 7 suggests fillet radii and key chamfer dimensions [1]. Peterson [8] refers to typical fillet radii of $r/d = 1/48$ in, where d is the shaft diameter. In either source, these dimensions do not correspond to common radii on bull nose end mills.

Shaft Failure at Keyseat

Keyseats are a common point of failure for shafts because they act as stress raisers. In power transmission, a shaft essentially carries an alternating stress due to bending and a mean stress due to torsion.

Notch Sensitivity

The effective stress in a part is typically lower than what one would predict from theoretical stress concentration. The effective fatigue stress concentration is given by,

$$K_f = 1 + q(K_t - 1) \tag{1}$$

where q is the notch sensitivity and K_t is the theoretical stress concentration [8]. The notch sensitivity ranges between zero and unity, depending on material properties and the geometry of the stress raiser. In contrast to common practice, a notch sensitivity of $q = 1$ for keyseats should always be used for the following reasons.

- Notch sensitivity data [9] is derived from "tests on specimens with fillets, semicircular notches, V-notches, and transverse holes;" most of these geometries are axisymmetric and none resemble keyseats. Some variation in notch behavior was seen between these different geometries.
- Peterson [8] states that their reported notch sensitivity values were "not verified for notches having a depth greater than four times the notch radius because data are not available," which would be the case for keyseats.
- The data for notch sensitivity has large uncertainty so conservative calculations are merited.

A designer should realize qualitatively that the increased strength due to heat treatment would not be fully felt due to embrittlement making stress concentrations more significant. With that said, the lack of data on notch sensitivity for keyseats prevents quantitative use in design.

Stress Concentration

Peterson [8] gives stress concentration factors for bending and torsion for profile keyseats. Those factors are based on photoelasticity measurements for bending [10] and an electroplating method for torsion [11]. Those and more recent studies show that Peterson's values are worth continued use. With that said, these studies are 50 years old, they use different methods, and the keyseat proportions do not match the US standards; therefore, new measurements are called for.

Peterson offers the following formulas for stress concentration in bending K_t and torsion K_{ts} in a keyseat,

$$K_t = 1.426 + 0.1643\frac{0.1}{r/d} - 0.0019\left(\frac{0.1}{r/d}\right)^2, \tag{2}$$

$$K_{ts} = 1.953 + 0.1434 \frac{0.1}{r/d} - 0.0021 \left(\frac{0.1}{r/d} \right)^2, \tag{3}$$

where r is the fillet radius and d is the shaft diameter.

Stress concentrations in keyseats have been investigated more recently using finite element analysis [12, 13]. Pedersen [12] modeled keyseats according to the German standard DIN 6885-1; such keyseats have different proportions than those given by ANSI-ASME B17.1-1967; given that difference, Pedersen reports results that correspond well to those given by Peterson. Le and Le [13] studied keyseats that comply with ANSI-ASME B17.1-1967; they have good agreement with Peterson for bending but lower values for torsion so using their results is not conservative. These reports are worth reading for those interested in design of keyseats. Le and Le report stress concentration values for sled runner keyseats [13]; those values are not shown in most other sources on stress concentrations. Pedersen proposes cutting keyseats using elliptical rather than circular cutting tools and shows that doing so can reduce stress up to 50% [12].

Key Failure

Overview

Keys have been assumed to fail in compression prior to shear failure because the mean stress in compression along a bearing surface is higher than the mean shear stress. However, model results reported here show that the distribution of stress within a key is non-uniform and complex. Where the shaft and hub push into the key, the sharp corner of the material causes a singular contact stress, similar to what would be seen if a cube were pushed onto a flat surface.

This section shows finite element results for a key failure, reviews textbook formulas on key failure, accounts for discrepancies in these models, and recommends use of larger fillet radii.

Model Results for Stress Distribution in Keys

To study stress distribution in keys, a finite element model was developed in Ansys. To focus on the keyed joint itself, parallel plates joined by a key were modeled; the top plate was fixed and the bottom plate could slide under it. The geometry was 2D. The key was 0.25 in square, with 0.0125 in chamfers, and the keyway and keyseat had 0.01 in fillets.

All three parts were modeled as 1020 cold-rolled steel, with a yield strength of 57 ksi and ultimate tensile strength of 68 ksi. However, the material was modeled as undergoing multilinear isotropic hardening with plastic stress rising linearly from the yield point to failure at 15% elongation. Without accounting for plastic response, singular stresses arose that prevented the model from converging. A load of 3206 lbf was applied, which would cause an average 28.5 ksi on bearing surfaces and 22.2 ksi in shear (equivalent von Mises stress); that is, the key would have a factor of safety of 2 in compression against yielding.

Contacts were treated as frictionless. Large deflection was modeled. The mesh was automatically generated and refined using convergence on total deformation; convergence to within 1% was obtained.

The maximum stress is limited by the plastic response of the material, and much higher than one would predict if stress were uniform. This stress is reached at the edges of the lateral contacts

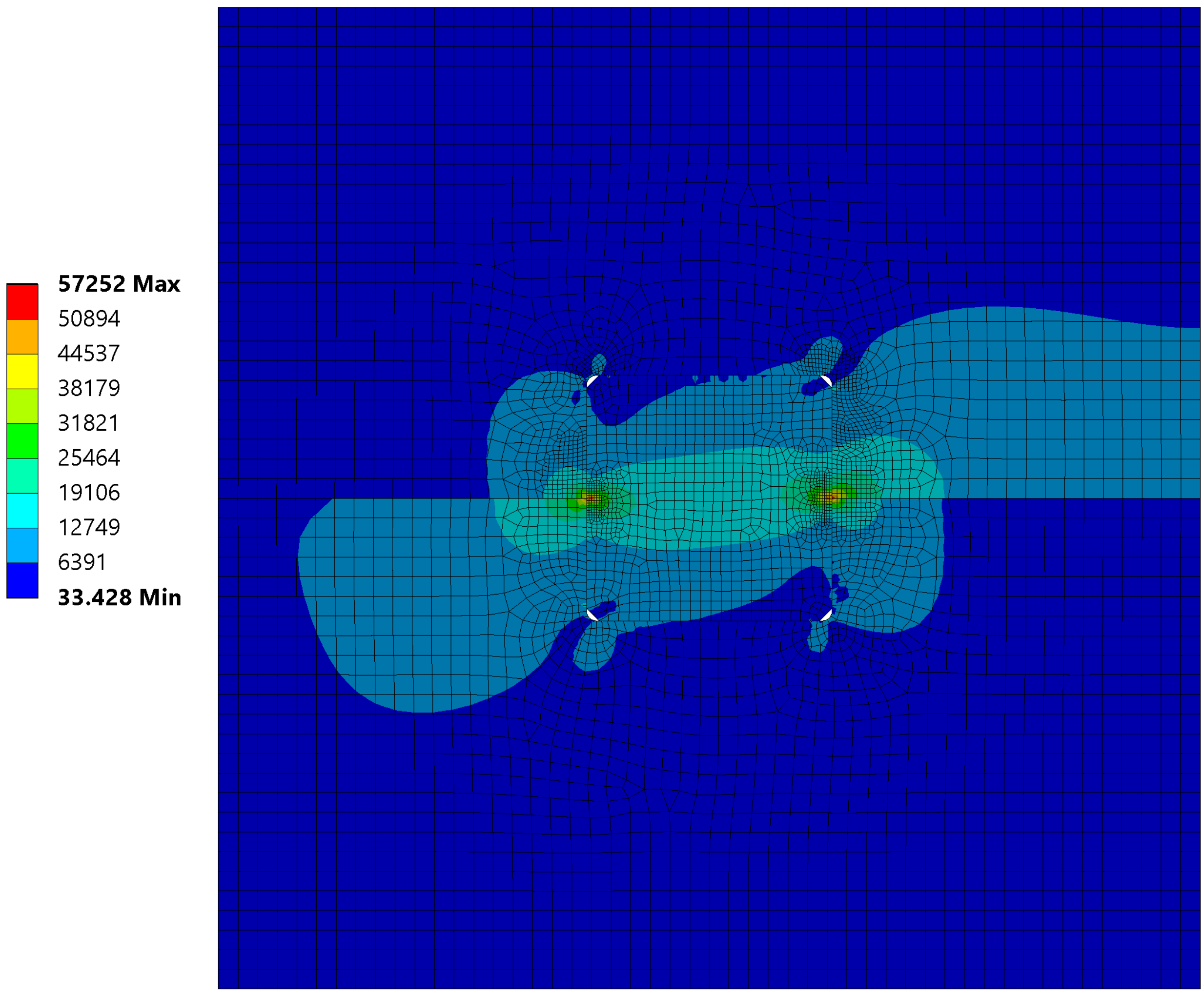

Figure 2: Von Mises stress on the full geometry modeled.

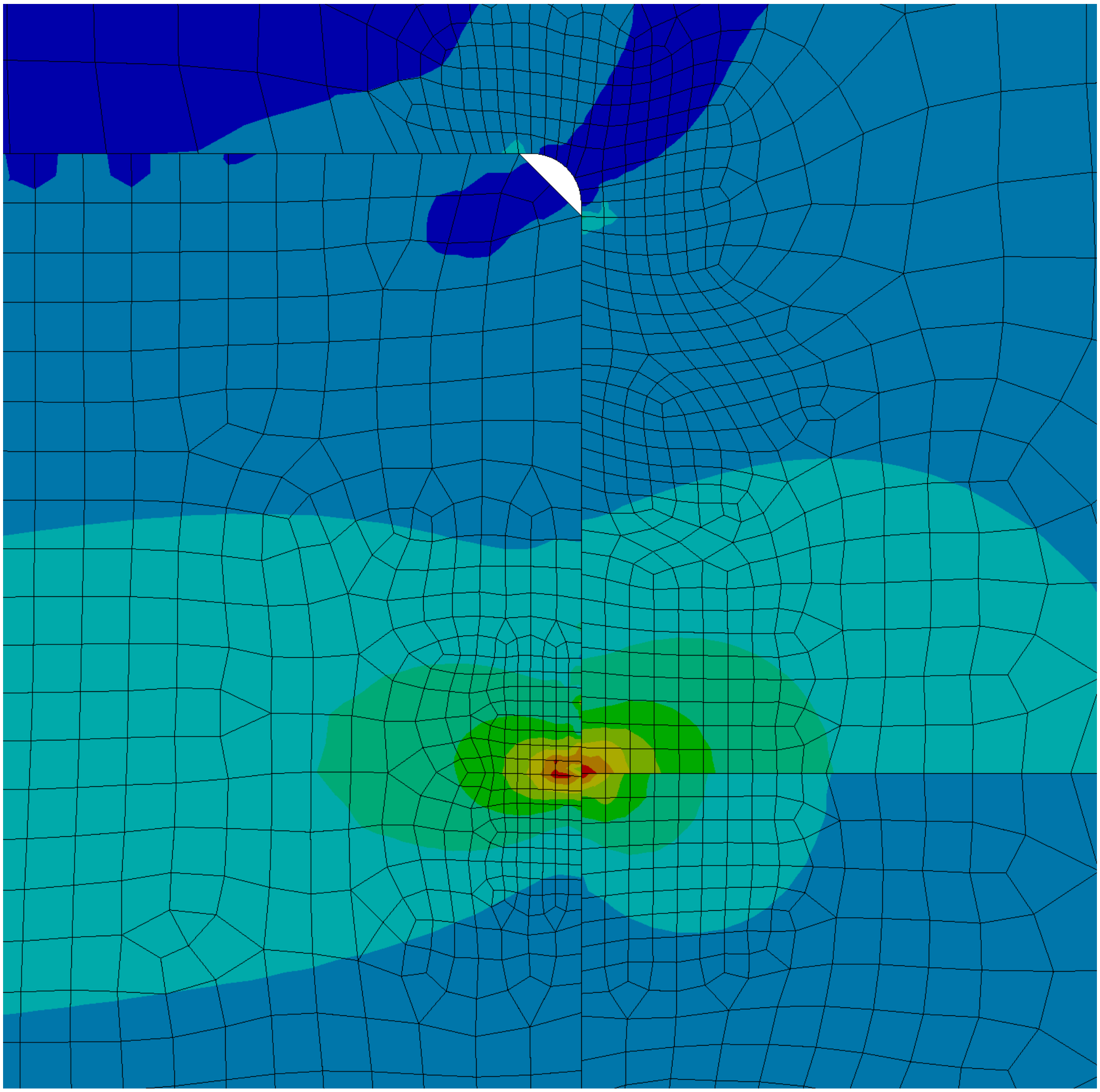

Figure 3: von Mises stress shown for a portion of the model, focusing on the contact between the key, keyseat, and keyway. The stress has peaks near the intersection between each part, and where the key corners dig into the surroundings. The colormap is the same as for Fig. 2.

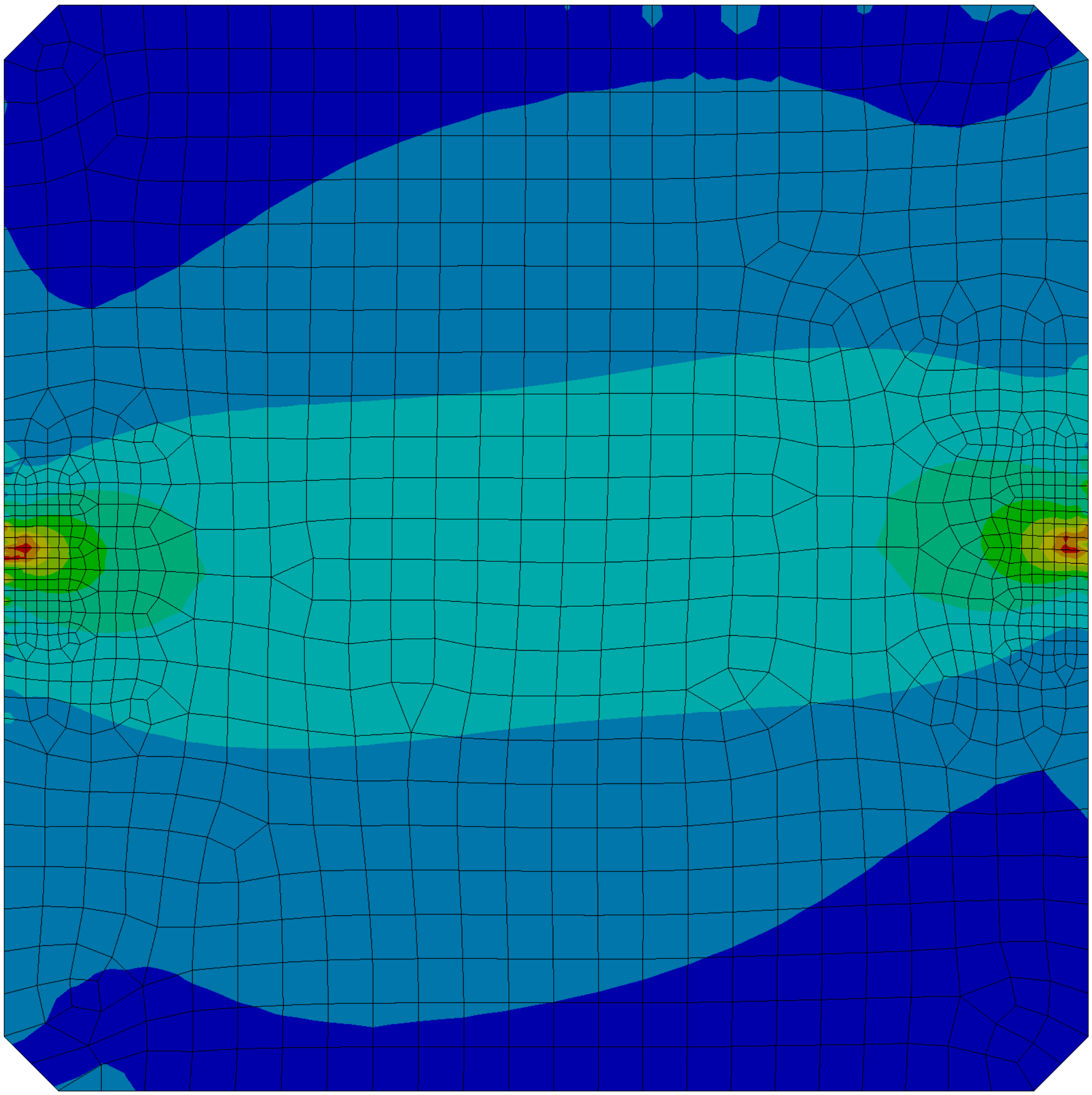

Figure 4: von Mises stress within the key. Again, stress peaks are evident and exceed textbook formula predictions. The colormap is the same as for Fig. 2.

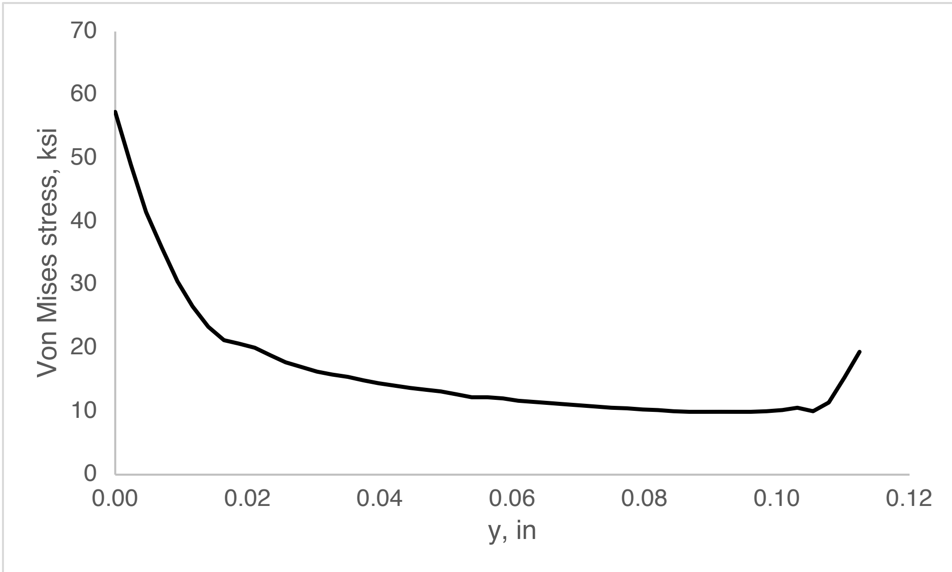

Figure 5: von Mises stress along the right bearing face of the key, going from $y = 0$ at the intersection of the keyseat and keyway to $y = 0.1125$ in at the chamfer on the key. Stress is non-uniform and highest near the shear plane.

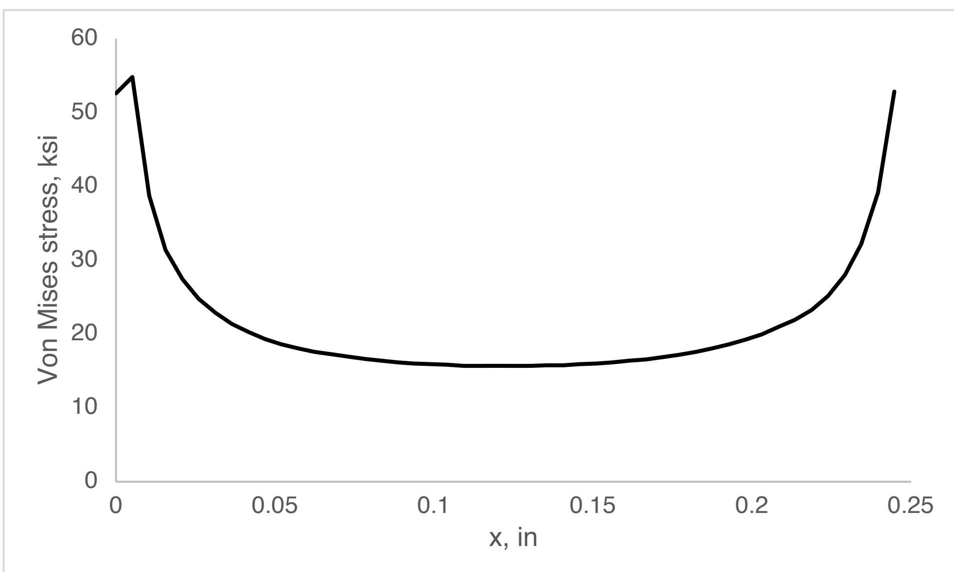

Figure 6: von Mises stress along the shear plane. Stress is non-uniform, with peaks near the contact points between the key and the surrounding objects.

with the key, as well as in two corners of the key. Stresses at these points would be singular in the absence of plastic response, due to the sharp corners.

These results are the beginning of a more detailed study of stress distribution in keys. Even so, it is clear that stress is nonuniform across both the bearing surface and the shear plane. Strain hardening may limit that variation. The textbook formulas assume uniform stress distributions, which is clearly not correct under the elastic limit. However, as a material yields, stresses throughout a yield surface may enter the plastic region, which would then make the stress more uniform and thus existing formulas may indeed give reasonable estimates of stress. Although existing formulas may be acceptable, clearly, more investigation is called for.

Shear
Assuming that shear stress is uniformly distributed across the key,

$$\tau = \frac{F}{\ell W}, \tag{4}$$

where ℓ is the length of the rectangular face of the key. With a factor of safety N, yield strength S_y, and using the von Mises failure theory,

$$\ell = \frac{\sqrt{3}NF}{WS_y}. \tag{5}$$

Compression
Compression failures of keys are typically predicted by dividing the force on the key by the bearing area and applying this stress to the key material. This approach is used by Mott [14], Norton [15], and Shigley [16].

Assuming that compressive stress is uniformly distributed across the bearing surface on the key,

$$\sigma = \frac{2F}{\ell H}, \tag{6}$$

where ℓ is the length of the rectangular face of the key. With a factor of safety N and yield strength S_y,

$$\ell = \frac{2NF}{HS_y}. \tag{7}$$

Summary and Recommendations
These textbook shear and compression formulas for key failure all predict compressive failure because the bearing surface for a square key is taken to be half of the shear area. However, it seems likely that keys typically fail in shear, in practice.

There are two likely explanations for this discrepancy. First, the stress across the bearing surface is typically taken to be uniform while the results reported here show that to be incorrect. If no yielding has occurred, compressive stresses would not be uniformly distributed along the face of the key; thus, keys in service probably undergo some yielding.

Second, the key is not just subject to transverse loads from the shaft and hub; the moment generated by those loads is counteracted by vertical loads from the top and bottom of the keyseat and keyway. The result is a stress state with compression along multiple axes; under the von Mises failure criterion, the radial compressive stresses render the transverse ones less significant.

Third, the practical consequences of compressive yielding should be re-thought. Typically, compressive yielding of the key is calculated. However, it's unclear why that would be problematic. The key is confined in a keyseat and keyway and would thus not undergo the continuous deformation observed in, say, cylindrical compression test samples. In some cases, compressive failure of the keyseat or keyway would increase backlash to an unacceptable degree. A shear failure, though, would correspond to the key snapping entirely, which is the more relevant limiting factor for most applications.

One should note that key design often focuses on finding key length for a desired factor of safety. However, it is often desirable for the key to have the same length as the hub to balance the machine; sizing on that basis often leads to a very high factor of safety. This may account for why ubiquitous failure of keys is not observed.

In summary, this finite element study shows that key failure is not due to compressive failure in the way previously thought, so larger fillet radii are allowable. Clearly, this problem needs further study, which may lead to key sizing guidelines that reflect this non-uniform stress distribution. Until then, the existing formulas for sizing keys to prevent shear or compressive failure are probably adequate and the compressive formula, being more conservative, should be used.

Conclusion

This paper collects guidance for design of keyed joints, including dimensioning, shaft failure in fatigue, and failure of keys. This guidance is generally consistent with common practice and textbook coverage. By collecting this information and clarifying some points—namely suggesting fillet radii that can be realized with common keyseat cutters—this paper is of use to instructors of this subject.

Moreover, this paper makes a novel technical claim, namely that keyseat fillet radii should be more generous than is common. Keyseat fillet radius should manage a trade-off, with a large radius being bad for the key and a small radius weakening the shaft. Stress concentration is highly sensitive to fillet radius, with a stress concentration of 4 being likely for very tight fillets; a more generous fillet radius could easily drop that factor to 3 or even 2. The main reason to have a tight fillet is to maximize the bearing area for the key, which would prevent compressive failure. The analysis presented here shows that that concern is probably misplaced. As shown in the section on stress distribution in keys, failure occurs at the shear plane so compressive failure is probably not dominant. Practically, if one had to choose between having a weak key and a weak shaft, a weak key is probably preferable, because keys do not cost much and are easy to replace, whereas stepped shafts are often custom parts and difficult to replace. With keys often being sized to fit the entire bore of a rotating part, their factor of safety may well be much higher than needed; in contrast, keyseats are among the likeliest points of failure for shafts. Finally, cutting tight interal square angles is difficult, so using larger fillet radii would not only strengthen shafts but make them easier to manufacture.

More experiments are needed. Notch sensitivity in keyseats should be measured. Modern measurements of stress concentrations in keyseats should also be taken. Key failure should be measured directly.

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