2006-497: BOTTLE ROCKETS AND PARAMETRIC DESIGN IN A CONVERGING-DIVERGING DESIGN STRATEGY

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Abstract

The Sophomore Engineering Clinic covers two semesters in an eight-semester design sequence. The course integrates engineering with writing and public speaking. In the past the course has used two semester-long design projects to teach design through a series of problems of increasing complexity. Though the course has been effective at teaching students to prototype devices, it has been less effective at teaching design as evidenced by written project documentation and observation of students' decision making processes. In the fall of 2005 the course was revised to incorporate a convergent-divergent framework to teach design. In addition, the semester-long project in the fall was replaced in favor of one four-week project followed by a ten-week project.

The initial four-week project was structured to formalize an approach to making choices using parametric studies in a diverging-converging design process. The design and construction of water-propelled bottle rockets from 2 L soda bottles was chosen as the initial four-week project. Students built and tested rockets which were limited in materials and construction. The limitations allowed the rockets to be characterized by three parameters: the mass of water used, the aspect ratio of fins used, and the mass of Playdoh used in a nose cone. Students generated parametric testing schedules and took data on the performance of their designs with respect to the variables in order to inform design choices. During the course, these steps towards a final design were linked to a diverging-converging framework for design thinking. At the end of the course all students had performed parametric studies of their rockets and rocket performance was improved both over the four weeks and as compared to previous years. In addition, the following ten-week project showed improvements in quantitative metrics of performance over the previous years.

Introduction

All students in the engineering curriculum at our university are required to take an eight-semester design sequence called the *Engineering Clinics*. The sophomore year of the Engineering Clinic series is devoted to Design and Communication. The course is team taught by faculty from multiple departments within the College of Engineering and the College of Communication. The students spend 160 minutes in an engineering lab period and 150 minutes in a communication class period per week. Two sections of the lab are run with about 60 students in each and six sections of the communication class with about 20 students in each. Assignments and grading are integrated through both communication- and engineering-specific sections, a trend that is gaining national acceptance[1,2,3]. In previous years the Sophomore Clinic has tasked students with various semester-long projects including the design and construction of residential bridges, music effects pedals, golf-ball launchers, motorized cranes, and load bearing truss systems. While these projects were successful at following the national trend of integrating design into the curriculum at this early stage [4,5,6,7,8] they were not as successful in *teaching* students to be good designers. In other words, students could competently apply the design skills learned in connection with the projects, but they were not thinking like designers.

Design Thinking

Dym et al. link this problem to the difference between the "engineering science" model of engineering education, which views acquisition of analytical competency as foundational, and the "project based" model, which views active participation in learning experiences as foundational. Most students are extremely practiced in the modes of learning associated with the engineering science model: they can readily perform mathematical and scientific analysis. Simply placing them into a project-based setting such as the engineering clinic sequence does not alter their thinking. Students continued to approach their work as though they were doing assigned homework problems. In fact, engineering design calls for a radically different way of utilizing mathematical and scientific analysis:

> Engineering design is a systematic, intelligent process in which designers generate, evaluate, and specify concepts for devices, systems, or processes whose form and function achieve clients' objectives or users' needs while satisfying a specified set of constraints.[9]

When mathematical and scientific analysis are practiced to achieve competency, the emphasis is on finding the right answers. When they are applied to engineering design, the emphasis is on the many higher order skills embodied in the above definition: *generating, evaluating, and specifying* ideas that meet human needs within various constraints. To define what kinds of thinking are required to engage in engineering design and to shed light on how it might more effectively be taught, Dym et al. propose a framework they call Divergent-Convergent Thinking.

In brief, the diverging-converging framework breaks the design process into two interrelated phases. Convergent thinking uses the analytical skills learned in physics and mathematics courses—for example, experimental methodologies and observations, and other quantitative and qualitative methods—to assess various design solutions. The results of these analyses and observations are then used to enhance subsequent design iterations. Divergent thinking is used to generate initial design concepts and to widen the range of thinking when a particular design strategy has reached a road block.

This paper details the use of parametric design in the context of a diverging-converging design process as related to the design of water-propelled bottle rockets. The diverging-converging framework is laid out after which parametric design is discussed as a tool within that structure. Methods of classroom implementation are discussed and comparisons are made to the abbreviated version of the bottle rocket project from previous years. The paper concludes with qualitative and quantitative indicators of success. The new structure has just completed its first semester in fall 2005 and its effects on the following spring's design course are currently being reviewed.

Incorporating Design Thinking into Project-Based Learning

Making the leap from the more concrete thinking skills of analysis to the more abstract thinking skills of design would be a challenge for students, and it was decided that it would be too difficult to incorporate explicit instruction in divergent-convergent thinking into the existing semester-long project. Instead, the strategy was to use a simpler project to introduce design thinking. Then, it was hoped, when the students turned to the longer, more complex project, they would be more likely to engage in design thinking.

In the fall 2005 offering of Sophomore Clinic, the course was revised to introduce a more structured approach to teaching design. In previous years, a single, semester long project was used in both the fall and the spring. The new approach utilizes two projects in the fall, to be followed by a single project in the spring semester. In the new structure, students are given a well defined, well constrained problem during the first four weeks of the fall course, followed by a more open-ended ten-week design problem with fewer constraints. For the following spring semester, students work on a very open-ended project. The three projects of increasing complexity are used to introduce a diverging-converging framework for design[9]. It is intended that this progression will enable the students to be more conscious, and therefore more capable, of engaging in the cycles of diverging-converging thought that effective design teams undergo.

Overview of the Project

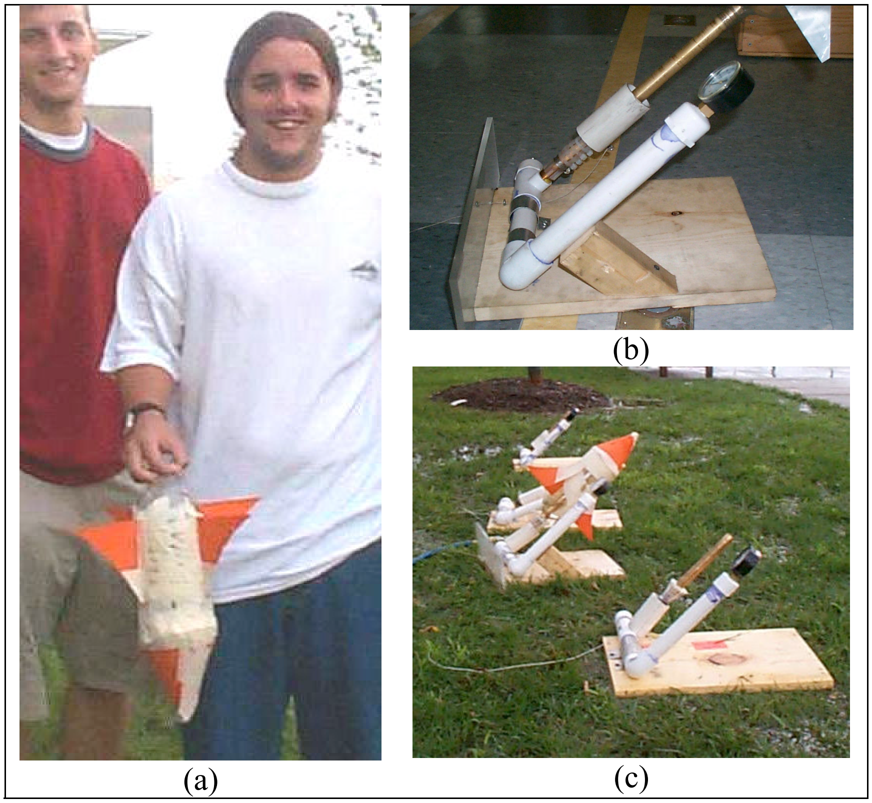

Figure 1: The bottle rocket project. (a) students with a water bottle rocket, (b) the launcher, (c) three launchers, one with a rocket and two without.

The design of water-propelled bottle rockets was chosen as the initial project to introduce parametric design. This project has been implemented by other college-level programs to teach core engineering concepts[10]. Briefly, students are given foam board, duct tape, a 2L Coca-Cola bottle, and a can of Playdoh along with the instructions:

> *For this project your goal is to design a bottle rocket which travels the farthest distance of any rocket in the class. Your rocket must be fabricated from only the set of materials we have given you.*

The rockets are launched from a nozzle oriented at 45 degrees that imparts an internal pressure of 60 psi to the water. Figure 1 shows a finished rocket and two views of the launch apparatus.

In previous years, this project had been assigned as an introductory ice-breaker type activity. Students were given sixty minutes to design, test, and optimize their designs. In this offering of the course, the same one-hour challenge was issued on the first day to kick off the course, but the instructors returned to the project for an additional four weeks to lay out the design framework. In the first lab period, teams of students designed, built and launched rockets that were propelled by pressurized air and water made of the materials listed above. They were told to build a rocket that could fly as far as possible, but were given little advice on how to go about design, and no formal instruction in the mechanics of rocket propulsion involved. Students were asked to keep a log of distance traveled versus amount of water used, however, to determine the optimal amount of water without fins. After the first day, all the teams had some measure of success in designing a rocket that could fly and developed some intuition for the rockets. Rocket flight distances ranged from 50 to 200 feet, which is comparable to previous year's rockets whose best performers flew 100 to 300 feet but with 90 psi initial pressure, 30% more than used in this semester.

During the next class period the formal design project was introduced. The formal, four-week project was more restricted than the one-hour version, as follows:

- Students could still choose how much water to use in the rocket.
- At most, one can of Playdoh could be used to help balance the rocket, but could be placed only on the nose of the rocket.
- Wings were to be placed at 120 degree intervals around the circumference (if wings were used at all) and the wing designs had to be parameterized by their aspect ratio, length/height.

The rocket design was then described by three parameters, which are illustrated in Figure 2: wing aspect ratio, weight in the nose, and amount of water at launch. At this point, the students had two additional lab periods to optimize their three-parameter design through repeated testing and a final class for the final launch. The role of parametric design in the diverging-converging approach to teaching design will be considered below.

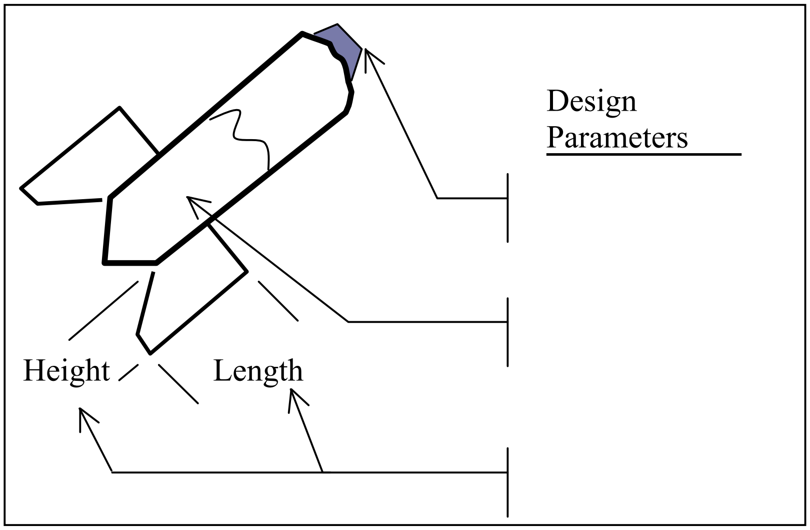

Figure 2: Bottle rocket design parameters.

The instructors also returned to a previous project, The Hoistinator,[11] for the more open-ended design project for the remaining ten weeks of the course. For the Hoistinator, students were tasked with developing truss systems attached to a pre-existing base and winch arrangement that could support a minimum of 420 lbs. In this project students were scored in proportion to the weight of material they could lift divided by the weight of material used in their trusses. Whereas the project had been a semester-long endeavor in previous years, the pairing with the initial bottle rockets project left only ten weeks for completion. The final project in the sequence, spring 2006, will be the design of an electromechanical device. Students will design motorized vehicles that must traverse a prefabricated rail system while operating a winch to raise and lower objects with an electromagnet.

Implementing the New Design Philosophy

To guide students in thinking about the bottle rockets project within Dym et al.'s framework, two pairs of mental arenas were defined: Design vs. Analysis and Problem Solving, and Convergent vs. Divergent thinking. Within these arenas, Convergent thinking was discussed as applied to Analysis/Problem Solving and to Design, separately, and Divergent thinking as applied to Design. These three concepts are discussed below.

In the Divergent Design phase ideas are generated and recorded with almost no restrictions; ideas must be at least theoretically plausible given current technology. In course assignments related to the rocket's project, students were given this explanation:

> *Divergent thinking is contrary to convergent thinking in that the ideas/choices do not have to lead directly to the best solution and they do not have to necessarily fall within the constraints. It helps, though, if the ideas are technically feasible.*

To illustrate divergent thinking, students were asked to brainstorm modes of propulsion / transportation of the rocket which were completely different from water propulsion. In this exercise, blanket "teleportation" was not acceptable whereas using electron tunneling had some merit; the combustion of liquid hydrogen, though arguably an unsafe approach, was still an acceptable proposed mode of propulsion and therefore a valid divergent design thought. The divergent design phase is most associated with brainstorming. Its importance lies in its ability to overcome design hurdles by, to use the cliché, thinking outside the box (this important concept has similarities to powerful nonlinear numerical solution techniques which will be discussed more later).

The ideas generated within the Divergent Design phase, though not necessarily bound by constraints, must still be analyzed to determine their efficacy. The convergent thinking aspect provides both a means of assessment of the choices made in the divergent phase and a rationale for making additional choices in order to find the optimal solution, a concept introduced as convergent design. In this framework, convergent thinking was described for students as follows:

> *In brief, convergent thinking may be thought to include (a) the generation of constructive design ideas that work within the constraints and (b) analysis and problem solving that assesses a particular design's efficacy.*

Convergent thinking involved analysis and problem solving, which were separated along a thin line. Convergent Problem Solving was discussed as the textbook-type homework problems with which engineering students were most familiar. The problems are extremely well defined and constrained to the point that they offer, usually, a single correct answer. While this is not necessarily in line with a design problem, such textbook-type problems highlight the link between engineering and design – engineers use the physical laws, chemistry, physics, etc. to solve problems in an effort to learn something about their designs. Convergent Analysis was discussed as the assessment of a particular design with respect to the given set of goals and criteria. Convergent analysis, then is slightly more open-ended in that one must formulate the problem as well as apply the physical laws. With these definitions, however, there will invariably be some amount of overlap in the process.

The lynchpin in this approach was formalizing the link between Convergent Analysis/Problem Solving and Convergent Design as a means to refine design decisions and choices based on analytical work, experimentation and observation. The need for emphasis on this aspect of the process was clear. Previous design projects within Sophomore Clinic showed that while students were comfortable and performed well in the problem-solving and to a lesser degree the analysis phases of the project, they rarely linked results of this convergent analysis/problem solving with decisions/choices which would form the basis for convergent design. They remained "unaffected by their education" [12]. For example, a student developing a truss system for the Hoistinator project commented to the lead author that his calculations showed his truss could hold 6000lbs (over four times the maximum weight tested!) but still asked if he should add more strengthening supports. In relation to the bottle rockets project, previous years' students would choose wing shapes based solely on aesthetics, not performance.

Parametric Design and the Bottle Rockets Project

In the first one-hour-to-build meeting, students were introduced to the notion of parametric design by collecting data on the rocket's performance versus the amount of water used with no fins or stabilizers. The data collected had the general trends shown in Figure 3. The data based on results from the entire class showed a clear trend toward an intermediate optimal amount of water which many teams recognized.

In the following class periods, after the three-parameter design space was defined, multi-variable parametric design was discussed as a means of performing convergent analysis. Students were tasked with developing an experimentation schedule that listed a matrix of design parameter combinations they would test *and* how the results of those tests would inform future design choices. The latter portion of the assignment closed the loop on the design process.

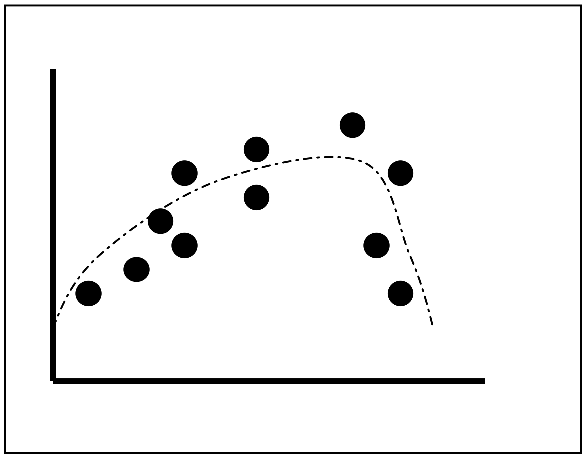

Figure 3: Representative plot of volume of water versus distance with no fins.

Initially students brainstormed the rocket fin shapes, amounts of water to use, and amounts of Playdoh to use in their parametric tests. Choosing parameter values to test encompassed the initial divergent design phase since the students had relatively little information with which to make decisions. Next, parametric testing over several variables to determine the rocket's performance with respect to each parameter constituted a convergent analysis approach where the efficacy of each design choice was assessed through direct measurement of performance. Finally, searching for trends in the results of the convergent analysis phase to inform the next set of parameter values for the next prototype rocket, convergent design, closed the parametric design loop.

The Limitations of Parametric Design Methods

The utility of parametric design as a teaching tool is that it builds on students' pre-existing strengths—problem solving and analysis—as a means to formalize decision-making in the design process. In class, the limitations as well as the strengths of parametric design were discussed. Students were given the following example: if we consider the parameter space as defined in only one dimension (for example, with only the amount of water) then the resulting performance could be described by the curve OD given in Figure 4. However, if the performance was actually governed by two (or more) variables, then a true(er) objective function might be given by the set of curves, OA, OB, OC, and OD (see Figure 4). The design could be optimized along OD but without including a larger parameter space, including fin aspect ratio as shown, the design could not reach the optimal point along OA. In a class-wide discussion after the final launch students were led to the understanding that some rocket designs, chosen on the first day of class without much thought or analysis, were bound by a parameter space that included a locally optimal configuration but not the globally optimal set of parameters. For example, one group realized that though they had optimized their particular rocket's design, the best performing rockets were fundamentally different. They would require a round of divergent thinking to generate new ideas, a new/wider parameter space.

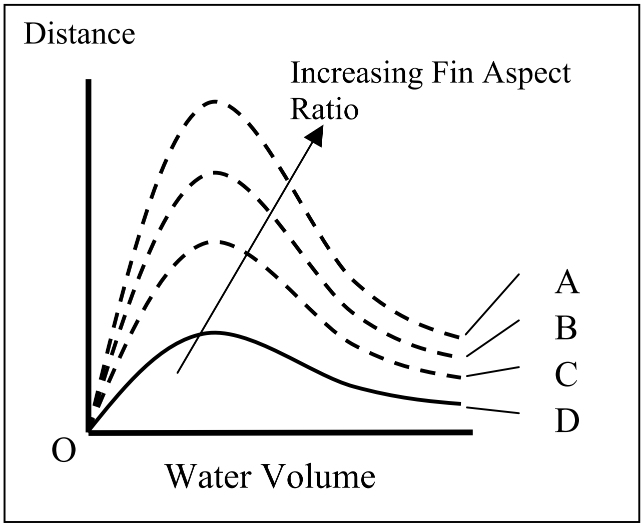

Figure 4: Parametric design space with multiple dimensions.

It is interesting to note that in a broader sense, the diverging-converging framework, including parametric design, is akin to well known nonlinear numerical solution techniques such as the Metropolis algorithm[13]. In the Metropolis algorithm, a multivariate objective function is maximized or minimized by the successive iteration of dependent variables. At each iteration the objective function is calculated and the new configuration is either accepted or rejected. The algorithm is unique in that while the method will always accept a new configuration when the objective function is improved, there is a probability, proportional to the error, that it will accept configurations that worsen the objective function. The Metropolis algorithm has been used to solve what have been previously considered intractable numerical problems such as the traveling salesmen problem and gives support to methodologies that allow thinking outside the box, or, in the design terms, divergent design choices given some amount of convergent analysis for assessment.

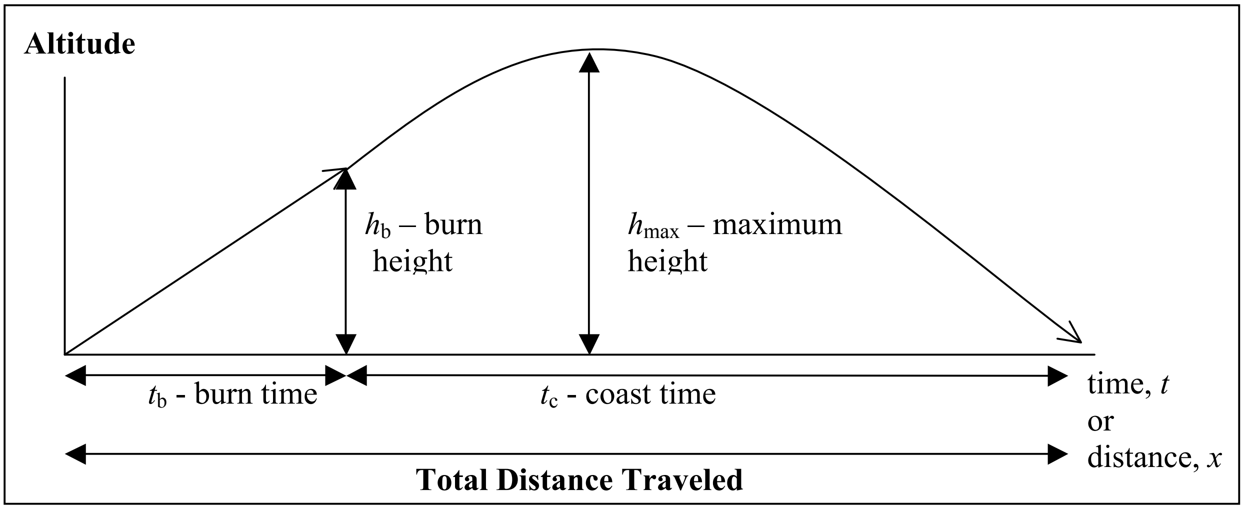

Figure 5: Phases of rocket flight.

Convergent Problem Solving

In addition to parametric experimentation as a form of convergent analysis, lectures on the physics of ballistic projectiles and the so-called rocket equation governing propulsion were presented as well. These lectures and associated assignments required the students to use the problem solving aspect of convergent thinking to generate predictive equations governing the rocket's behavior. Students were then tasked with using information gathered from their analytical predictions to inform design choices, another form of convergent design.

During its flight, the rocket experiences two phases: the burn phase where it is expelling water and a coasting phase after all the fuel (water) is spent. The two phases are shown in Figure 5.

The following assumptions were made to facilitate derivation of an analytical solution to the problem:

- Air acts as an ideal gas.
- There is no air drag.
- The air supply tube does not interfere with ejection of water.
- The air inside the rocket remains at a constant pressure during the burn phase.
- The burn time is much smaller than the coast time.

With these assumptions, during the burn phase the velocity of the rocket along its axis is governed by

$$(1) \qquad v_f = -g\cos\theta t_b + v_e \ln\frac{M_0}{M_f}$$

which is known as the Rocket Equation. In this equation g is gravity, θ is the angle with respect to the ground, t_b is the burn time, v_e is the exit velocity of the water , M_o is the initial mass of the rocket (water plus bottle with fins plus Playdoh) and M_f is the final mass of the rocket (bottle, fins, and Playdoh *sans* water). In this equation everything except t_b and v_e were known, Assuming that $PV = nRT$ holds for the air inside the rocket and that no energy is lost as the water is expelled, then it can be shown that:

$$(2) \qquad v_e = \left[\frac{2\left(P_{bottle} - P_{atmosphere}\right)}{\rho\left(1 - \frac{d_{nozzle}}{d_{bottle}}\right)^4} \right]^{1/2}$$

where pressures are measured in absolute terms inside the bottle and in atmosphere, the *d's* are diameters of a cross-section of the bottle (assumed to be 5") and the nozzle (assumed to be 0.85"), and ρ is the density of water. In the bottle rockets project for this class v_e was fixed[14].

With a known value for the velocity of the exiting water, the burn time could be found from

$$(3) \qquad t_b = \frac{M_{H20}}{\rho A_{nozzle} v_e}.$$

where M_{H20} is the initial mass of water.

Given equations (2) and (3), students could determine the initial velocity of the rocket from (1). In reality, the rocket reached this velocity rather quickly (about 0.2 seconds), so it was also assumed that the rocket had the velocity, v_f, immediately after it left the launcher.

With a known initial horizontal velocity, the horizontal distance traveled by the rocket could be found from the physics of ballistic trajectories. Students were assigned the derivation of the total distance traveled as a homework assignment. They then used the results of

$$(4) \qquad d = v_f \cos(\theta) t$$

where

$$(5) \qquad t = v_f \sin(\theta) \sqrt{\frac{2}{g}}$$

to chart the relationships between mass of Playdoh and mass of water to the distance traveled by the rocket. This information was intended to inform the choices of those parameters in a convergent problem solving sense.

Results and Observations

The diverging-converging framework incorporated into the four-week bottle rocket project was intended to give the students exposure to a formalized approach to decision-making in engineering design. There are several qualitative factors that suggest success in this area:

- On the first day, half the student teams felt they had found an optimum value for the amount of water. Many used this same or a similar amount throughout the four weeks.
- All groups engaged in a directed course of parametric evaluation of their bottle rockets, testing over each parameter individually while keeping the others constant.
- Many groups charted their data, looking for trends in flight distance with respect to a given rocket parameter.
- Several groups completed successive optimizations over each parameter then retested the initial parameter to assess interdependency.
- The best performing rockets flew 30% farther with 25% less pressure than rockets designed by clinic students in previous years.
- The bottom performing designs after day 1 were much improved after 4 weeks. While the shortest distance traveled the first day was less than 50 feet, the shortest distance on the final launch was 150 feet.
- The ten-week design project following the bottle rocket's introduction to design showed quantitative improvements over last year in terms of the ratio of the weight-lifted to the weight of materials used even with the reduced time for design and construction. This supports the claim that students' designs were improved over last year with respect to the scoring criteria.

In addition to these positive factors, however, troubling behaviors were observed. Some groups changed their designs (the amount of water was the most common choice) on the day of the final launch with no substantive rationale. One group remarked, "We found that 750 ml was the optimal amount of water, but we think it will work better with a little less." In addition, though written reports were assigned and students were given repeated written and verbal instructions to include data to support all final design choices, many student groups included data in their reports but did not actually link the data to design choices, making it hard to determine how certain parameters were chosen. The overall enhancement of rocket performance, though, suggests that students may have put parametric design into practice more so than into their documentation.

Summary

The four-week bottle rocket project will likely be repeated in the fall 2006 offering of Sophomore Clinic and the instructors will incorporate feedback from this teaching of the course and from the follow-on project to improve students' application and written communication of parametric design techniques.

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