

## **AC 2010-214: BRIDGING MATHEMATICS CONCEPTS TO ENGINEERING CONTEXTS: JUST-IN-TIME REVIEW MODULES**

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## **Bridging mathematics concepts to engineering contexts: Just-in-time review modules**

### **Abstract**

The basic problem addressed in this paper is that many engineering students do not remember the essential mathematics skills and concepts they learned in earlier math courses when they enter higher level engineering courses, and yet they need to apply these earlier math abilities to advanced engineering coursework. The challenge facing the instructor is how to bring all students up to mathematical mastery level as quickly as possible at appropriate points during the semester when core mathematical skills are needed, including during the first week of class, so that they can be successful in achieving the required learning outcomes in engineering courses. It is a challenge because the typical response by the instructors to poor pre-test scores is to lose valuable class time doing review of the pre-requisite material.

In a junior level biomedical engineering course, students were given a diagnostic pre-test covering essential math skills and concepts on day one of the course. This focused on functional substitutions, graphing equations, complex algebra, and evaluating improper integrals. It was found that very few students achieved a passing grade on this test as they entered the engineering course. After the pre-test, students were directed to a set of review materials for them to study to refresh their knowledge and skills. They then undertook a second in-class test a week later. The overall results, detailed results by test question and student responses to a set of survey questions will be reported in this paper. We will also compare the performance of this group of students on a selected final exam question with the performance of students from previous years, on a similar question.

These experimental data show that providing students with review material at the beginning of a semester helps them to achieve mastery in essential mathematical knowledge and skills. Using this study as the prototype, we started to develop a series of just-in-time mathematics modules which will provide a bridge between targeted mathematics concepts and applied engineering problems. Students will complete these modules prior to encountering these topics in engineering courses. The starting point would be to assess their entry level of mathematical competence on specific topics and then, as appropriate to their level of competence to provide individualized mathematical review material (quizzes, video, review notes, animations practice problems, etc) in an interactive online system. So, we are looking at designing a process of test, individualized review, and re-test. Students would be expected to demonstrate mathematical competence after reviewing the material by taking an in-class test. The elements and functionality of such a technological tool will be described in the paper.

**Key Words:** Math for engineering, Calculus, Math review, student performance, pre-requisite knowledge and skills.

## Introduction

The mathematics knowledge and skills gap encountered by undergraduate engineering students when they enter engineering courses requiring the use of mathematics abilities which were taught in the three semester calculus sequence has been well documented <sup>1,2,3</sup>. However, there is 'widespread agreement among academics and practicing engineers that a good grounding in mathematics is essential for engineers' <sup>4</sup>. The challenge facing the engineering instructor is how to bring all students up to mathematical mastery level as quickly as possible at appropriate points during the semester when core mathematical skills are needed, including during the first week of class, so that they can be successful in achieving the required learning outcomes in engineering courses.

The Australia Mathematical Sciences Institute developed 10 recommendations to improve mathematics education for engineering students including the development of on-line formative assessment, with an item bank of formative test questions, automatic grading and feedback, so that compulsory online quizzes can be conducted during the semester for large engineering mathematics classes <sup>4</sup>. Similarly, **Helping Engineers Learn Mathematics (HELM)**, a curriculum development project undertaken by a consortium of five English universities uses computer aided assessments (CAA) to encourage formative self-assessment by students, so they can verify that the appropriate skills have been learned in their math classes <sup>5</sup>. Students can take a formative test multiple times to review material prior to completing a summative test, the latter score counting as coursework. In both these projects, the focus is on using computer aided processes to support student learning of mathematics in mathematics classes, rather than in engineering courses.

Such online computer aided assessment and learning packages have been shown to be an effective tool for increasing engineering student's knowledge of experimental design <sup>6,7</sup> with students with access to interactive online materials scoring significantly higher on tests than students in control groups without access to such materials <sup>7</sup>. A similar effect has been seen in mathematic classes, where students who completed web-based homework assignments performed significantly higher on final exams than did control groups <sup>8</sup>. Clearly, online materials have the potential to increase student performance.

No literature was found on the impact of online modules in enhancing student's knowledge of mathematics in the context of engineering courses. As seen in the literature reviewed, many engineering faculty at our university have noted that students entering engineering courses are not able to use or recall essential pre-requisite mathematics knowledge and skills. Our hypothesis is that online assessment, review of mathematics material, and the practice of essential mathematical skills in the context of engineering courses will assist students in reaching the mathematical mastery needed to be successful in those engineering courses. This is where our study differs from previous ones because we are examining engineering student's mathematical abilities in the context of engineering courses, and providing just in time modules for review of pre-requisite material needed to be successful in achieving the specific engineering learning outcomes. The study is based on a strongly built partnership between the Department of Mathematics and the College of Engineering at this university, combining expertise to work on the enduring problem of preparing engineering students to be mathematically competent. It

builds community among mathematicians and engineering educators who can, together, create learning materials that are both mathematically rigorous and relevant for the education of future engineers.

## **Methodology**

Linear Systems for Biomedical Engineers (BME 311) is a junior level required course for Biomedical Engineering students. This course also prepares students in the bioinstrumentation emphasis area for their required course Introduction to Digital Signal Processing. By the end of the course, students should be able to classify biomedical systems, find the outputs of linear, time invariant biomedical systems for arbitrary input signals and represent these signals and systems in the frequency domain using various transforms.

BME 311 has the prerequisites of Differential Equations from the Department of Mathematics and Electric Circuits from the Department of Electrical and Computer Engineering. While it is not unusual for Biomedical Engineering students to take several out of department courses, these two courses from two different departments treat similar mathematical concepts with different notations and contexts, which also contribute to the problem of lack of ability to integrate similar concepts treated from different perspectives in different contexts. In addition to basic differential and integral calculus, students are expected to know how to solve first and second order differential equations, how to manipulate and graph functions, and how to work with complex numbers.

On the first day of class, students were given a diagnostic pre-test covering the essential math skills and concepts needed for the course. The test focused on piecewise functions, graphing sinusoidal signals, complex algebra, and evaluating improper integrals.

After students had completed the test, they were directed to a set of materials posted on the course website and were asked to go through all the documents posted to review the essential concepts and skills necessary to successfully complete the test, and the course. They were told that a week later they would have a second test, but not the same test, covering the same material. The better of the two scores were to be used as a homework grade.

The review materials (pdf files) that were made available to the students on the course webpage were prepared based on the needs for this particular course. These documents contained a review of the theory, various examples emphasizing key points and subtleties of each topic, and a few problems for practice. It also included examples with complete solutions, pointing out stages in a problem where they may encounter difficulties. For instance, when converting a complex number from Cartesian form to polar form, it is vital to identify the quadrant where the complex number lies so the angle is correctly identified.

The following four topics were tested, reviewed, and re-tested:

- Sinusoidal functions – standard algebraic form; elements of a sinusoid: period, amplitude, vertical and horizontal shift; graph of a sinusoid (8 pages);
- Unit step function used to express piecewise defined functions that appear in applied fields to represent quantities that change abruptly at specific values of time; graphing piecewise defined functions (3 pages);
- Complex algebra - complex numbers in Cartesian and polar forms, operations, graphical representation, notation specific to engineering fields, such as electronics (3 pages);
- Improper integrals of the type (1 page).

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The mean student score and standard deviation on each question in the pre- and post-test were calculated, as well as the overall score for the tests. Then, the pre- and post-test scores were compared for statistical differences using a matched sample t-test to determine if the two scores were significantly different, for each question, and for the overall score on the test.

After the post-test, student also completed a short 5 point likert scale survey about their experiences of the pre- and post-test and in reviewing the materials. The survey also included two open ended questions about the most and least useful elements of the review material. The comments were content analyzed for common themes.

## Results

Ninety eight percent of students said that they had reviewed the material provided, spending an average of 1 hour 20 minutes reviewing material.

Table 1 below shows the mean and standard deviation of the pre- and post-test scores. The maximum score for each question was 5, with a total of 20 for the test.

Table 1: Average pre- and post-test scores.

Test Questions	Mean Pre-test	S.D.	Mean Post-test	S.D.	T-test <i>p</i> value	Significance * = yes
Question 1	2.22	1.62	4.37	1.23	0.000	*
Question 2	1.88	1.72	2.84	1.58	0.002	*
Question 3	1.63	1.85	4.33	1.03	0.000	*
Question 4	1.76	2.05	2.33	1.71	0.077	n.s.
Total	7.49	4.54	13.96	3.65	0.000	*

In rating the level of difficulty of each question, the instructor rated them as follows: Q1 = easy, Q2 = reasonable, Q3 = easy, Q4 = difficult. The results show that there was a significant

improvement in student performance for questions 1, 2 and 3, as well as for the overall test, suggesting that the review material helped improve student performance on the post-test.

While there was a slight improvement in student scores on question 4, this was not significant. We think that this is due to asking them to evaluate an integral of a composite function with a unit step function in it, something they had not seen in the review materials. They were required to integrate information from topic 2 (piecewise defined function) and from topic 4 (improper integrals), so by embedding topic two into question 4, this created a brand new problem, which most of them did not know how to tackle. This suggests that they needed to be given an example of how to creatively integrate these two topics in the review materials.

The biggest gains were seen on questions 1 and 3. In these problems, they were asked to do the same procedure they had seen in the review materials. In question 1, they were asked to graph a sinusoidal signal with different amplitude and frequency. Question 3 was similar to the pre-test one, where they were asked to do the basic operations (addition, multiplication, division) with different complex numbers. In question 3 the students encountered unfamiliar notation that is customarily used in the engineering fields, that is, the polar form of a complex number that the students were used to manipulate in their math classes is changed into a different notation in their engineering courses. This caused some confusion in the pre-test, but the review material introduced them to the new notation, enabling them to better solve the problem in the post-test.

Question 2 was not procedural and the examples presented in the review materials gave them some practice but not a method that they can apply to the new problem. The practice helped them to do better in the post test but did not produce such excellent results as in questions 1 and 3. In short, procedural problems were mastered very easily, while critical thinking topics require more in-depth review.

Table 2 shows the student ratings on a 5 point likert scale for 6 questions about the review process.

Table 2: Student ratings about the review and test process

<b>Rate yourself on each of the following statements. Circle one from the options. 1=Strongly Disagree, 2=Disagree, 3=Neutral, 4=Agree, 5= Strongly Agree</b>	Mean	S.D.
1. I experienced difficulty with the notation while taking the first test.	1.24	0.87
2. The score on the first test is not a reflection of my true ability.	3.55	0.99
3. After reviewing the materials, I remembered that many of the topics were covered in previous classes.	4.14	0.7
4. It is a good idea for faculty to review essential math concepts at the start of the semester.	4.56	0.5
5. The review material will enhance my learning and success in this course.	4.14	0.67
6. I will perform better if I take the same test again.	4.06	0.74

Survey questions 3, 4 and 6 show that students found that the review process helped them to recall important math concepts and that they felt it would help their performance in the course.

The themes that emerged from student responses to the two open ended questions, 'What was most useful about the review material', and 'What was least useful about the review material' are contained in Figures 1 and 2 below.

Students valued the examples provided, specific areas that needed attention, the general organization of the material and also that the material was directly related to the quiz. The least useful features were that they would like to have seen some more complex examples worked out, needed more specific instruction on certain topics (like integrals) and in some cases that the instructions were not clear.

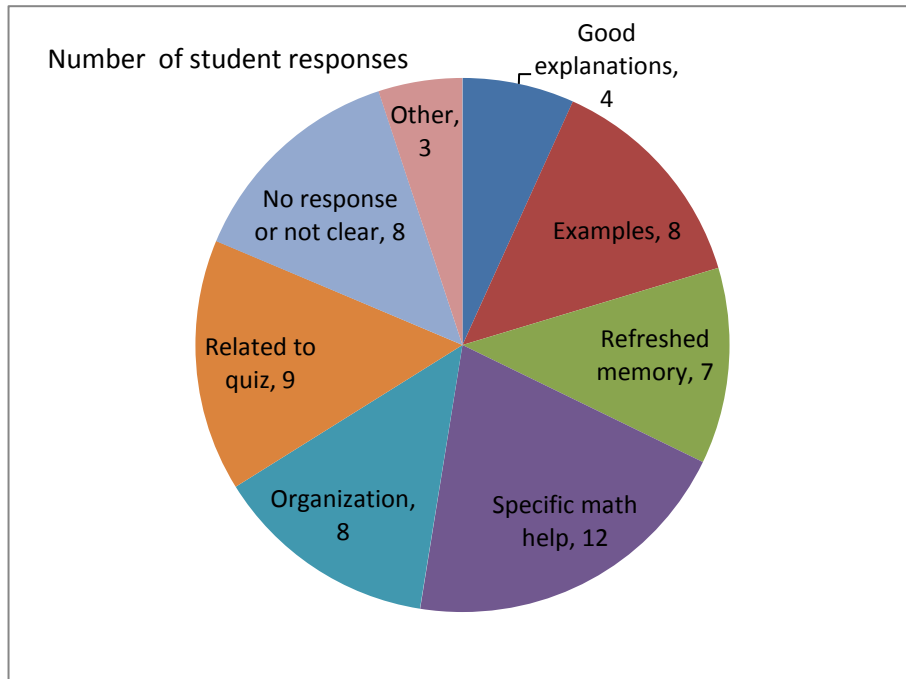


Figure 1: Student perceptions of what was most usefulness about the review process

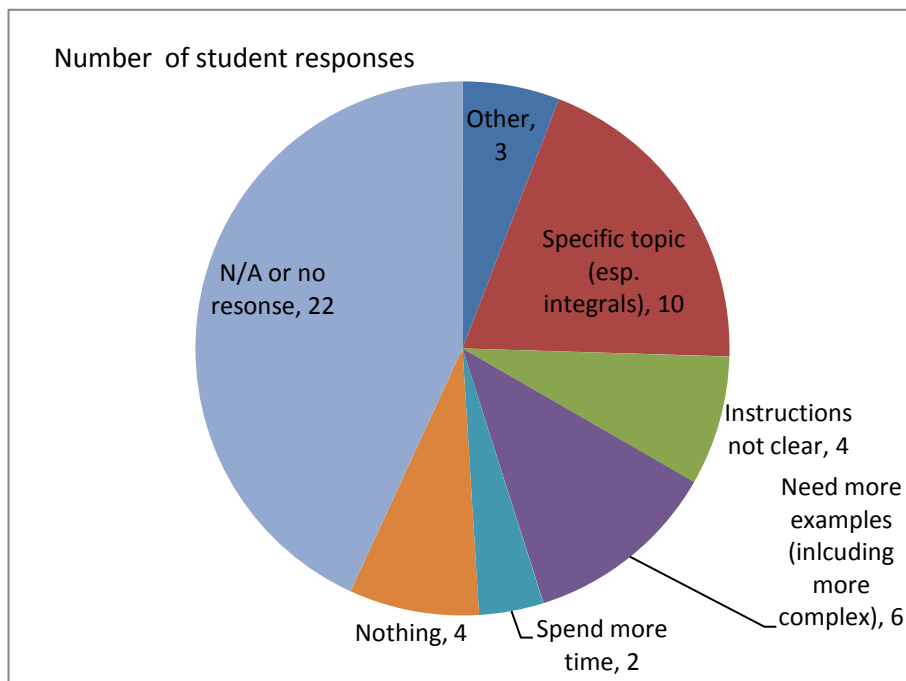


Figure 2: Student perceptions of what was least usefulness about the review process

Further evidence that this approach has improved student's mathematical ability comes from the comparison of student performance on a common exam question contained on the 2008 and 2009 final exam geared to assessing the course objective, 'students will be able to find the output of a linear time invariant system for a given input signal and frequency response'. The 2008 group



did not have any review material and 75% of them scored 70/100 or higher on the same test question, in comparison to 93% of the 2009 students scoring 70/100 or higher.

## **Discussion**

Our results show that having students review relevant mathematical concepts and procedures helped them to improve their mathematical competence and confidence, thus increasing their chances for success in the course, as indicated by the improved score on a common final exam question.

While the materials were made available online as pdf files, they were not interactive in nature, unless students chose to do the suggested practice items using pencil and paper. These were not submitted for grading and feedback. We are currently exploring ways to develop a web based system that is interactive, and that provides an element of artificial intelligence by evaluating student's level of competence on particular concepts or procedures, then providing appropriate review materials and practice items that build on the student's existing individual ability and competence level.

To this end, we are currently evaluating different software programs and developing a rubric of essential elements needed for an artificially intelligent system that can determine student's existing capabilities on a pre-test, provide review material, give appropriate practice items and feedback on responses. We are evaluating existing software programs including Maple TA, WeBWork with a possible integration into Moodle, WebAssign, ALEKS and Wolfram Mathematica for the Classroom. This aspect will be elaborated more in the presentation and elements of the rubric for evaluating different software programs will be made available.

An example of our design ideas for this individualized test and review online system is given in figure 3 below, showing a possible structure of how students would navigate through the online testing and review system. The system will also have value in gathering assessment data for use in course and program improvement, as well as for ABET accreditation purposes to demonstrate student competencies relating to ABET outcome A, 'An ability to apply knowledge of mathematics, science and engineering'.

The development of an online system has the potential to impact over 5000 undergraduate students on our campus alone, and if successful, could be expanded to other schools in the country. In addition to helping to improve the understanding of mathematical concepts and increase students' abilities, engineering faculty have also expressed interest in using the process to improve other areas in the engineering curriculum, such as statistical knowledge and skills.

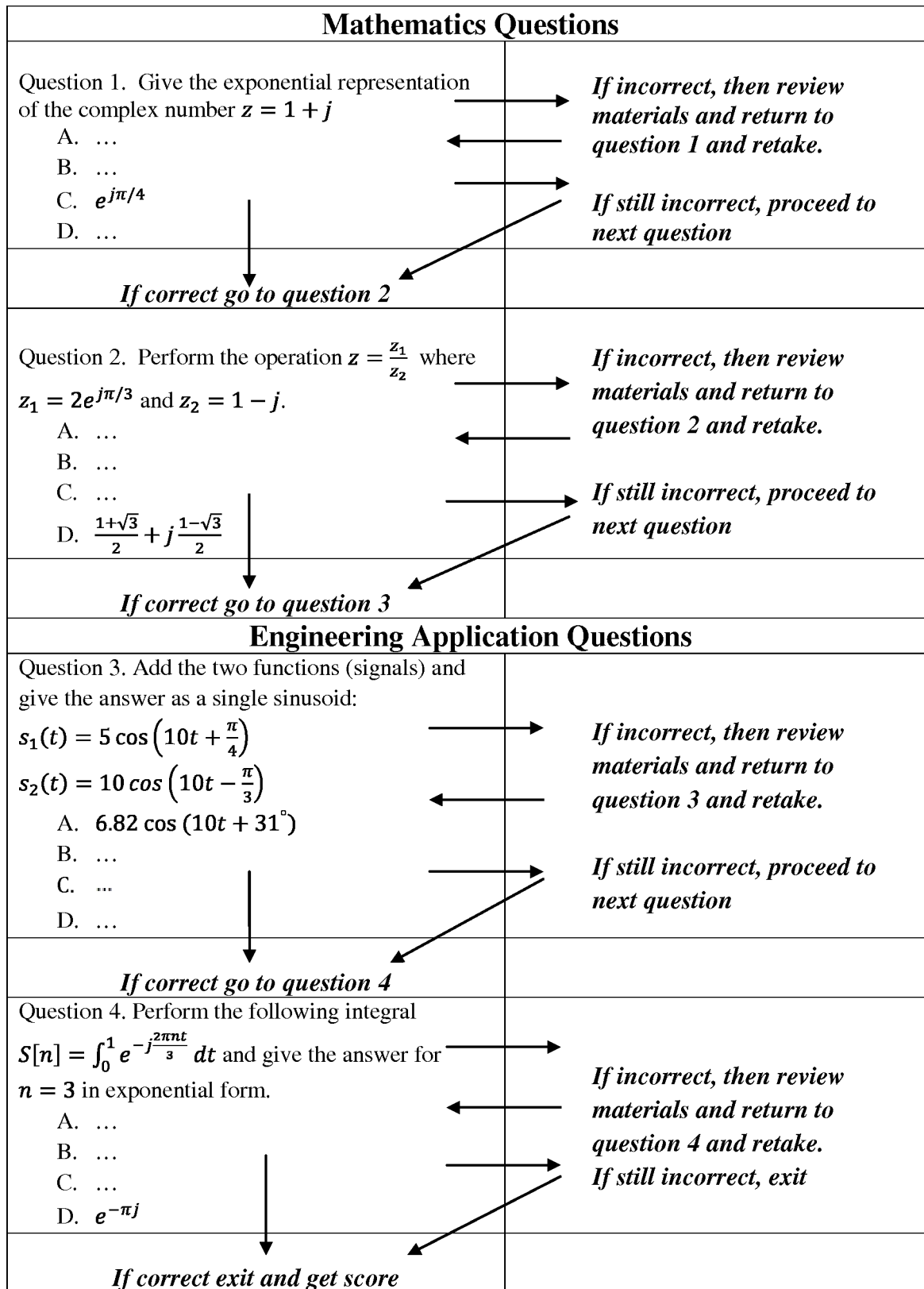


Figure 3: Flow diagram showing how students will navigate the system.

## Bibliography

1. James, G. (1995). *Mathematics matters in engineering*. Southend-on-Sea, United Kingdom: The Institute of Mathematics and its Applications.
2. Lopez, A. (2007). *Mathematics education for 21<sup>st</sup> century engineering students: Literature review*. Melbourne, Australia: Australian Mathematical Sciences Institute.
3. Ozturk, H. & Spurlin, J. (2006). Assessing the connectivity of a program's curriculum. Paper presented at the ASEE Conference, Chicago, IL, June 18-21, 2006.
4. Boardbridge, P. & Henderson, S. (2008). *Mathematics education for 21<sup>st</sup> century engineering students: Final report*. Melbourne, Australia: Australian Mathematical Sciences Institute.
5. Green, R., Harrison, A. S., Podcock, D. & Ward, J.P. (2004) The role of CAA in helping engineering undergraduates learn mathematics. *Maths CAA Series: Nov 2004*. Downloaded from <http://ltsn.mathstore.ac.uk/articles/maths-caa-series/nov2004/index.shtml#abstract>
6. Cetty, M. (2000). A scheme for online Web-based assessment. *Engineering science and education journal*, 9(1), 27-32.
7. Nahi, H.B., Charturvedi, S., Akan, A.O. & Pickering, J.W. (2007). Engineering education: Web-based interactive learning resources. *The Technology Teacher*, 67(3), 9-14.
8. Hirsch, L. & Weibel, C. (2003). Statistical evidence the web-based homework helps. *Focus: Newsletter of the Mathematical Association of America*, 23(2), 14.