Bringing Graphs Alive in Structural Dynamics

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Jim's experience and expertise is in structural engineering, and primarily focused on buildings. He has worked as a structural engineer in Baltimore and Pittsburgh, and worked on projects ranging from residential construction to Westinghouse's AP1000 nuclear power plant.

Jim spent four years in the Navy's Civil Engineer Corps after graduating with a Bachelor's in Mechanical Engineering from Villanova University. While in the Navy, he spent 15 months on Adak in the Aleutian Islands as a Public Works Officer, and two years in Jacksonville, Florida as the Officer in Charge of Construction Battalion Unit 420.

Jim earned a Master's at Johns Hopkins and a Ph.D. at Lehigh University. Jim taught at Ohio University before landing at Carnegie Mellon University.

When he is not doing engineering, Thompson sings in a barbershop chorus and quartet.

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Abstract

Textbooks on Structural Dynamics do an excellent job of presenting graphs that show how a dynamic system responds. However, the graphs themselves, being printed in a text, are static. Bringing the graphs alive in the classroom with Mathcad allows the instructor to vary the parameters and explore directly the effect of each parameter on the solution. This provides the opportunity for greater insight on the solution.

Introduction

Bringing Graphs Alive in Structural Dynamics is the vehicle for a much broader assertion, that since we can easily use computers in the classroom, and have software that can actively perform math, we can use the ability to do live math in the classroom to better illustrate what we are trying to show our students, and to lead them to insights on their own.

This paper presents examples of how the author uses Mathcad in a Structural Dynamics class to illustrate points in presenting the material.

Using Live Graphs in the Classroom

Example 1: The Relationship Between the Period of Vibration and the Natural Circular Frequency.

The texts on Vibrations, Dynamics, or Structural Dynamics with which the author is familiar typically present the relationship between the period of vibration, $\boldsymbol{T}$, and the natural circular frequency, ω_n with a graph like Figure 1. Vertical markers are typically used in the text to show the distance between two successive peaks, which represents 1 complete cycle of harmonic motion, and is called the period, $\boldsymbol{T}$. The text will then typically state the relationship between $\boldsymbol{T}$ and ω_n with an equation like Equation (1).

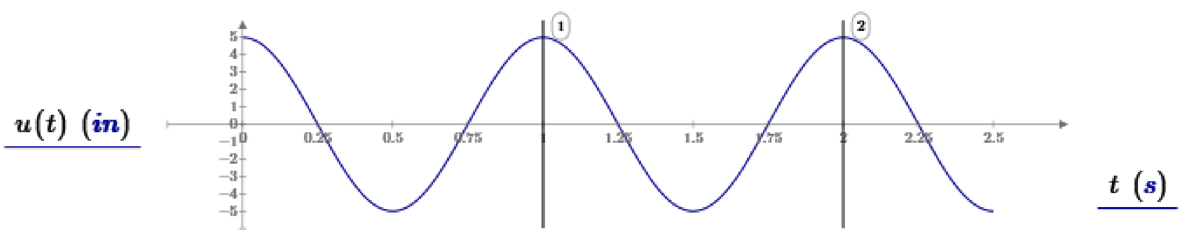

Figure 1:Free Vibration of an Undamped System - Displacement vs. Time

$$T = \frac{2\pi}{\omega_n} \qquad \text{Equation (1)}$$

In the Structural Dynamics course taught by the author, a graph like Figure 1 is presented on the screen using Mathcad, and the value of ω_n is varied, leading the students to deduce the relationship between $\boldsymbol{T}$ and ω_n. Figures 2 – 5 show the plots that are successively displayed in class. Note that in class, only one plot is displayed on the screen and the value of ω_n is varied. The vertical marker gives the value of $\boldsymbol{T}$ in seconds.

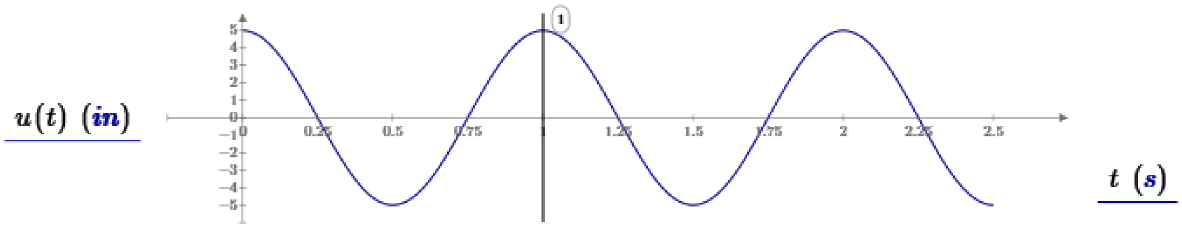

Figure 2: System Response with ω_n = 2π rad/s

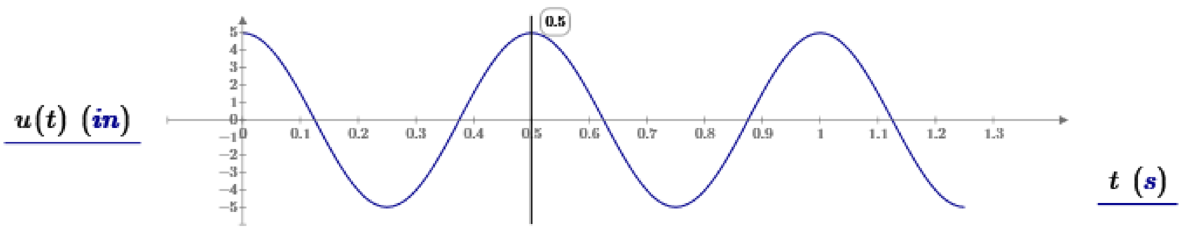

Figure 3: System Response with ω_n = 4π rad/s

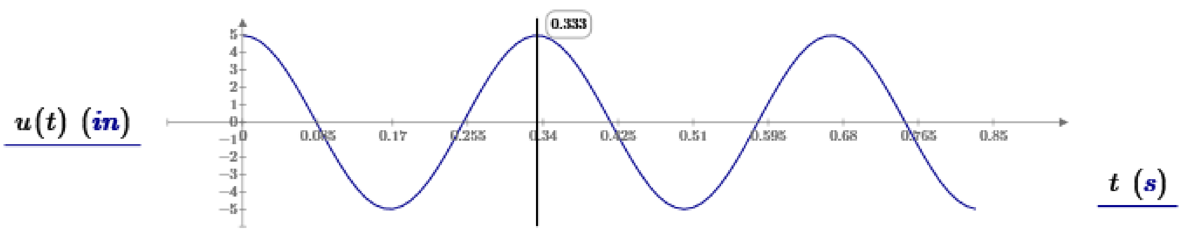

Figure 4: System Response with ω_n = 6π rad/s

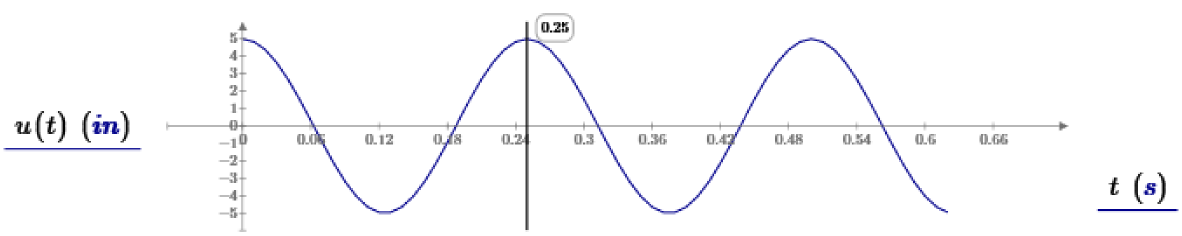

Figure 5: System Response with ω_n = 8π rad/s

Example 2: Fourier Series Representation of Non-Harmonic Functions

The response of a vibrating system to a periodic but non-harmonic forcing function is typically part of the study of Structural Dynamics. Being able to characterize a periodic but non-harmonic forcing function as the sum of harmonic functions through the Fourier series also allows characterization of the system response in terms of a Fourier series.

The first graph a text will typically show for the Fourier series representation is a graph like the one in Figure 6, which shows the first 4 terms in the Fourier series for a sawtooth forcing function. The text can note that the first term has the largest contribution, with increasingly smaller contributions from higher order terms.

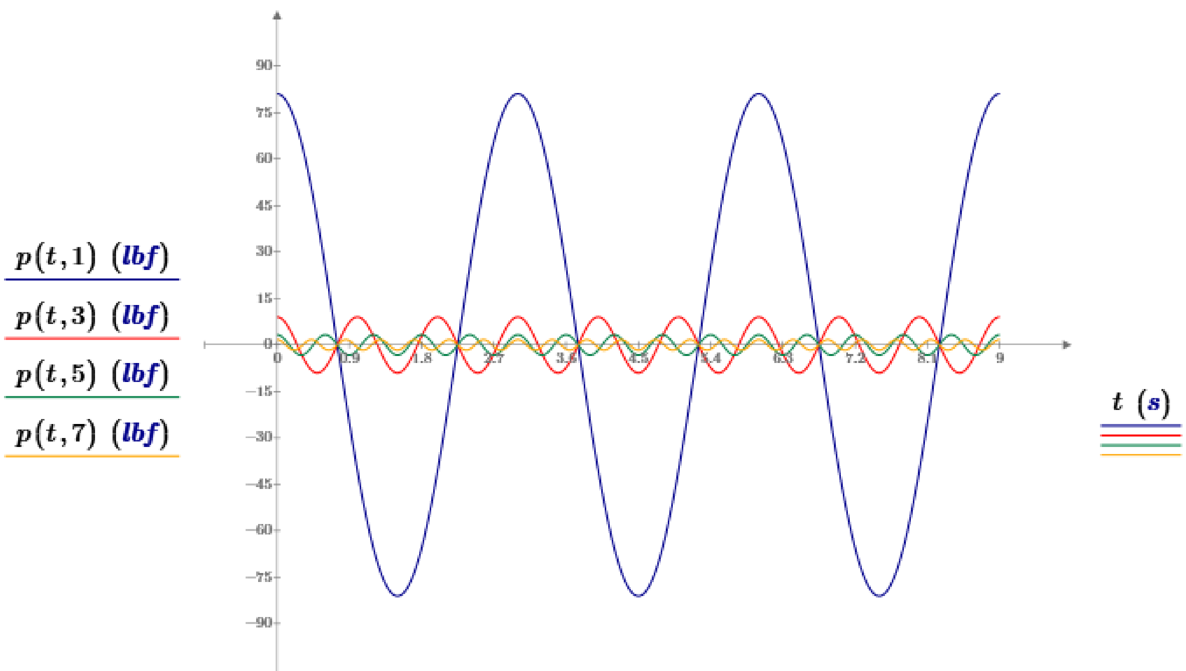

Figure 6: Sawtooth Forcing Function – Fourier Terms

What the text can't do is then omit the first term to look at the relative contributions of the next four terms, as Figure 7 shows. By changing the graph on the fly in the classroom, the instructor can look at how the contributions of the terms vary. The overall conclusion is the same, but the path to get there is different.

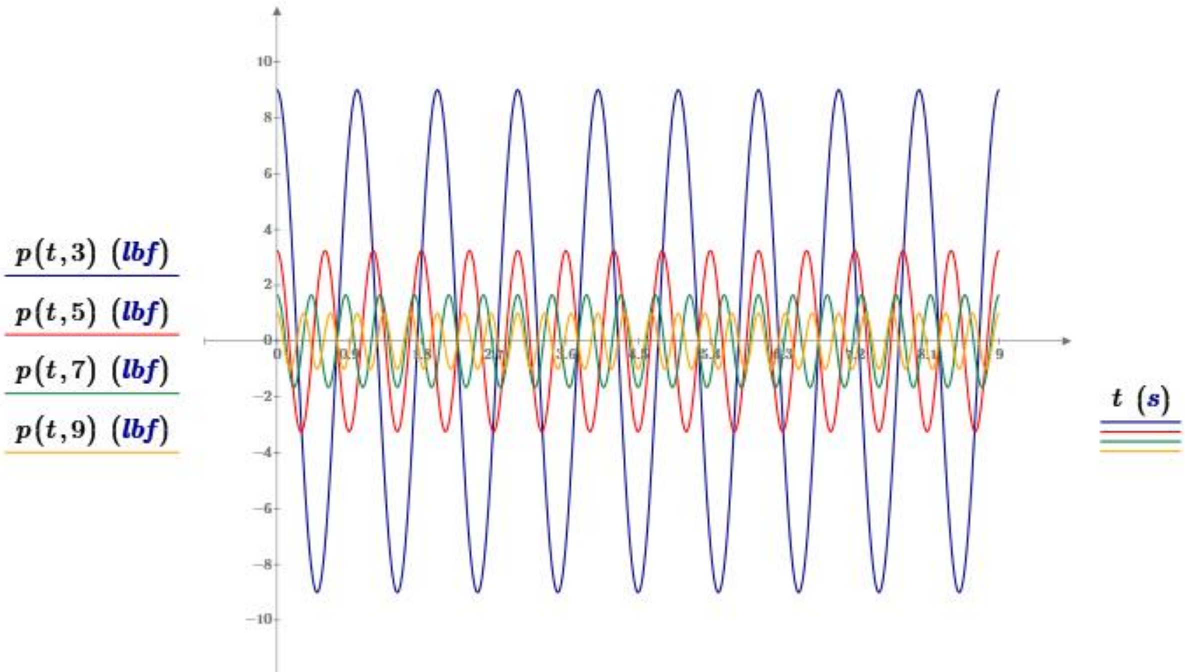

Figure 7: Sawtooth Forcing Function – Fourier Terms

When discussing the Fourier series for a non-harmonic forcing function, the next plot the text will typically show is one that shows the summation of the first n terms, where n is a number that gets the summation close to the desired shape of the function. For example, for the sawtooth forcing function, a text would likely show a graph like Figure 8, with four terms, and note that this is a reasonable approximation of the forcing function.

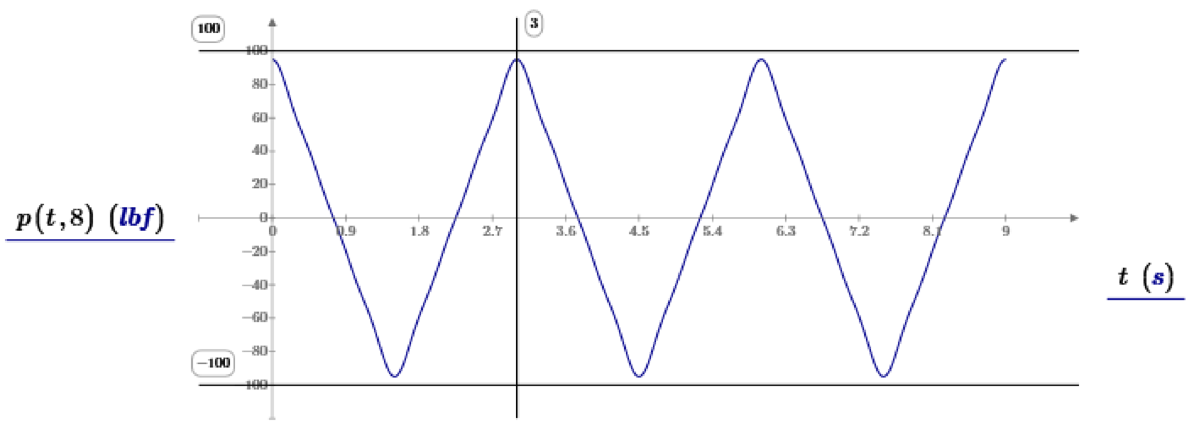

Figure 8: Sawtooth Forcing Function with Four Terms (Even terms are zero)

Using live math in the classroom, the instructor can vary the number of terms included in the series and show how additional terms bring the graph closer to the sawtooth.

Additional Advantages of Using Mathcad in the Classroom

One of the other advantages of using Mathcad in the Classroom is that the math that is displayed on the screen looks exactly like what the instructor has likely already written on the board or included in their PowerPoint slides. On the board or in a PowerPoint presentation, while the equation looks great, it cannot do anything. If Mathcad is up simultaneously with either a board or PowerPoint, the instructor can show how the math works as well as displaying the equation.

As an example, some of the equations used to create the graphs in this paper are displayed below. Figure 9 was used to create Figures 1 – 5 by varying the value of ω_n.

$$u(t) := u_0 \cdot \cos(\omega_n \cdot t) + \frac{v_0}{\omega_n} \cdot \sin(\omega_n \cdot t)$$

Figure 9: Equation for the Free Vibration Response of an Undamped System

Figure 10 was used to create Figures 6 and 7. In this equation, t is the time variable which is defined from 0 to 3T, and j is a parameter passed to the function to pick out the jth term in the series.

$$p(t,j) := \frac{8 \cdot p_0 \cdot \sin\left(\frac{\pi \cdot j}{2}\right)^2}{(\pi \cdot j)^2} \cdot \cos\left(\frac{2 \cdot \pi \cdot j}{T_0} \cdot t\right)$$

Figure 10: Equation for Individual Terms of a Sawtooth Forcing Function

Adding a summation to the equation in Figure 10 gives Figure 11, which was used to create Figure 8.

$$p(t,k) := \sum_{j=1}^{k} \left(\frac{8 \cdot p_0 \cdot \sin\left(\frac{\pi \cdot j}{2}\right)^2}{(\pi \cdot j)^2} \cdot \cos\left(\frac{2 \cdot \pi \cdot j}{T_0} \cdot t\right) \right)$$

Figure 11: Equation for the Fourier Series Representation of a Sawtooth Forcing Function

Summary

The examples presented in this paper only scratch the surface of what an instructor can do with live math in the classroom. In addition to having a plan for changing some parameters in a systematic fashion as part of a lesson, the instructor can use Mathcad to make changes that were not planned, spurred by a suggestion or a question from the class, or an idea that occurs to the author during their explanation of the planned presentation.

Using live math in the classroom works in any class where an instructor would write an equation on the board and explain to the class how it works in words. With Mathcad, the instructor can show them how the math works, and what the effect of varying the parameters is on the results.