# AC 2007-198: BROADENING STUDENT KNOWLEDGE OF DYNAMICS BY MEANS OF SIMULATION SOFTWARE

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# Broadening student knowledge of dynamics by means of simulation software


## Abstract

Some examples are given with the aim of broadening undergraduate student knowledge and understanding of dynamics. Typically the examples involve non-linear equations and numerical methods must be employed. Here, because of its wide availability and of its increasing use in undergraduate mathematics courses, MAPLE[1] is employed. The central goal of the work is to introduce new phenomena, and the examples treated are: (i) The effect of viscous damping on the stability of an inverted pendulum. It is shown that with a linear model viscous damping does not stabilize an unstable state, whereas damping plays an important role when a non-linear model is considered. (ii) Forced harmonic motion of a non-linear hardening spring-mass system. The numerical simulation of the response illustrates the "jump phenomena" in which the steady state amplitude undergoes a jump in passing through frequencies close to the linear resonance frequency. (iii) A simple pendulum with an oscillating support, illustrating parametric resonance. Depending on the system parameter values instabilities can occur (parametric resonance). This is shown numerically and confirmed with an available analytic expression. The associated MAPLE files are given in an appendix.


## Introduction

The availability of commercial codes such as MAPLE[1] has made it possible to numerically treat problems in dynamics which are analytically intractable. Of course other codes such as MATHCAD® and MATLAB®, for example, are also available. However the thrust here is not to debate the relative merits of finite difference schemes in various software packages. The students should be aware of the nature of finite difference schemes (a simple illustrative example is given in a previous work[2]) then, at least in dynamics classes, the software may be treated as a "black box". Several examples that broaden student physical knowledge and understanding were given previously[2], namely: a non-linear pendulum subjected to various initial conditions, showing how the period depends on the amplitude; a non-linear softening spring showing the existence of instabilities; an undamped inverted pendulum restrained by a spiral spring, illustrating the existence of multiple equilibrium states and their stability; a simulation of a sweep test (forced motion of a single-degree-of-freedom system in which the forcing frequency varies with time), showing that if the sweep rate is too fast, no resonances will be observed. Here several new examples are presented (for convenience both sets are included in TABLE 1 in an appendix, which has MAPLE worksheet objects included). The examples are: (i) the effect of viscous damping on the stability of an inverted pendulum; (ii) forced harmonic motion of a non-linear hardening spring-mass system; and (iii) a simple pendulum with an oscillating support, illustrating parametric resonance. Examples (i) and (iii) can readily be handled in a beginning course, whereas example (ii) may be more suitable for an intermediate course.

**Physical Examples**

***Effect of viscous damping on the stability of an inverted pendulum***

The equation of motion for an inverted pendulum with a torsional spring and torsional damper at the base is given below (see FIGURE 1).

$$ml^2\ddot{\theta} + C\dot{\theta} + G\theta - mg\frac{l}{2}\sin(\theta) = 0$$

**( 1 )**

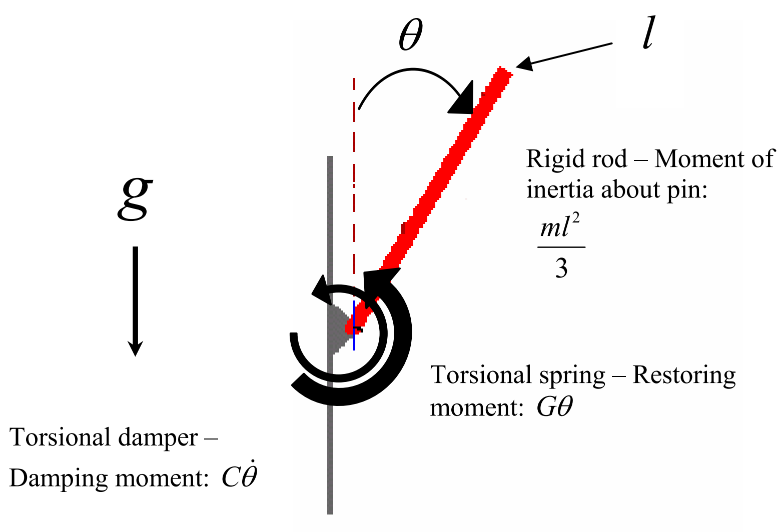



**FIGURE 1 – INVERTED PENDULUM**

This equation can be written in the dimensionless form:

$$\frac{d^2\theta}{d\tau^2} + J\frac{d\theta}{d\tau} + B\theta - \sin(\theta) = 0$$

**( 2 )**

Here $\tau$ (dimensionless time) $=\sqrt{\frac{g}{l}}t$, $B$ is a dimensionless spring constant defined by

$G = mgBl$ and $J$ is a dimensionless damping constant defined by $J = \frac{C}{ml^2}\sqrt{\frac{l}{g}}$.

It should be pointed out to the students that equation ( 2 ) has two possible equilibrium states, namely the roots of:

$$B\theta - \sin(\theta) = 0 \tag{3}$$

This has the solutions $\theta = 0$ and, for $B = 0.95$, $\theta = 31.62^\circ$ ( $0.5519 rad$ ). Such states will not be seen in practice if they are unstable. Consider the $\theta = 0$ case. Equation ( 2 ) is solved numerically (with initial conditions $\theta = 5^\circ \approx 0.0873 rad$ and $d\theta/d\tau = 0$ ) using MAPLE's finite difference scheme. (The students / reader could consult reference[2] for an illustration on a simple finite difference scheme.)

Shown in FIGURE 2 is the response for the linearized system ( $\sin(\theta) = \theta$ ) for zero damping. The response (and hence the $\theta = 0$ equilibrium state) is clearly unstable. An interesting question is whether this state can be stabilized by adding damping to the system.

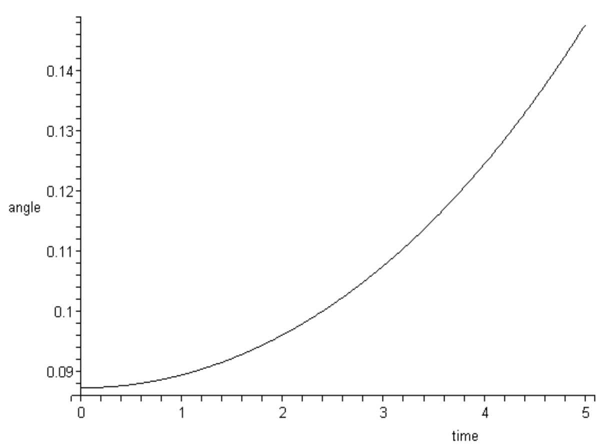



**FIGURE 2 – RESPONSE FOR LINEAR SYSTEM AND ZERO DAMPING**

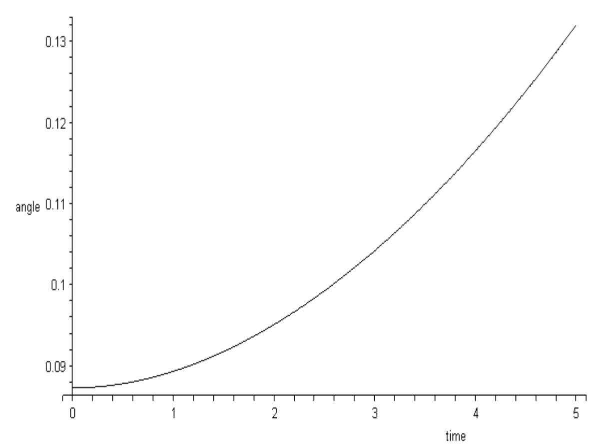
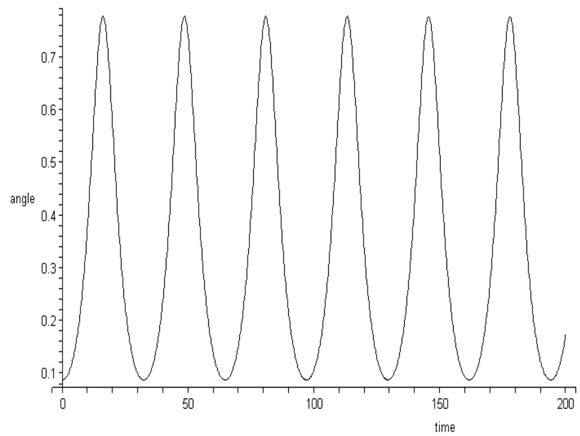



**FIGURE 3 – RESPONSE FOR LINEAR SYSTEM WITH DAMPING**

Shown in FIGURE 3 is the response for the case where the damping value (non-dimensional) is 0.20. The instability still persists; damping plays no significant role. However this response is not what would actually occur. Consider now the non-linear model in which $\sin(\theta)$ is retained.



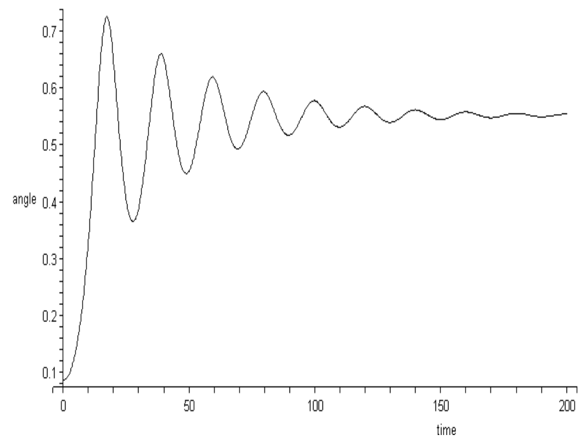
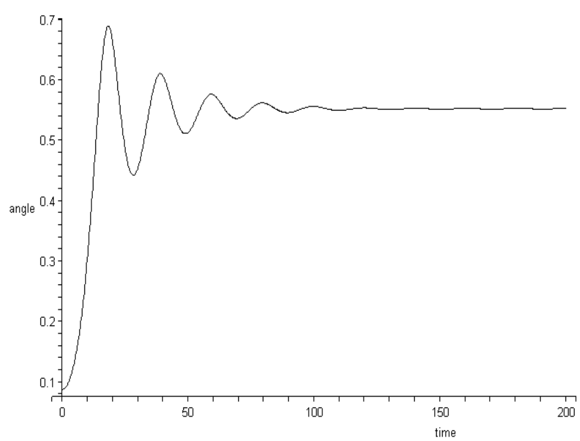




**FIGURE 4 – RESPONSE FOR NON –LINEAR SYSTEM AND ZERO DAMPING**

**FIGURE 5 – RESPONSE FOR NON –LINEAR SYSTEM WITH DAMPING**

Shown in FIGURE 4 is the response for zero damping (and initial conditions close to zero). Note that the response grows but finally becomes bounded, oscillating about the $\theta = 31.62^{\circ} \approx 0.5519 rad$ state. FIGURE 5 depicts the response for a damping value (non-dimensional) of 0.05. Here damping is seen to play a role. The larger the value of the damping coefficient the faster the approach to the $\theta = 0.5519 rad$ state. This can be seen by comparing FIGURE 6 to FIGURE 7. The responses for non-dimensional damping values of 0.10 and 0.20 are shown, respectively.



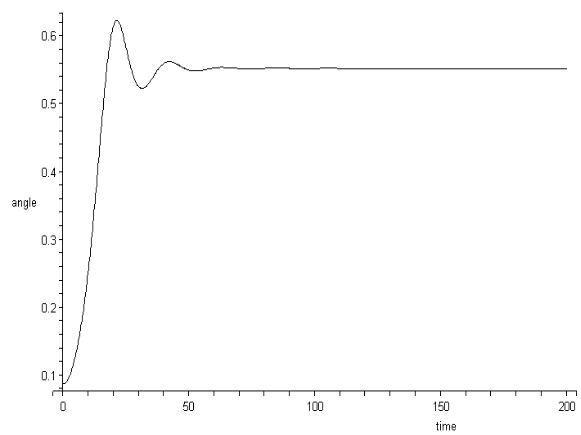




**FIGURE 6 – RESPONSE FOR NON –LINEAR SYSTEM – DAMPING = 0.10**

**FIGURE 7 – RESPONSE FOR NON –LINEAR SYSTEM – DAMPING = 0.20**

***Harmonic motion of a hardening spring-damper-mass system***

An important consequence of non-linearity can be illustrated with the following example. The equation of motion for a spring-damper-mass system with a hardening spring is given by:

$$\frac{d^2x}{dt^2} + 2\beta\omega_0\frac{dx}{dt} + \omega_0{}^2 x + \frac{k_1}{m}x^3 = \frac{Q}{m}\sin(\omega t)$$

**( 4 )**

where the spring force is $kx + k_1x^3$, $m$ is the mass, $\beta$ is the damping ratio of the system, $\omega_0$ (given by $\sqrt{k/m}$) is the undamped linear natural frequency and $Q$ and $\omega$ are the amplitude and frequency, respectively, of the external harmonic excitation. Setting $k_1 = \delta k$ and introducing the following dimensionless quantities: $\tau = \omega_0 t$ , $\chi = \frac{x}{Q/k}$ , $\nu = \omega/\omega_0$ ; equation ( 4 ) becomes:

$$\frac{d^2\chi}{d\tau^2} + 2\beta\frac{d\chi}{d\tau} + \chi + \delta x^3 = \sin(\nu t)$$

**( 5 )**

Equation ( 5 ) is a harmonically forced Duffing equation[3]. A numerical solution to the problem is given in the following.

Taking, $\delta = 0.25$ (weak non-linearity) and $\beta = 0.10$ (light damping) the response can be obtained with the aid of MAPLE (initial conditions are set to $\chi = 0$ and $d\chi/d\tau = 0$). The goal is to illustrate the "jump phenomena", where the steady state response amplitude undergoes a jump when the frequency of the excitation approaches the linear resonant frequency ($\nu = 1$). The response is obtained for several values of the excitation frequency and the steady state values are plotted versus the frequency ratio $\nu$ . FIGURE 8 through FIGURE 11 illustrate the responses and the steady state values for excitation frequencies close to the linear resonance frequency. Note the increase of the steady state values and the sudden drop ("jump") at $\nu = 1.5$ .

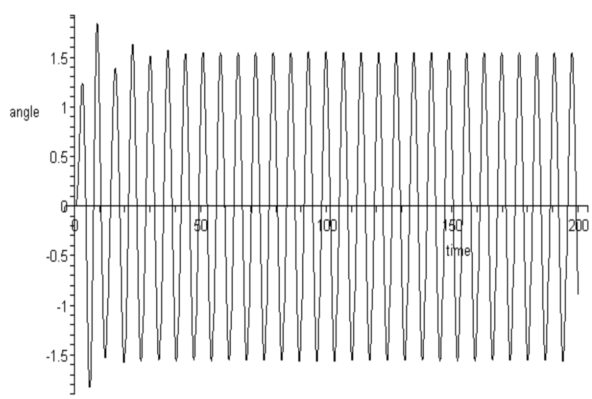



**FIGURE 8 – RESPONSE FOR** $\nu = 0.9$

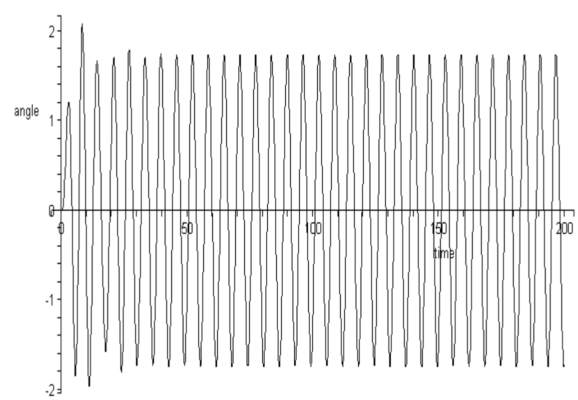



**FIGURE 9 – RESPONSE FOR** $\nu = 1$

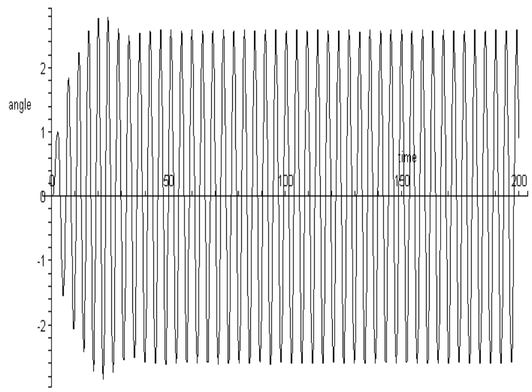



**FIGURE 10 – RESPONSE FOR** $\nu = 1.4$

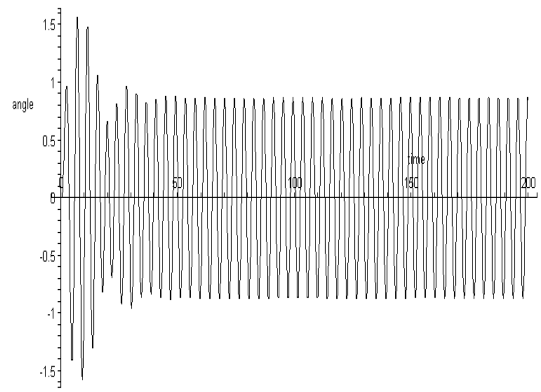



**FIGURE 11 – RESPONSE FOR** $\nu = 1.5$

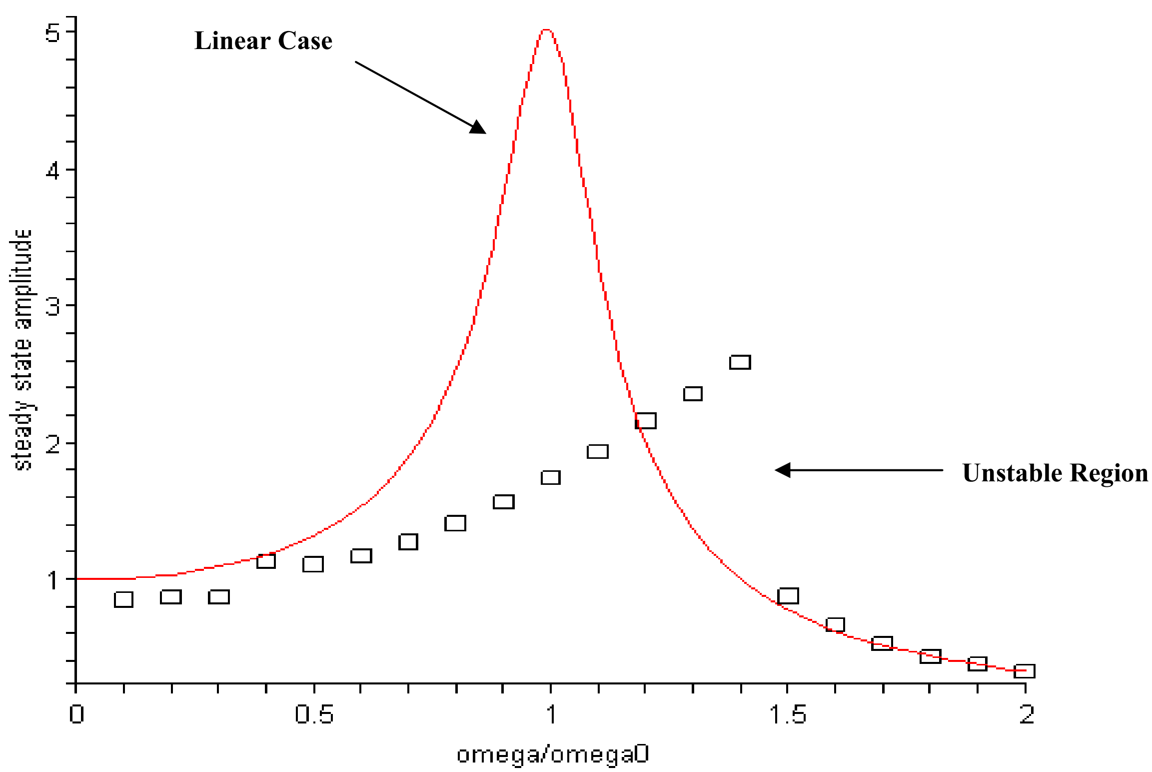



**FIGURE 12 – NUMERICALLY OBTAINED FREQUENCY RESPONSE**

FIGURE 12 depicts the frequency response for the system obtained numerically. A "jump" is clearly observed in the vicinity of $\nu = 1.5$. Also included in the figure is the frequency response for the system when $\delta = 0$ (linear system, response peaks at the resonant level $\nu = 1$). Students should notice the "bending" of the resonant peak in the case of a non-linear system. Here the peak is bent to the right since the non-linearity is of a hardening type (in the case of a softening non-linearity the peak would bend to the left). No steady state values were obtained for the unstable solutions highlighted in the figure. These are unstable stationary solutions and will not be observed in practice.

***Motion of a pendulum on an oscillating support***

Resonances other than forced motion ones can be illustrated with the following example. Consider the motion of a pendulum which is connected to a support that undergoes a harmonic translational motion. The pendulum is subjected to gravity and to a viscous damping moment at the support ($cl^2\dot{\theta}$). The amplitude of the translational motion and its frequency are prescribed, $ql$ and $\Omega$, respectively. The system is shown in FIGURE 13.

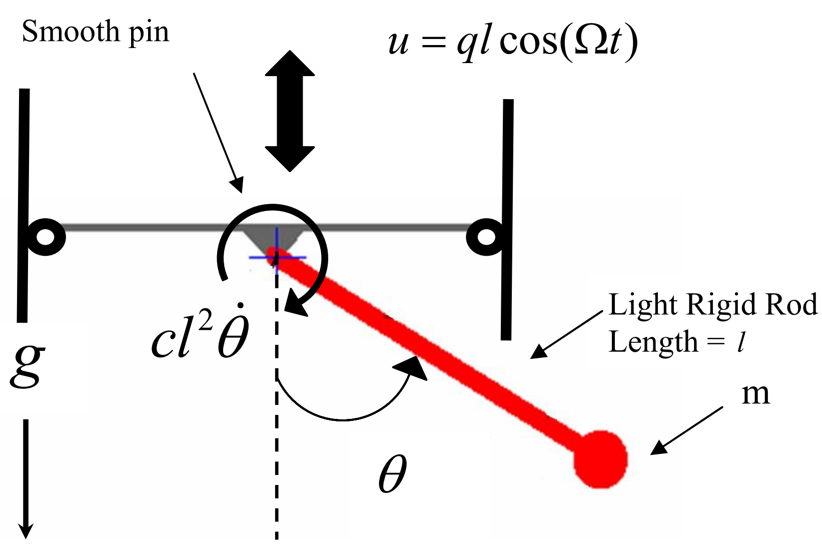



**FIGURE 13 – PENDULUM WITH AN OSCILLATING SUPPORT**

Deriving the equations of motion may be challenging from a student viewpoint. One approach is to use a reference frame translating with $u$. An observer in that frame sees the mass $m$ undergoing circular motion. Then it can be seen that the acceleration of the end mass is given by: $\vec{a} = (\ddot{u}\cos(\theta) + l\dot{\theta}^2)\vec{n} + (\ddot{u}\sin(\theta) + l\ddot{\theta})\vec{t}$ where $\vec{n}$ and $\vec{t}$ are the unit vectors directed along the normal and tangential directions to the circular motion, respectively. Using $\sum F_t = ma_t$ leads to: $-mg\sin(\theta) - R\cos(\theta) = m(\ddot{u}\sin(\theta) + l\ddot{\theta}) = 0$, where $\vec{R}$ is the pin reaction force. Taking moments about the center of mass (here the end point mass) leads to:

$R\cos(\theta)l - cl^2\dot{\theta} = 0$. Solving the first equation for $R$ and substituting into the second one leads to the equation of motion:

$$\ddot{\theta} + \frac{c}{m}\dot{\theta} + \frac{1}{l}(g + \ddot{u})\sin(\theta) = 0 \tag{6}$$

After substituting the function for the harmonic translation and assuming small pendulum oscillations ($\sin(\theta) \approx \theta$), a non-dimensional version of equation ( 6 ) can be written as:

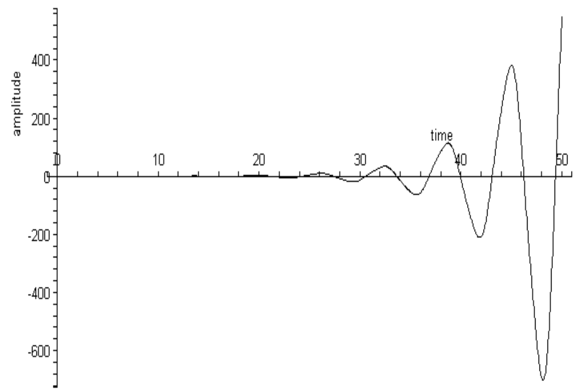
$$\frac{d^2\theta}{d\tau^2} + C_1\frac{d\theta}{d\tau} + (1 - \upsilon^2 q\cos(\upsilon\tau))\theta = 0 \tag{7}$$

where the following non-dimensional coefficients were employed.

$$\tau = \Omega_0 t\ ,\ \ \Omega_0^{\ 2} = g/l\ \ C_1 = c/\Omega_0 m\ ,\ \upsilon = \Omega/\Omega_0$$

Equation ( 7 ) is a damped Mathieu-Hill[4] equation. Students should be made aware that, although the system is homogeneous, with no apparent forcing function, there are situations when the response of this type of system can be unstable. This is due to the presence of the time-dependent coefficient in the equation. It can be shown that the coefficient frequencies that will cause instabilities are related to the undamped natural frequency of the system by $\upsilon = 2\upsilon_n/j$, $j = 1,2,3...$ ( where $\upsilon_n$ is the non-dimensional undamped natural frequency, see reference[4] for more details). Note that here $\upsilon_n = \sqrt{g/\Omega_0^{\ 2} l} = 1$, then the condition becomes $\upsilon = 2/j$, $j = 1,2,3...$. When this is satisfied, a so-called "parametric resonance" is possible. For $j = 1$ the condition leads to the primary parametric resonance at $\upsilon = 2$. This is different from a forced resonance condition, where a resonance is expected when the frequency of the forcing function approaches the natural frequency of the system ($\upsilon = 1$ not $\upsilon = 2$). This parametric instability is verified numerically in the following.

FIGURE 14 shows the response for the following parameters: $\theta(0) = 0.1rad$, $d\theta(0)/d\tau = 0$, $q = 0.2$, $C_1 = 0.01$ and $\upsilon = 2.0$ which, as mentioned above, is expected to lead to unstable response. The numerical simulation shows an exponential growth of the response and, consequently, instability. FIGURE 15 shows the numerically obtained response for the following parameters: $\theta(0) = 0.1rad$, $d\theta(0)/d\tau = 0$, $q = 0.2$ and $\upsilon = 1.6$. In this case the condition for parametric instability is not satisfied and no resonance is expected. This is confirmed by the response shown in the figure.



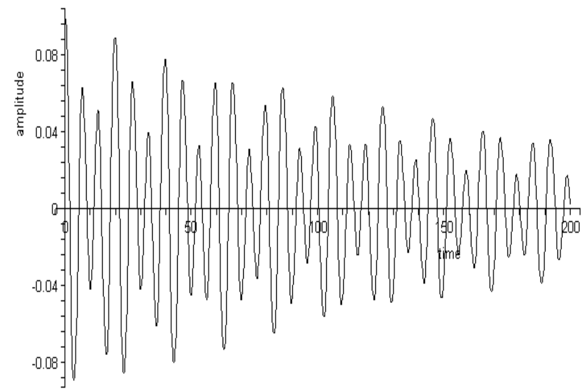




**FIGURE 14 – PENDULUM RESPONSE TO INITIAL CONDITIONS – UNSTABLE**

**FIGURE 15 – PENDULUM RESPONSE TO INITIAL CONDITIONS – STABLE**

There are several approaches to investigating the conditions that drive parametric systems unstable. Their discussion is beyond this text. Nevertheless, a simple approach, known as Hill's infinite determinant[4], is shown here. This approach can lead to the "boundaries" of the instability zones in the space defined by the parameters $\upsilon$ versus $q$ . Students can verify, numerically, whether the response of the system to initial conditions is stable or not (and confirm the predictions given by the approach).

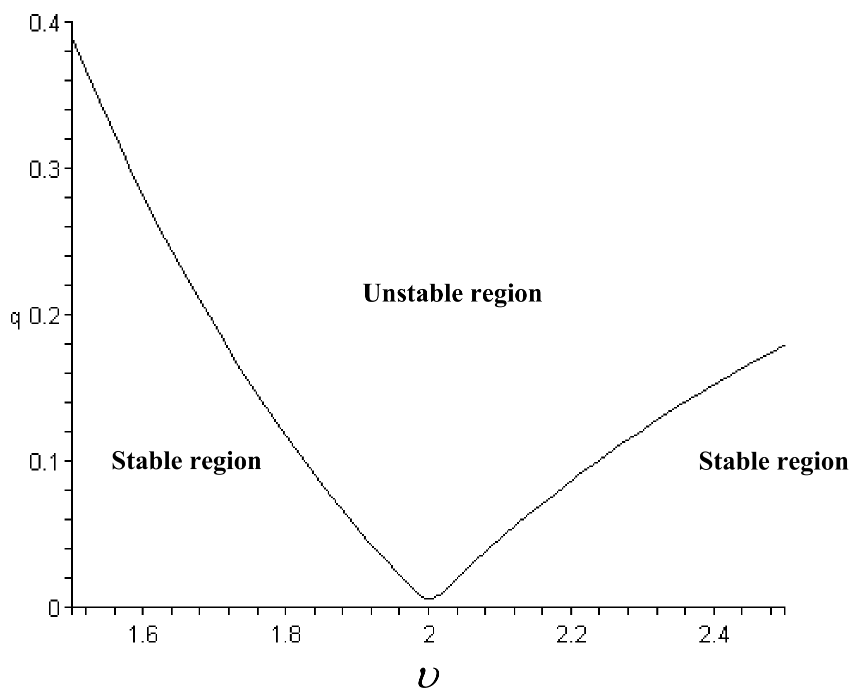



**FIGURE 16 – PENDULUM STABILITY BOUNDARIES**

The primary instability zone ( $j = 1$ ) for the pendulum system is given in FIGURE 16 (using the same numerical values as before – worksheet is given in Appendix A). When the system parameters $q$ and $\upsilon$ lead to a point falling inside the “unstable region” the response to small disturbances will be unstable. Note that the unstable response obtained above is for the point $q = 0.2$ and $\upsilon = 2.0$, which can be seen to fall inside the unstable region. On the other hand, the stable response is for a point falling in the stable region ( $q = 0.2$ and $\upsilon = 1.6$ ). Damping does play a role here and enough damping could stabilize an “unstable condition” (the instability zone moves off the $\upsilon$ axis ).

## Conclusions

In most undergraduate engineering courses students are introduced to mathematics software such as MAPLE®. For dynamics courses, some intractable problems can then be explored in order to demonstrate interesting and important physical phenomena. The examples presented here were: (i) The effect of viscous damping on the stability of an inverted pendulum. It was shown that with a linear model viscous damping does not stabilize an unstable state, whereas, damping plays an important role when a non-linear model is considered. (ii) Forced harmonic motion of a non-linear hardening spring-mass system. The numerical simulation of the response illustrates a “jump phenomena” in which the steady state amplitude undergoes a jump in passing through frequencies close to the linear resonance frequency. (iii) A simple pendulum with an oscillating support, illustrating parametric resonance. Depending on the parameter values of the system instabilities can occur (parametric resonance). This is shown numerically and confirmed with an available analytic expression.

### References


[1] www.maplesoft.com

[2] A. Mazzei and R. A. Scott, "Enhancing student understanding of mechanics using simulation software," *Proceedings of the 2006 American Society for Engineering Education Annual Conference & Exposition, Chicago - IL*, 2006.

[3] J. J. Thomsen, *Vibrations and Stability*, 1st ed: McGraw-Hill, 1997.

[4] V. V. Bolotin, *The Dynamic Stability of Elastic Systems*. San Francisco, California: Holden-Day, Inc., 1964.

## Appendix A

```
############ inverted pendulum stability ######################
restart:
with(linalg):with(plots):with(DEtools):
eq10:=(diff(x(t), `$`(t, 2)))+J*(diff(x(t), t))+B*x(t)-sin(x(t));
B:=0.95;J:=0;
for i from 1 to 20 do
eq10;
sol001:=dsolve({eq10,x(0)=evalf(convert(5*degrees,radians)),D(x)(0)=0},{x(t)}
, type=numeric, method=gear,output=procedurelist):
odeplot(sol001,[t,x(t)],0..200,numpoints=1000,color=black,labels=["time","ang
le"]);
J:=J+0.01;
end do;
restart:
with(linalg):with(plots):with(DEtools):
eq10:=(diff(x(t), `$`(t, 2)))+J*(diff(x(t), t))+B*x(t)-(x(t));
B:=0.95;J:=0;
for i from 1 to 20 do
eq10;
sol001:=dsolve({eq10,x(0)=evalf(convert(5*degrees,radians)),D(x)(0)=0},{x(t)}
, type=numeric, method=gear,output=procedurelist):
odeplot(sol001,[t,x(t)],0..5,numpoints=1000,color=black,labels=["time","angle
"]);
J:=J+0.01;
end do;
####################### duffing equation ###############################
restart:
with(linalg):with(plots):with(DEtools):
eq01:=(diff(x(t), `$`(t, 2)))+2*beta*(diff(x(t), t))+x(t)+delta*x(t)^3-
sin(nu*t);
delta:=0.25;beta:=0.1;
eq01;
nu:=0.1;
for i from 1 to 20 do
eq01;
sol001:=dsolve({eq01,x(0)=0,D(x)(0)=0},{x(t)}, type=numeric,
method=gear,output=procedurelist):
odeplot(sol001,[t,x(t)],0..200,numpoints=1000,color=black,labels=["time","ang
le"]);
nu:=nu+0.1;
end do;
restart:
with(stats[statplots]):
fig01:=plot([[0.1,0.8495],[0.2,0.8720],[0.3,0.8676],[0.4,1.1246],[0.5,1.1078]
,[0.6,1.1728],[0.7,1.2727],[0.8,1.4091],[0.9,1.5621],[1.0,1.7450],[1.1,1.9307
],[1.2,2.1593],[1.3,2.3589],[1.4,2.5885],[1.5,0.8779],[1.6,0.6639],[1.7,0.533
3],[1.8,0.4393],[1.9,0.3800],[2.0,0.3290]],style=point,symbol=box,color=black
):
#fig02:=xscale((1/0.0032),fig01):
#fig03:=yscale(1/0.0873,fig02):
plots[display](fig01,labels=["omega/omega0","steady state
amplitude"],labeldirections=[horizontal,vertical]);
restart:
with(linalg):with(plots):with(DEtools):
```

```
fig01:=plot([[0.1,0.8495],[0.2,0.8720],[0.3,0.8676],[0.4,1.1246],[0.5,1.1078]
,[0.6,1.1728],[0.7,1.2727],[0.8,1.4091],[0.9,1.5621],[1.0,1.7450],[1.1,1.9307
],[1.2,2.1593],[1.3,2.3589],[1.4,2.5885],[1.5,0.8779],[1.6,0.6639],[1.7,0.533
3],[1.8,0.4393],[1.9,0.3800],[2.0,0.3290]],style=point,symbol=box,color=black
):
 eq01:=(diff(x(tau), `$`(tau, 2)))+2*beta*(diff(x(tau),
tau))+x(tau)+delta*x(tau)^3-sin(omega/omega0*tau);
 amp:=1/(sqrt((1-(omega/omega0)^2)^2+(2*beta*omega/omega0)^2));
 omega0:=1;delta:=0;beta:=0.10;
 eq01;
 amp;
 plot(amp,omega=0..2);
fig01:=plot([[0.1,0.8495],[0.2,0.8720],[0.3,0.8676],[0.4,1.1246],[0.5,1.1078]
,[0.6,1.1728],[0.7,1.2727],[0.8,1.4091],[0.9,1.5621],[1.0,1.7450],[1.1,1.9307
],[1.2,2.1593],[1.3,2.3589],[1.4,2.5885],[1.5,0.8779],[1.6,0.6639],[1.7,0.533
3],[1.8,0.4393],[1.9,0.3800],[2.0,0.3290]],style=point,symbol=box,color=black
):
 fig02:=plot(amp,omega=0..2):
 #fig03:=yscale(1/0.0873,fig02):
 plots[display]([fig01,fig02],labels=["omega/omega0","steady state
amplitude"],labeldirections=[horizontal,vertical]);
 ############### pendulum on an oscillating support #####################
 restart:with(linalg):
 eq01:=diff(g(x),x$2)+c*diff(g(x),x)+(nu0^2-nu^2*q*cos(nu*x))*g(x);
 g(x):=a1*sin(nu*x/2)+b1*cos(nu*x/2);
 eq01;
 eq1:=combine(eq01,trig);
 eq1:=sort(eq1,[sin(nu*x/2),cos(nu*x/2)]);
 eq1a:=collect(eq1,[sin(nu*x/2),cos(nu*x/2)]);
 coef01:=coeff(eq1a,sin(nu*x/2),1);
 coef02:=coeff(eq1a,cos(nu*x/2),1);
 m1:=collect(coef01,[a1,b1]);
 m2:=collect(coef02,[a1,b1]);
matriz:=matrix(2,2,[coeff(m1,a1,1),coeff(m1,b1,1),coeff(m2,a1,1),coeff(m2,b1,
1)]);
 equacao:=det(matriz);
 raizes:=solve(equacao,q);
 c:=.01;nu0:=1;
 raizes;
 raizes[1];
 raizes[2];
 #raizes[3];
 #raizes[4];
 plot([raizes[1],raizes[2]],nu=0..5,0..10);
plot([raizes[1],raizes[2]],nu=1.5..2.5,0...0.4,labels=["nu","q"],color=black)
;
 restart:with(linalg):with(DEtools):with(plots):
 eq01:=diff(g(x),x$2)+c*diff(g(x),x)+(nu0^2-nu^2*q*cos(nu*x))*g(x);
 c:=0.01;q:=0.2;nu0:=1;nu:=2;
 eq01;
 sol001:=dsolve({eq01,g(0)=0.1,D(g)(0)=0},{g(x)}, type=numeric,
method=gear,output=procedurelist);
odeplot(sol001,[x,g(x)],0..50,numpoints=1000,color=black,labels=["time","ampl
itude"],labeldirections=[horizontal,vertical]);
 c:=0.01;q:=0.2;nu0:=1;nu:=1.6;
 eq01;
```

```
 sol001:=dsolve({eq01,g(0)=0.1,D(g)(0)=0},{g(x)}, type=numeric,
method=gear,output=procedurelist);
odeplot(sol001,[x,g(x)],0..200,numpoints=1000,color=black,labels=["time","amp
litude"],labeldirections=[horizontal,vertical]);
```

## Appendix B

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|---|---|---|
| (i) The effect of viscous damping on the stability of an inverted pendulum. It is shown that with a linear model viscous damping does not stabilize an unstable state, whereas, damping plays an important role when a non-linear model is considered | Dynamics, Vibrations | C:\Documents and Settings\amazzei.KET |
| (ii) Forced harmonic motion of a non-linear hardening spring-mass system. The numerical simulation of the response illustrates the "jump phenomena" in which the steady state amplitude undergoes a jump in passing through frequencies close to the linear resonance frequency | Dynamics, Vibrations | C:\Documents and Settings\amazzei.KET |
| (iii) A simple pendulum with an oscillating support, illustrating parametric resonance. | Dynamics, Vibrations | C:\Documents and Settings\amazzei.KET |
| (iv) a finite difference scheme | Mathematics, dynamics, vibrations | C:\Documents and Settings\amazzei.KET |
| (v) a non-linear pendulum subjected to various initial conditions, showing how the period depends on the amplitude | Dynamics, Vibrations | C:\Documents and Settings\amazzei.KET |
| (vi) a non-linear softening spring showing the existence of instabilities | Dynamics, Vibrations | C:\Documents and Settings\amazzei.KET |
| (vii) the stability of an inverted pendulum restrained by a spiral spring, illustrating the existence of multiple equilibrium states and their stability | Dynamics, Vibrations | C:\Documents and Settings\amazzei.KET |
| (viii) a numerical simulation of a sweep test (forced motion of a single-degree-of-freedom system in which the forcing frequency varies with time), showing that if the sweep rate is too fast, no resonances will be observed. | Dynamics, Vibrations | C:\Documents and Settings\amazzei.KET |

**TABLE 1**