

## **Building Middle School Teacher Mathematics and Science Content Knowledge through Engineering Design (Fundamental)**

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## Building Middle School Teacher Mathematics and Science Content Knowledge through Engineering Design (Fundamental)

We present a model for professional development that emerged from our work with 24 middle school mathematics and science teachers in rural Appalachia. The model utilizes iterative design/redesign to address “the engineering problem” of building teacher content knowledge for teaching mathematics and science. Project TESAL (Teachers Engaged in Science And Literacy) is a three year Math Science Partnership providing proximal context for developing this model. Project TESAL involved two weeks professional development each summer, two days each semester, and classroom observations/support. Teachers participated all three years and created then implemented and refined two lesson plans per year. Project TESAL involved 24 participating teachers from four counties with 41% to 67% low-income students, less than 80% highly qualified mathematics or science teachers, and below average mathematics and science test scores in a state well below the national average.

Our model includes the following steps:

Step 1: Identify mathematics and science knowledge gaps utilizing established standardized assessments (e.g., Diagnostic Teacher Assessments in Mathematics and Science [DTAMS], Force Concept Inventory [FCI]).

Step 2: Engage teachers in productive struggle as learners in design tasks requiring that knowledge and including mathematical modeling for prediction prior to building and testing designs.

Step 3: Evaluate teacher learning through observations, focus group interviews, and pre-post testing with standardized assessments.

Step 4: Redesign design tasks to further support teacher learning, and require participating teachers to implement design lessons addressing related knowledge gaps with their students.

As an example, we focus here on Real World Newtonian Physics (Next Generation Science Standard *MS-PS2-2*) where 12 of 24 teachers incorrectly identified relative motion of a dropped object and how to consider friction forces in initial DTAMS testing. Teachers experienced a roller coaster design project lesson as learners with the task to build a paper roller coaster so a marble took 45 seconds to traverse the track. Conversations during the coaster project and later classroom observations highlighted content knowledge gaps where teachers had misconceptions about how mass of a marble influences travel on the track; confusing how potential energy, kinetic energy, force, and speed differentiate. We developed new design modules for teachers requiring them to build and test ramps at various heights to launch small and large marbles first to hit a target and later to hit target with enough force to break a napkin. Measurements from designs with small marbles were used to build mathematical models predicting mechanics with large marbles.

We collected supporting evidence for the model from classroom observations, focus groups, and content knowledge tests. Briefly, teachers improved targeted FCI items and overall DTAMS

scores. In focus groups, teachers talked about productive struggle with critical mathematical and science content, understanding the process deeply enough to guide their students effectively, the importance of redesign, and how this professional development experience was dramatically different and more valuable than others they had experienced.

## **Context**

Difficulties attracting students to STEM careers are enhanced in Appalachia and West Virginia (WV) <sup>1-2</sup>. WV is far below the national average in percentage of STEM degrees (21% vs. 30%). 17% of adults over 25 in WV have a Bachelor's degree (lowest nationally); many communities have much lower rates <sup>3</sup>. Thirteen of 55 WV counties are "low education counties" where "25 percent or more of residents 25-64 years old had neither a high school diploma nor GED."<sup>4</sup>.

Project TESAL is a three year Math Science Partnership providing proximal context for developing the model described here. Structurally, Project TESAL involved two weeks of professional development each summer, two days each semester, and classroom observations/support (see Figure 1). Participating teachers remained in the program all three years and created then implemented and refined two lesson plans per year. Project TESAL involved teachers from four counties with 41% to 67% low-income students, less than 80% highly qualified teachers in mathematics or science, and below average mathematics and science test scores in a state well below the national average. The 24 participating teachers had 1 to 32 years teaching experience (median = 8 years) and considered themselves science educators (n=11), mathematics educators (n=8), special educators teaching math or science (n=4), or technology educators (n=1). All participants had a bachelor degree; 17 (70%) were highly qualified per federal definitions.

A key strength of Project TESAL is that the collaborative project team involved WV Regional Education Service Area personnel who have authentic long-standing relationships with key schools and teachers in the area working closely with university faculty who have deep engineering, science, and mathematics content knowledge, education pedagogy, curriculum resource, literacy, and educational evaluation/research expertise. This sort of team is quite unusual in the mostly rural Appalachian area where we work.

Project TESAL incorporates characteristics of effective professional development in mathematics and science <sup>5-10</sup>. Teachers engage in significant mathematics and science content related to the work of teaching as they develop, design, implement, and refine modules to address middle grade content standards and objectives (CSOs) in mathematics, science, literacy, and engineering design. Teachers collaborate with peers and experts in engineering design, literacy, science, and mathematics education as part of a team moving through learning, development, and implementation cycles. This work is aligned with research in that is ongoing, content-focused, embedded in the work of teaching, and aligned with WV CSOs.

National standards documents have made clear that mathematics is an essential tool for scientific inquiry, and science is a critical context for developing mathematics competence <sup>11-12</sup>. Mutually reinforcing science and mathematics understandings while teaching either discipline is a pragmatic and readily available interdisciplinary opportunity <sup>13-14</sup>. *A Framework for Science Education* <sup>15</sup> gives engineering and technology a greater focus. In our approach, *Common Core*

*State Standards for Mathematics*<sup>16</sup> content domains (e.g., ratios and proportional relationships, statistics and probability), and standards for mathematical practice (e.g., making sense of problems and persevering in solving them, modeling mathematics, choosing appropriate tools) are integrated with science and engineering practices from next generation standards (e.g., “asking questions/defining problems”, “using mathematics/computational thinking”), as well as crosscutting concepts focused on “systems/system models”<sup>17</sup>. Engineering design projects provide extensive opportunities for engaging in practices common to both the *CSSM* and *Framework*: defining problems, constructing explanations, developing models, and attending to precision.

Middle grade CSOs include engineering design in the science framework, but the design process is not easy to learn. This is at least partially because design is a dynamic iterative process rather than a specific skill or piece of content knowledge. Such processes are less often part of traditional teacher training. Therefore, teacher preparation and scaffolding are key to implementation of design based learning and related student learning gains<sup>18</sup>. Project TESAL addressed teachers’ knowledge of pedagogy and their content knowledge<sup>19</sup>. Teachers need to know how students develop understanding of content, how to set significant learning goals, how to select/implement appropriate instructional tasks, and how to assess learning<sup>20</sup>. In order to successfully impact student learning, teachers must have deep understanding of mathematics and science they teach. Well-designed professional development experiences are integral to developing such knowledge and skills<sup>5</sup>. Project TESAL targets improved mathematics and science content knowledge in an engineering design based approach<sup>21</sup>. We strive to shift students and teachers from being processors of information toward becoming creators of mathematics and science models as tools to help solve societally relevant scientific challenges through design/development of appropriate technologies<sup>22</sup>.

### **The Project TESAL Model**

Our model utilized iterative design/redesign to address “the engineering problem” of building teacher content knowledge (see Figure 2). We identified knowledge gaps, engaged teachers as learners in design tasks requiring that knowledge, evaluated teacher learning, redesigned design tasks to further support teacher learning, and required teachers to apply the model with their students. What follows is a concrete application of this model focused on middle school physical science and related mathematics standards.

Teachers correctly answered 81-88% Diagnostic Teacher Assessments in Mathematics and Science (DTAMS)<sup>23</sup> items in their content area, but 56-65% outside their content area (Table 1). Mathematics item analysis revealed incorrect responses across mathematical topics (e.g., data analysis, expressions and equations, rational number, proportional reasoning). While teachers did well applying an algorithm or formula, they struggled with items requiring analysis, reasoning, and application. Science item analysis revealed two areas of weakness described here in the context of Next Generation Science Standards<sup>17</sup>.

- 1) Real World Newtonian Physics (*MS-PS2-2= Plan an investigation to provide evidence that the change in an object’s motion depends on the sum of the forces on the object and the mass of the object.*) 12/24 teachers incorrectly identified relative motion of dropped object and how to consider friction forces.

- 2) Thermal Transport by Convection (*MS-PS3-3= Apply scientific principles to design, construct, and test a device that either minimizes or maximizes thermal energy transfer.*) Only 7/24 teachers correctly identified wall insulation traps pockets of air to prevent heat flow by convection.

We focus here on Real World Newtonian Physics. Teachers experienced a roller coaster design project lesson as learners with the task to build a paper roller coaster so a marble took 45 seconds to traverse the track. The coaster was used to demonstrate the design process and emphasize redesign. Mathematical modeling was used to predict time based on coaster design components. Redesign led to a literacy assignment to write an instruction manual on how to build their redesigned coaster. Each group then built another group's coaster using the instruction manual.

Conversations during the coaster project and later classroom observations highlighted content knowledge gaps for teachers, especially understanding how gravity, mass, and speed interact. Teachers had misconceptions about how mass of a marble influences travel on the track; confusing how potential energy, kinetic energy, force, and speed differentiate. We developed new design modules for teachers requiring them to build and test ramps at various heights to launch small and large marbles first to hit a target and later to hit target with enough force to break a napkin. Measurements from designs with small marbles were used to build mathematical models predicting mechanics with large marbles. Scaffolding for modelling was an Excel file with embedded equations and dynamic trajectory graph. Model were tested against observations.

We knew scientific and mathematical content of the modules would challenge teachers. Teachers individually completed web-based versions and experienced productive struggle similar to that experienced by their students. Teachers completed modules a second time in groups during professional development where peers and content experts provided scaffolding as needed and worked to adapt portions of modules to middle grade students.

### **Force Concept Inventory Findings**

We focus here on three Force Concept Inventory (FCI) items (see Table 2). These items were designed to uncover misconceptions related to gravitation, specifically “gravity intrinsic to mass”, “heavier objects fall faster”, “gravity increases as objects fall”, and “gravity acts after impetus wears down”; misconceptions notoriously difficult to change<sup>24-26</sup>.

Table 2 displays means, standard deviations, significance, and effect size statistics from paired samples t-tests. There was a medium-large significant improvement [ $t(14)=3.05$ ,  $p<.05$ ,  $d=.77$ ] on the first item, a small-medium non-significant improvement [ $t(14)=1.38$ ,  $p>.05$ ,  $d=.41$ ] on the second item, and no improvement on the third. The first two items are most similar in context to the ramps and marbles modules as they involve questions about speed or acceleration of metal balls of the same size but different weight and a stone, respectively, being dropped from a roof. The third item requires respondents to identify forces acting on a ball as it moves through a frictionless channel, requiring transfer to a substantially different context than that provided by the ramps and marbles modules. One iteration of our model remediated a stubborn misconception in a specific context. We focus future iterations on transferring that learning more broadly.

## **Focus Group Themes**

End of summer focus group discussions with working teams of participating teachers were recorded and transcribed for coding and analysis in NVIVO, a qualitative data analysis software package. Discussion prompts focused in year 1 on perceived impact on teachers, anticipated impact on teaching practice, and anticipated challenges, and in year 2 focused on changes in teacher perspective and experienced impact on teaching practice and student learning.

To accomplish participant checking, preliminary description of themes from year 2 were shared with the 17 teachers who participated in focus groups that year (see Table 3). All teachers indicated these themes described them “very well” or “extremely well”, and no teacher had substantive additions or contradictions.

The most relevant theme was tied to experiencing design lessons as learners. Teachers talked about productive struggle with critical mathematical and science content, understanding the process deeply enough to guide their students effectively, the importance of redesign, and how this professional development experience was dramatically different and more valuable than others they had experienced.

## **Discussion**

Valid and reliable assessment of teacher content knowledge coupled with available content expertise of project personnel is a strength that gives rise to a challenge in determining how to address and scaffold content needs of prospective groups. How much do middle school mathematics teachers need to know about science, and how much do science teachers need to know about mathematics, in order for them to meaningfully plan integrated instruction? In the context of somewhat low content knowledge scores and specific content deficiencies, especially outside of teachers’ primary content area, how do we address content needs in safe and authentic ways? Many teachers are uncomfortable opening their content knowledge gaps to remediation. We found sustained engagement with our teachers critical, and teachers were more open to remediating gaps in the context of design projects focused on similar gaps their students are likely to have; gaps that just happen to overlap with content knowledge teachers need to develop more deeply themselves.

A compelling issue across focus group themes was productive struggle—in particular, that of teachers—seen in their comments about themselves, student effects, and parent responses. Project TESAL teachers are experiencing productive struggle authentically, and their misconceptions are similar to those of their students. Design based learning provided an experimental framework that was familiar to them and enabled further, richer experimentation that was targeted at understanding misconceptions and could be adapted for use in their classrooms.

Productive struggle is a key feature in learning that is conceptual, robust, and transferable<sup>25-27</sup>. The idea of struggle leading to learning is not new<sup>28-33</sup> and research supports the benefits of learning through some form of struggle<sup>34-36</sup>. In mathematics, productive struggle has been noted as a fundamental “feature of teaching that consistently facilitates students’ conceptual understanding”<sup>34</sup> and is highlighted as an essential practice for strengthening the teaching and

learning of mathematics”<sup>37</sup>. Yet is it difficult to understand and implement. Many teachers have not experienced this as a learner themselves. Part of that difficulty is the dominant cultural view of mathematics and science as only for “some people” or as a static body of knowledge that must be learned rather than created. Such beliefs often lead to instructional practices that tend to remove struggle from students rather than leverage it for learning<sup>38</sup>. Productive struggle hinges on instructional tasks that investigate content and create knowledge in meaningful ways. Therefore, instructional approaches that engage students in productive struggle also challenge existing notions of what it means to do mathematics and science; this can be uncomfortable for parents, other teachers, administrators, and some students. Making these experiences and related difficulties explicit may support teachers developing understanding of how to effectively engage their students in productive struggle.

*Table 1: Science and mathematics initial content knowledge means (standard deviations).*

	DTAMS Science Content Score	DTAMS Math Content Score
Science Teachers n=11	16.18 (4.24)	13.09 (3.73)
Mathematics Teachers n=8	11.38 (3.96)	17.50 (2.88)
Special Education and Technology Teachers n=5	11.20 (4.94)	12.20 (3.70)
Total N=24	13.54 (4.72)	14.38 (4.02)



*Table 2: Force Concept Inventory (FCI) items pre/post means (standard deviations), significance level (p), and effect size (Cohen's d).*

FCI Item	Pretest	Posttest	p	d
Two metal balls are... dropped from the roof...	.53 (.52)	.93 (.26)	.01	.77
A stone dropped from the roof...	.33 (.49)	.53 (.52)	.19	.41
A ball is shot at high speed into the channel...	.07 (.26)	.00 (.00)	.33	.27

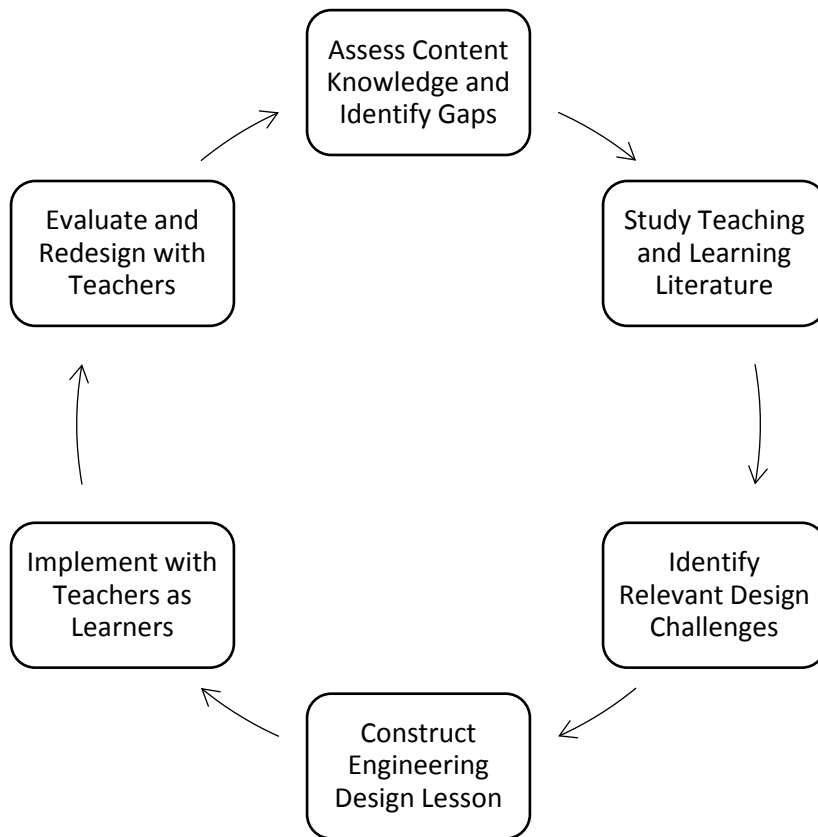
Note: Two-tailed significance from paired samples t-test; n=15 as not all teachers were present for both pretest and posttest.

Table 3: Focus group themes.

<p>Barriers anticipated from last year for the most part were not actually problematic. In fact some teachers described finding this approach actually saved time because multiple CSOs are addressed in clusters rather than one at a time. In a couple cases, perceptions from their colleagues were challenging and in several cases their colleagues were very supportive. The main barrier turned out to be physical space to house the student projects themselves.</p>
<p>Anticipated student impact on motivation and learning did occur. Teachers universally agreed on this. Impact on motivation was most pronounced for inclusion students and those who had in the past been difficult to engage. "Advanced" students often struggled with the lack of directions but teachers all agreed this was productive struggle even if their students may not have seen it that way. In some cases teachers experienced pushback from parents who thought their children should be learning the way that they had learned.</p>
<p>Most teachers shared this approach with other teachers in their building, most often with their team members and curriculum coaches. In a few instances, colleagues or administrators were skeptical at first but then saw the benefits of the approach in how students responded. There was unevenness in support experienced from administrators ranging across consistently positive from the start, to starting skeptical but beginning to be won over as they saw impacts, to not being present or engaged in teachers classrooms to even see what was happening.</p>
<p>Teachers perspectives coming in to this year were generally more focused as compared to the first year. Most and maybe all teachers experienced productive struggle, especially in year 1 and 1st part of 1st week this summer. This was often very uncomfortable for them, to the point that many considered dropping out of the program, especially when it was not clear to them why we were asking them to do things. They felt their struggle sometimes went beyond what was productive. There was general consensus that now they understand the big picture of experiencing struggle as learners so that they can guide their students through the process more effectively. They almost uniformly see great value in this and talked at some length about how this PD is dramatically different and more valuable than most other PD they have experienced.</p>
<p>Many, but not all, teachers described the approach spreading in at least one of the following ways. A. To other lessons beyond those they were required to do. B. To other teachers in their teams. C. To other teachers across most or their entire school.</p>
<p>Suggestions for future: -More detailed feedback on lessons, both in planning stages and in observations. They want both validation and constructive criticism. -Some more communication on "why" and the big picture from our perspective, although they understand there are good reasons for not explaining everything.-More connections of specific math CSOs to the science and engineering. - More opportunity for them to see each other's lessons. One suggested videotaping observations and showing those to the group, or even them visiting each other's rooms although that may not logistically be possible. -Scaffolding their presentation of this approach to their colleagues, possibly with a PowerPoint that includes general description of approach, literature base for its effectiveness, and evidence from multiple teachers in this program of its effectiveness. They could then share this with their entire school at one of their early out PD afternoons. -Scaffolding development of parent involvement and buy-in in some way.</p>



*Figure 1: Project Project TESAL Program Design and Participating Teachers Building Roller Coaster*



*Figure 2: Engineering Design Process Applied to Teacher Content Knowledge*

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