BYOE: Activities to Map Intuition to Lumped System Models

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Abstract

The objective of this series of experimental activities is to create a stronger qualitative connection between observed behaviors of simple systems and the equations, terminology, and graphical methods used to describe and represent them. This work is motivated by an observed inability of students to qualitatively model and predict how a real-life system will behave, despite an understanding of such models in homework and lecture settings. Thus, there is a disconnect between understanding how real-life physical behaviors map to the elemental and system equations of idealized models. The experiments, presented herein, and corresponding qualitative discussion among peers are designed to precede and pair with subsequent course discussion of the concepts involved. During the follow-on lectures, the instructor references, and may repeat, these demonstrations to link the student's observations to the appropriate terminology, equations, and graphical representations being taught. The four short experiments presented in this paper are described briefly below; a selection of these activities will be demonstrated at the ASEE conference.

Canoe Coast-down: Students study video taken of a canoe in "coast-down", as its velocity decays. The canoe exhibits a first-order response to this initial condition. Students hypothesize models for their observations, and thus begin developing the skill of system identification. During a subsequent class the instructor leads the students through a more complete analysis.

Playdough Hot Potato: Students are given playdough that represents a "hot potato" and asked to come up with ways to make it cool down as fast as possible. In a follow-on lecture the instructor introduces the parameters of thermal resistance, thermal capacitance, time constant, and step input size; and links the cooling methods proposed by students to the corresponding parameter(s). The open-ended rich solution set of this challenge offers to open discussion in many directions, including the limitations of lumped system modeling.

Fluid in a Tube: This experiment illustrates the step response of a second order fluid system as a function of its damping ratio. Students are asked to observe fluid oscillations in a tube and explore how the size and duration of oscillations varies with restrictions to air flow at the end of the tube. During a follow-on lecture the instructor shows plots of the oscillations observed in this activity for both high and low damping ratios. This provides a lead-in for more extensive discussions on the characteristics and behavior of second order systems.

Slinky and Mass: A small mass attached to a mini-slinky forms a simple spring-mass system which is used to map the observed time domain response of a minimally damped second order system to the graphical representation of its frequency response. Students move one end of the spring up and down, observe the response of the mass on the other end, and qualitatively describe the system behavior for a range of frequencies.

Introduction, Background and Motivation

Observing students working on projects for general design classes, both introductory and capstone, and as a part of extracurricular student design teams, reveals that many students do not apply the analytical techniques learned in earlier coursework. Our goal is to better prepare students to integrate such analysis with the everyday engineering problems they face, outside of the classroom. Two possible explanations for failing to apply previously learned analytical techniques are: 1. students did not retain the knowledge, and 2. students do not recognize when it is appropriate to apply the “tools” in their analytical “toolbox” [1].

The importance of repetition in learning retention is well documented within the literature [2-4] and can be summarized using the forgetting curve [5]. The forgetting curve indicates that to maximize retention, any key concept must be repeated multiple times over the course of a term, beginning with repetition intervals of high frequency and gradually decreasing in frequency [6]. Therefore, the key concepts must be introduced and then reviewed several times within the first few weeks of the term to maximize long term retention. Unfortunately, some of the key concepts in an introductory lumped systems modeling class, such as frequency response and the behaviors of second order systems, are analytically complex and are typically taught after students have acquired the necessary mathematical skills, which is often very late in the term. This traditional approach does not provide adequate time for the necessary repetition within the course timeframe to maximize retention of these critical concepts.

The human capacity to recognize cause and effect relationships, and to associate a name to those observed behaviors, precedes our ability to create a physics-based mathematical model to precisely predict behavior. For example, a young child quickly develops an understanding that if you throw a ball up it will always fall down; that if you throw it harder it will stay in the air longer; and the angle you throw it at, combined with how hard you throw, determine how far the ball travels. This mental mapping of behaviors, termed “constructivism” [7], happens many years before a student can understand the equations for projectile motion. This same sequence of learning in young children, where the capacity for qualitative learning significantly precedes the ability to form a quantitative model, can facilitate learning at any age. Research in active learning through techniques such as model-eliciting activities, hands-on teaching, and predict-observe-explain, supports the value of qualitative experiential learning to enhance understanding [8]. Recognizing that topics can be understood qualitatively allows even the most complex topics to be brought to the beginning of a course allowing for more repetition and therefore a greater likelihood of retention. In addition, developing and checking conceptual understanding early in the term provides an opportunity to identify and address misconceptions that could otherwise persist throughout the course.

Assuming students retain the knowledge of what analytical “tools” they have available in their “toolbox” the next question is: Do they recognize when it is appropriate to use them [1]? For a lumped systems modeling course this requires the skills of *system identification* and *model formation*. Arguably, the skill of deciding how to model a system such as recognizing energy flow pathways and identifying what simplifying assumptions can be made, is a skill that takes years to develop, but a greater emphasis on *developing* these skills could be incorporated *throughout* such a course.

The intent of this series of experimental activities is to help students in an introductory lumped systems modeling course to create a simple mental model of system behaviors very early in the term [7], so later that term, when the mathematical modeling and graphical representation of the system behaviors are taught, students can relate this to the qualitative framework they have already formed [1]. This paper proposes that these simple activities presented early in the term will improve retention and understanding, and also improve utilization of course concepts in post-course design work.

Experiential learning techniques can be time consuming and thus challenging to incorporate into collegiate courses with a packed syllabus. Lab equipment can be expensive to purchase and maintain. Further, instructors may presume that students have already formed simple mental models of system behaviors from earlier coursework or life experiences. To minimize these possible implementation barriers, the criteria used to create these activities are that all students can experience the hands-on activity *concurrently*, within a traditional lecture hall. Thus, the experiments must be inexpensive, simple to set-up, and brief. Also the activities are purely qualitative and use common language, in addition to traditional systems terminology, so the activities can precede formal lessons on the topic. The intent is not to replace traditional quantitative modeling labs but rather to provide additional early-term concept exploration.

These experiments have been tested and are in the process of being implemented into an existing lumped systems modeling course. The efficacy of these activities will be measured and monitored over several years, with the outcomes presented in a follow-on paper. This paper is organized into 4 distinct experiments. For each experiment we include an overview, instructions for the students, teaching cues for incorporating the activity into the lecture, and some ideas for revisiting the experiment later in the term as a more traditional lab.

Experiment 1: "Canoe Coast Down"
First Order System Identification from Observed Step Response

Overview

The outcome of this activity is an introductory understanding of system identification. A video is presented where a canoe is paddled to a target speed and then coasts down, traveling perpendicular to a stationary on-shore video camera (Figure 1). The canoe exhibits the response of a first-order system, and in this case, the students are viewing its initial condition response. Small groups of students work together to hypothesize models for this system. This activity can be combined with the other activities in this paper as an introductory "mini-lab", used alone as a breakout activity within a lecture, or as a group homework assignment. In a follow-on lecture the instructor revisits this exercise using it as a basis for simple system identification, and analysis of a first-order system.

Figure 1: "Canoe Coast-down" video screenshot

Student Instructions
Watch the video provided [9]. Working with your group discuss why the canoe slows down, what parameters determine how quickly it slows down, and what the shape of the velocity verses time curve might look like. Once your group comes to an agreement record your answers on the response sheet (Figure 2).

(1) Why does the canoe slow down?

(2) What parameters determine how fast the canoe slows down?

(3) Qualitatively sketch your predicted shape of the velocity vs time curve as the canoe slows down, once paddling stops.

Figure 2: "Canoe Coast-down" Student Response Questions

Lecture Discussion
In a follow-on lecture the instructor revisits this exercise using it as a basis for simple system identification and analysis of first-order system step response. The responses from the student activity should be compiled in advance and summarized in preparation for the lecture. Open the lecture with the answer to question 3 "the shape of the Velocity-Time curve during canoe coast-down" by displaying velocity data (Figure 4) [9] collected using a submerged flowmeter that was mounted on the canoe (Figure 3).

Figure 3: Flow meter mounting for data collection

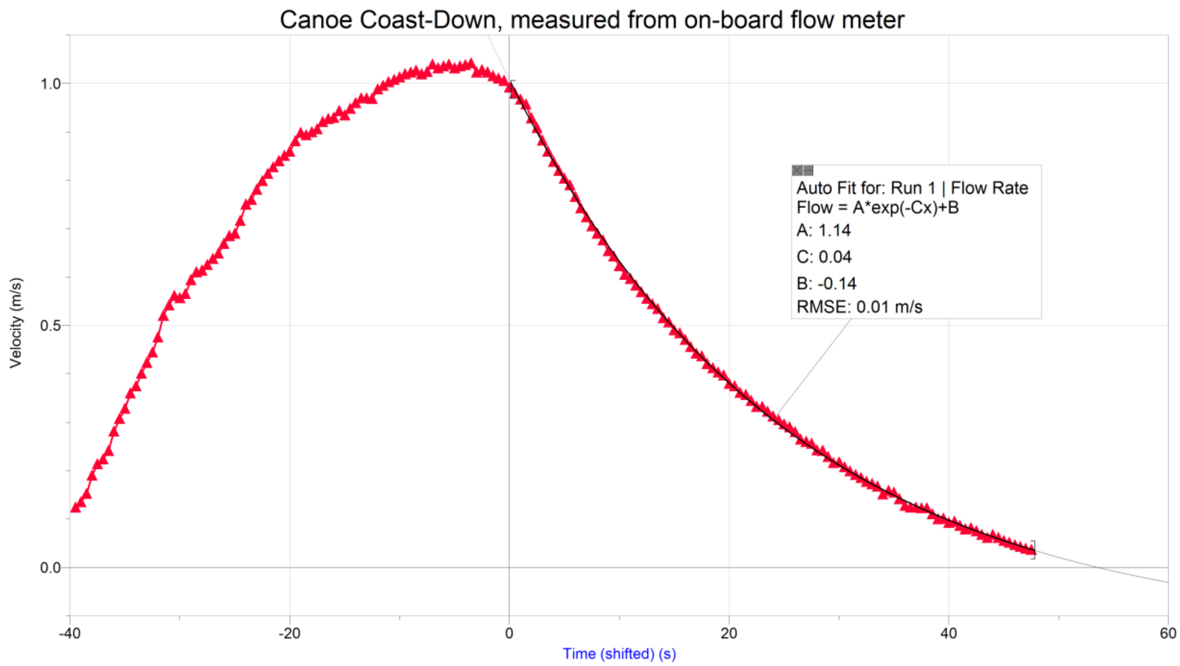


Figure 4: Canoe Coast-down starts at time 0. Data collected from onboard flowmeter. Exponential curve fit displayed as black line.

The data displayed closely matches an exponential curve fit to equation 1, with an RMSE of 0.01 m/s.

$$v(t) = 1.14\,m/s\ e^{-t/25s} - 0.14\,m/s \qquad (1)$$

Lead the students through the discussion to identify what each element of the equation represents to arrive at the generalized equation:

$$v(t) = (v_0 - v_{ss})e^{-\frac{t}{\tau}} + v_{ss} \qquad (2)$$

Where (v_0) is the initial velocity in m/s, (v_{ss}) is the steady state velocity, thus $(v_0 - v_{ss})$ is the magnitude of the step response, and τ is the time constant in seconds.

Once students are in agreement that this mathematical model closely fits the data, they can be asked how it lines up with their predictions from question 3. Returning to question 1, the discussion should lead to the conclusion that drag or friction is the reason the canoe slows down. Continuing to question 2, "parameters that determine how fast the canoe slows down" some potential student responses are velocity; mass of the canoe and passengers; area, shape, and/or surface roughness of the canoe; and drag coefficient or viscosity.

Depending on student's prior knowledge and if this activity is presented early in the term, as intended, students may have been puzzled on how to proceed from question 2 to 3. This brings the instructor to the crux of the activity. Explain that there are many ways to model a system with varying levels of fidelity. Sometimes a simplified model, such as assuming a constant drag force, provides an acceptable level of error in predicting behavior. If the drag is constant, -*c*, the form of the model's analytical solution is a straight line. If the drag scales linearly with velocity, -*bv*, the form of the model's analytical solution is a decaying exponential curve. And if the drag scales with velocity squared, $-av^2$ the deceleration is more rapid (Figure 5). The skill of *predicting* when drag will scale with velocity linearly vs quadratically is a topic for a fluid dynamics course, but *recognizing* the form of drag is well within the scope of a systems modeling course. As a topic wrap-up, introduce the concept that a system demonstrating exponential decay as a step-response to some non-zero initial condition is classified as a first order system, and behaves as such because it has just one energy storing element. It is also worth noting to students that the details of how to arrive at the analytical solution from a conceptual model will be covered later in the term.

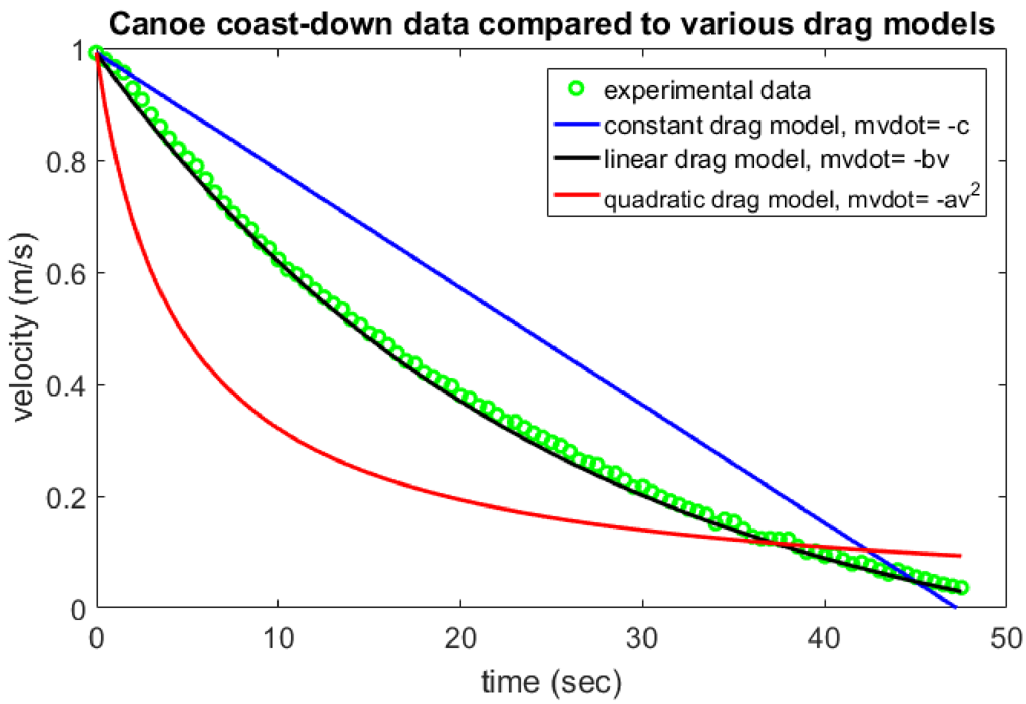


Figure 5: Canoe Coast-down data compared to various drag models

<u>Extension</u>

If time permits, the student activity can include students extracting velocity vs. time data from the video recording using open source tracking software [11]. A google site contains example handouts for the Canoe lab, videos and data acquired for use in activities described herein [9]. The extracted information can then be used to inform the formation of their hypothetical system model. This approach welcomes a rich discussion on experimental techniques because the video data provided to students shows the canoe velocity with respect to the shore, while the flow meter data presented in the follow-on lecture shows canoe velocity with respect to the stream.

An interesting side discussion is why the steady state velocity is not zero. Ask the students if that makes sense? If so what does it represent? Help students arrive at the conclusion that the flow meter is measuring the velocity of the canoe with respect to the water, and the curve fit indicates that at steady state the canoe velocity could be slightly slower than the stream velocity.

This activity can be revisited later in the term as a fully developed lab in which students derive data for modeling the canoe response using three different methods to measure velocity: GPS, video, and a flow meter [9] . There are a number of possible approaches and associated objectives for data analysis for this activity as a lab.

1) Compare models derived from the three measurement methods (video, GPS, flow meter). Identify sources of error/uncertainty. This could focus on concepts of precision vs. accuracy from an earlier class. Current from the river could be extracted from video data by also tracking a piece of debris in the river. Investigate upstream vs. downstream drag.
2) Determine whether a linear approximation of friction, bv, is valid and extract the system time constant from the data.
3) Introduce and use numerical integration to determine a drag coefficient, a, assuming nonlinear friction, av^2 best fits the data.

Experiment 2: "Playdough Hot Potato"
Parameters Governing the Response of a Thermal First Order System

Overview

The outcome of this activity is to have students map already existing intuition for cooling thermal systems to terminology used to describe a first order system, specifically thermal resistance, thermal capacitance, time constant, and input step size. Small groups of students are instructed to conceive a method for cooling an imaginary hot potato as quickly as possible. They are given a jar of playdough to create a physical representation of their ideas. As a group, they articulate what parameters they changed and why the changes cause the potato to cool down faster. This activity can be done with the other activities in this paper as an introductory "mini-lab", used alone as a breakout activity within a lecture, or as a homework assignment. A follow-on lecture finds the common denominators in the student's responses and applies the appropriate system modeling terminology.

Student Instructions

Imagine this playdough is a hot potato. You are hungry and want it to cool down as quickly as possible. Shape, mold or modify the playdough to a form that you think will cool down the fastest. Also modify the environment to accelerate the cooling. Use of additional equipment and props is encouraged. Identify what parameters you changed to increase the cooling rate and how those parameters change the cooling rate. Then, as a group, turn in responses to the following questions (Figure 6).

(1) Submit a photo of your final "potato sculpture" which you think would cool down quickest. Describe WHY you think the change would make the "potato" cool down faster

(2) Briefly describe (in words or sketches) how you modified *the environment* to make the "potato" cool down faster. What aides did you use? Describe WHY you think the changes would make the "potato" cool down faster

(3) List one real life example where designing a system with rapid cooling is important? (Besides cooling down hot food so you can eat it sooner)

Figure 6: "Hot Potato" Student Response Questions

Lecture Discussion

In a follow-on lecture the instructor exhibits the different playdough sculptures (Figure 7), also the proposed environmental modifications, and ask students to identify what features were common among these.

Figure 7: Example of Students' "Hot Potato sculptures"

This precursor exercise prepares students to make sense of what parameters govern the decaying exponential response of a 1st order thermal system cooling down:

$$T(t) = (T_0 - T_{ss})e^{-\frac{t}{RC}} + T_{ss} \qquad (3)$$

(T_0) is the initial temperature in °C, (T_{ss}) is the steady state temperature as time approaches infinity, thus $(T_0 - T_{ss})$ is the magnitude of the temperature step response, and RC forms the time constant (τ), in seconds. Thermal resistance (R) represents how easily heat moves from one object to another and thermal capacitance (C) represents thermal potential energy storage. In a lumped system modeling course if there is only one mode of heat transfer, regardless of whether it is convective or conductive, it can modeled as one lumped resistance value. For convection resistance R=1/(hA) where the variables are surface area (A) and convection coefficient (h). The details of the parameters that determine a convection coefficient are beyond the scope of the course, but recognizing that R scales inversely with surface area and also velocity (because h increases with velocity) is important to map the abstraction of thermal resistance to a student's already strongly-ingrained intuition that if it is spread out or if air velocity increases an object cools down faster. Similarly acknowledging that convection coefficient (h) is also a function of the fluid properties supports the intuition that if you put a hot object in water (even if the water is the same temperature as the surrounding air) it will cool down faster. The thermal capacitance (C) is derived from the mass (m) and material's specific heat (Cp). Students should already have a strong intuition that larger objects, with a larger C, take longer to cool down.

After listing the student's modifications on the board, the instructor can present the thermal step response equation (3), and then map system modeling terminology to the student's ideas by populating a table like the example provided in Figure 8.

Student's hot potato modification	**Lumped system terminology**	**Description of how they are mapped from student intuition to equation**
Dig a hole in the middle of the playdough	Decreases R value	R=1/(h*A) for convection Increasing surface area, A, decreases R
Flatten the playdough	Decreases R value	R=1/(h*A) for convection Increasing surface area, A, decreases R
Add ridges, grooves or cuts to the playdough	Decreases R value	R=1/(h*A) for convection Increasing surface area, A, decreases R
Blow on the potato	Decreases R value	R=1/(h*A) for convection Blowing increases the convection coefficient h
Dunk the potato in water	Decreases R value	R=1/(h*A) for convection Water has a higher convection coefficient, h, than air
Set on metal or other conducting surface	Decreases R value	R = L/(kA) for conduction Find a material with a large conduction heat transfer coefficient, k
Put it in the fridge/freezer	Increases magnitude of the step response	A larger temperature difference by changing the boundary condition will increase cooling rate (with no change to the time constant)
Cut playdough up into smaller chunks	Decreases C (Or Decreases R depending on how it is modeled)	Cutting the playdough into small masses can be modeled as making many separate systems with smaller C (C=m*Cp), although a somewhat larger R. It can also be modeled as a constant total C but an increase in the total surface area, which decreases R

Figure 8: Mapping intuitive concepts for cooling a "Hot Potato" to Lumped Systems Terms

<u>Extension</u>

This activity can also segue into a discussion on lumped versus distributed system modeling. Ask students to identify which playdough shapes would cool down uniformly and which would remain hot in the middle, while the outer surface cools. In lecture, the instructor introduces the concept of lumped vs distributed thermal systems and what types of shapes can be reasonably modeled as lumped systems and why.

With some additional equipment and planning this exercise could be repeated later in the course as a lab with data collection to monitor temperature response. It could also be repeated as a mini design competition to see which models cool down the quickest.

<u>Experiment 3: "Fluid in a Tube"</u>

<u>Step Response of a second order system with a variable damping ratio</u>

Overview

The outcome of this activity is to develop intuition for how the damping ratio effects the step response of a second order system. Pairs of students are asked to observe fluid oscillating in a tube and explore how the size and duration of oscillations varies with restricting air flow at the end of the tube. This can be combined with the other activities in this paper as an introductory “mini-lab”, used as an independent breakout activity within a lecture, or as a homework assignment. During the follow-on lecture the instructor shows plots of the oscillations observed in the activity for both high and low damping ratios, providing a lead-in for more extensive discussions on the characteristics and behavior of second order systems.

Student Instructions

Take the provided piece of clear flexible tubing and fill it about 2/3 full of water using a funnel and the colored water provided (Figure 9). Shift the water to one end of the tube by lifting one end and then firmly place your thumb over the end (Figure 10). Create a water level offset by holding the two ends of the tube at the same height (Figure 11). Release your thumb to observe the behavior as the water settles back to a common level. Describe the motion of the water as it settles. Explore changes in the response as you restrict the air exiting the tube by folding the opposite end of the tube over before releasing your thumb on the other end (Figure 12). You can also restrict the airflow by only partially uncovering the end of the tube with your thumb. Then, as a group, turn in responses to the following questions (Figure 13). Feel free to experiment with the fluid-in-a-tube system more to develop your question responses.

Figure 9: Filling Tube

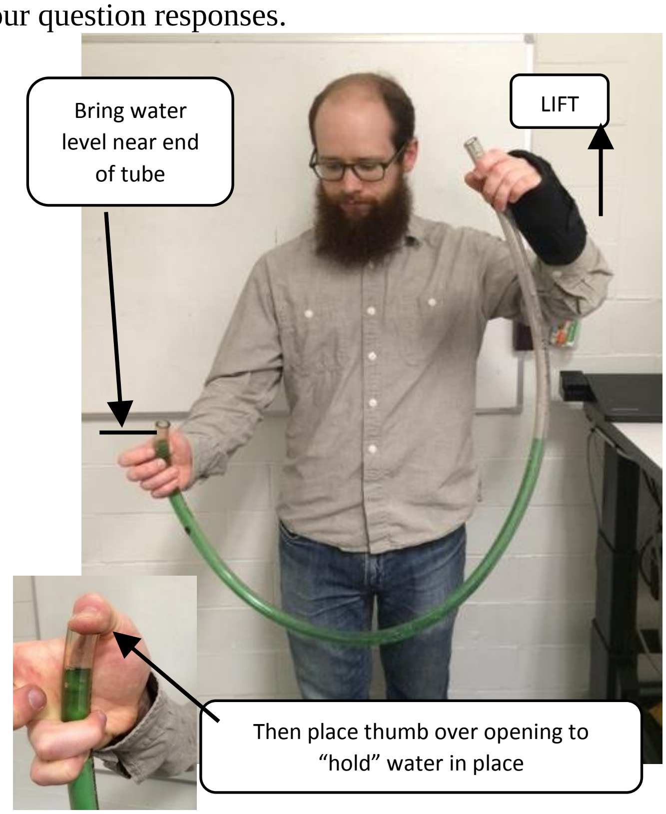


Figure 10: Shifting the water to one end of the tube

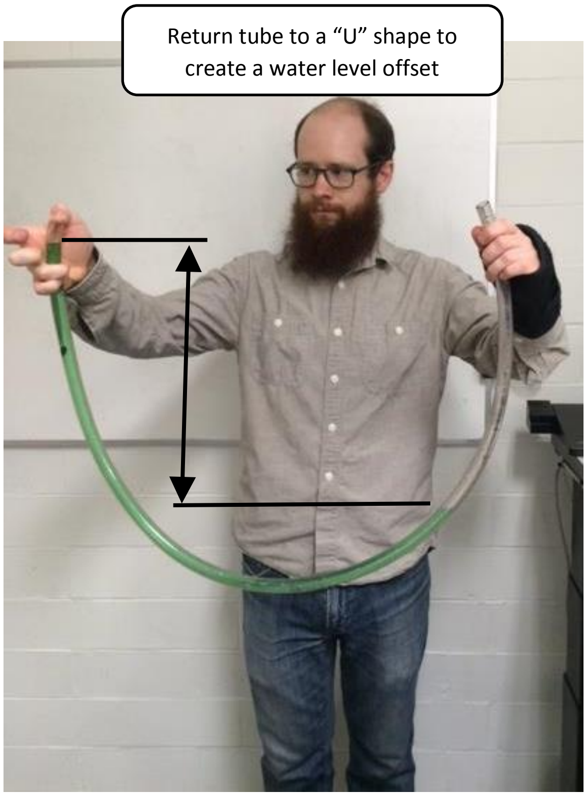


Figure 11: Creating the initial condition

Figure 12: To restrict airflow fold over the tube

(1) Describe the motion of the water after you release your thumb. How is it affected by restricting the airflow at the end of the tube?

(2) What parameters govern the behavior of this system?

(3) How is the step response behavior of this system different than the step response of the canoe? What aspect of a system might determine how it responds to a step input?

(4) List some real life examples of systems that display this type of oscillatory behavior.

Figure 13: “Fluid-In-A-Tube” Student Response Questions

<u>Lecture Discussion</u>

During a follow-on lecture the instructor shows plots of the damped oscillations observed in the activity (Figures 14 & 15). Note the plots should be prepared in advance, which can be done with motion tracking software. The open source “Tracker” software was used to create the plots in this example [10].

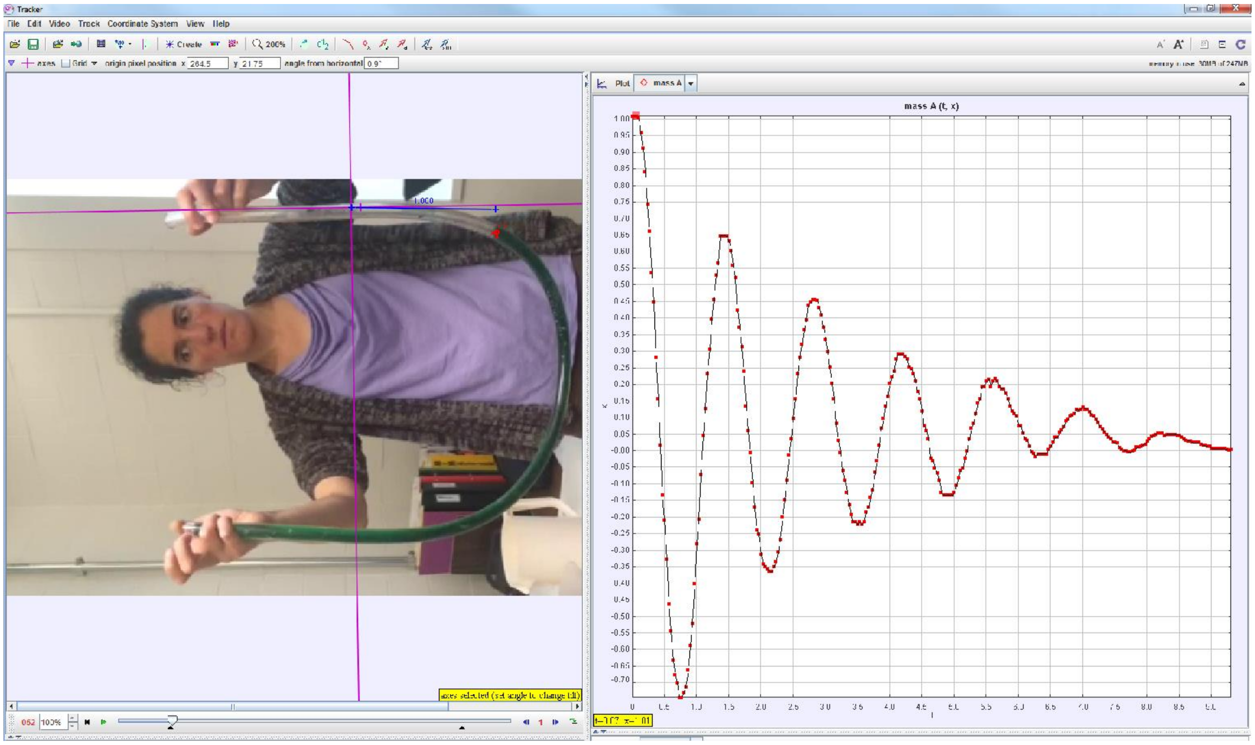


Figure 14: Response when the tube end is fully open demonstrates a low damping ratio

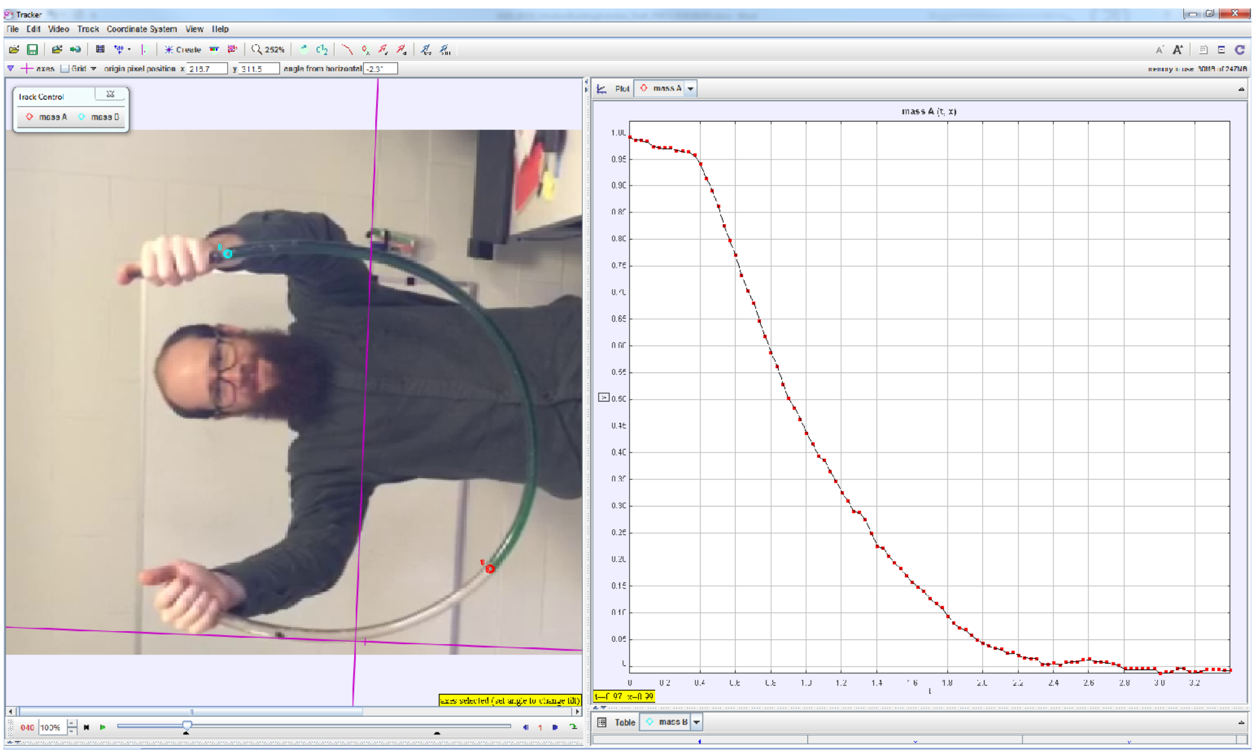


Figure 15: Response with just a pinhole opening at tube end shows a high damping ratio

Prompt the class to consider why this system oscillates after a step input but the canoe coast-down did not. Introduce the concept of a second order system and how system order relates to the number of independent energy storing elements in the system. Also introduce the term steady state, and damping ratio, ζ, which characterizes the amplitude and number of oscillations in a second order system before it reaches steady-state (Figure 16).

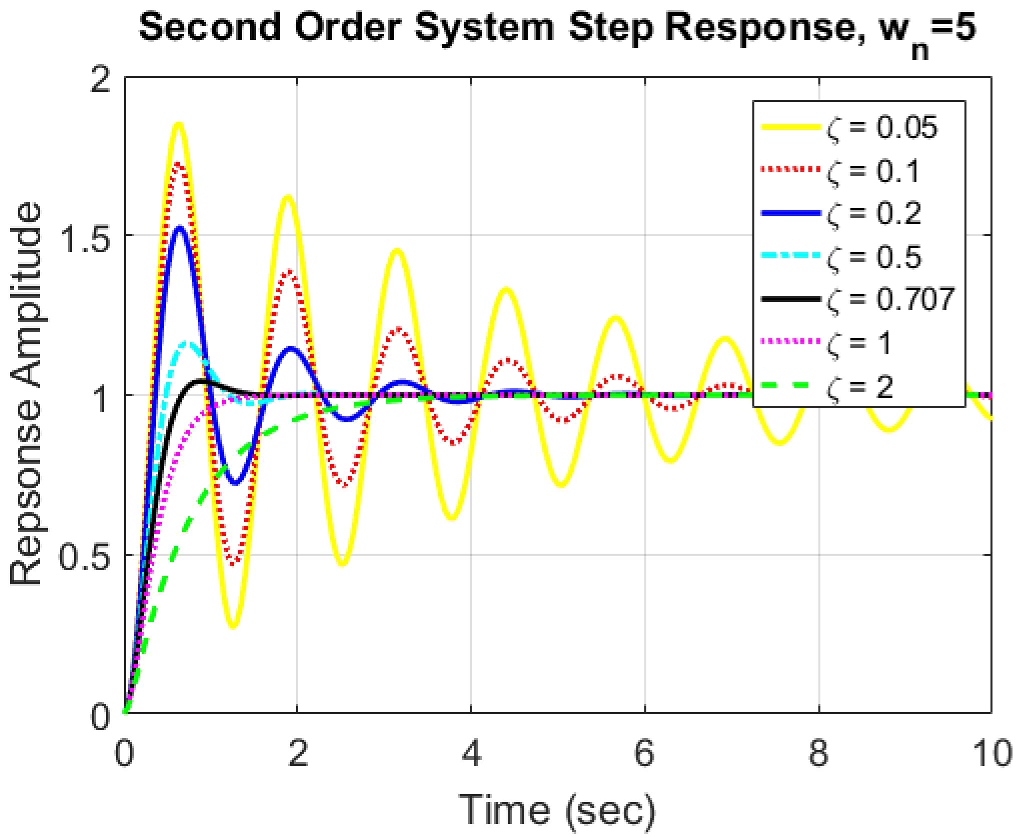


Figure 16: Second order system step response for a range of damping ratios

<u>Extension</u>
Natural and resonant frequencies can also be introduced here, as well as methods for determining damping ratio and natural frequency from experimental data.

Further extending this activity to a full lab could include analyzing the data from the motion tracking software to extract the damping ratio and natural frequency for this particular system. It may also include modeling the system analytically, comparing the measured natural frequency to predictions, and determining the resistance value from the measured damping ratio.

Experiment 4: Slinky and Mass
Mapping Observed Time Domain Response to Frequency Domain Plots

<u>Overview</u>
The outcome of this activity is an understanding of how a time domain response maps to its corresponding frequency domain representation. The system is a mass attached to a small slinky. A student holding the spring creates the *input* forcing function by moving it up and down, and the observed *output* is the position of the mass as a function of time. Three distinct behaviors for this minimally damped second order system can be identified with this simple activity: the pass

band region, the resonant response region, and the attenuation region. Students can work solo or with a partner as they explore. This activity can be aggregated with the other activities in this paper as an introductory "mini-lab", used as an independent breakout activity within a lecture, or used as a homework assignment. A follow-on lecture compares the crude graph developed from observations in this activity to a detailed frequency response plot for an underdamped second order system.

Student Instructions

Hold the spring at the open end so the mass is hanging down on the other end (Figure 17). Move your hand up and down to create an input to this spring-mass system and observe the motion of the mass, which is the output of system. Try moving your hand extremely slow, very fast, and then at several speeds in between. As you explore the effects of varying your hand movement speed (frequency) try to keep the distance your hand travels (amplitude) relatively consistent. Observe both the distance the mass moves relative to your hand movement (magnitude), and if the mass is moving in the same direction at the same time as the mass, or opposite (in phase, or out of phase, respectively). Complete the response questions (Figure 18)

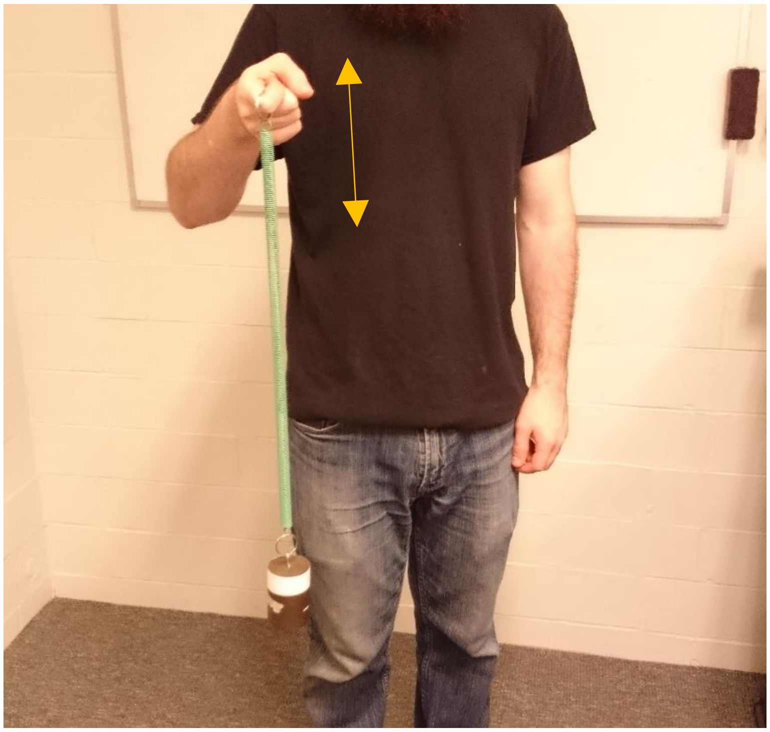
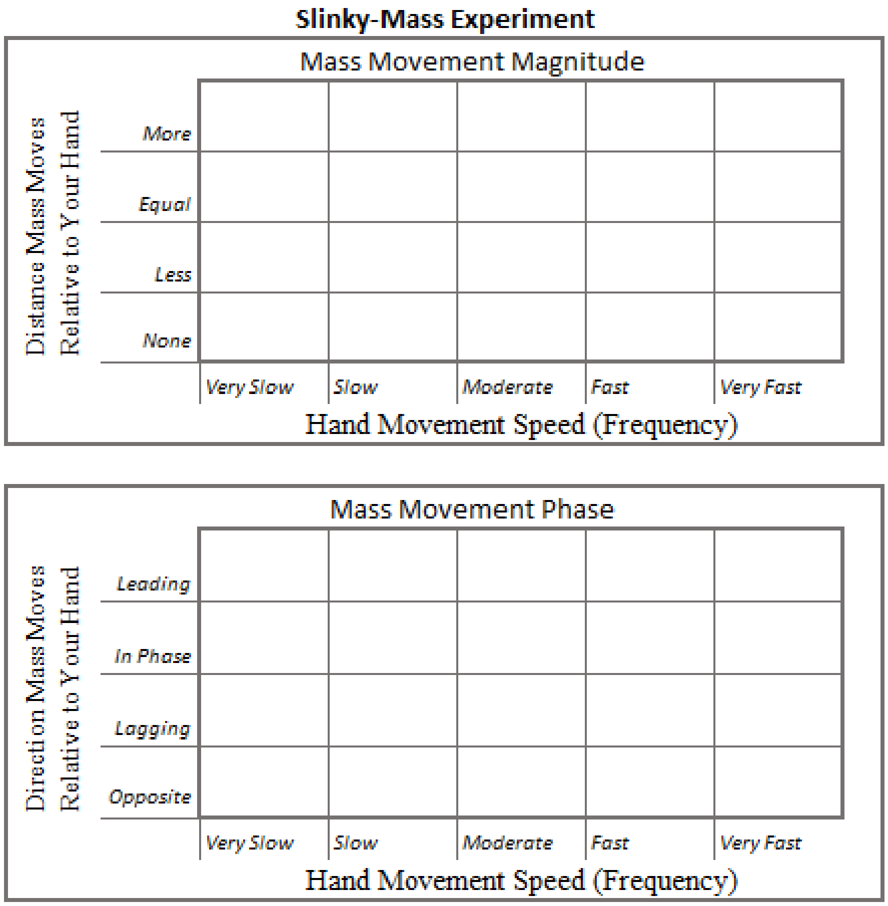


Figure 17: "Slinky and Mass" Experiment Set-up

(1) After exploring the slinky-mass system, test at the 5 speeds in the chart below (very slow to very fast) and populate the plot with *rough* quantification of the distance and direction of the mass movement for each speed.



(2) Apply a step function input to this spring-mass system by abruptly moving your hand from one height to another. How is the behavior of this system similar to the third experiment "Fluid-in-a-Tube"? How is it different?

Figure 18: "Slinky and Mass" Student Response Questions

<u>Lecture Discussion</u>

In a follow-on lecture use this experiment as a lead-in to discussing frequency response and bode plots. Display one or several of the student's graphs from the experiment (Figure 19). Lead the class to the conclusion that there were generally three types of behaviors observed: when the mass movement was equal to the input movements, when the mass did not move despite the hand movement, and when the mass moved much more than the hand movements. At this point introduce the terminology and graphical representations to illustrate the frequency response of a system mapped to their experience with the activity (Figure 20): The pass band region where the output matches the input both in phase and magnitude occurs at very low frequencies for this system. The attenuation region where the output moves very little despite the amplitude of the input occurs at very high frequencies. And the resonant region where the output amplitude is much larger than the input amplitude is at some mid-range frequency. It may be worth spending the time to repeat this exercise and bring students attention to the phase response for various frequencies as it is a more subtle observation.

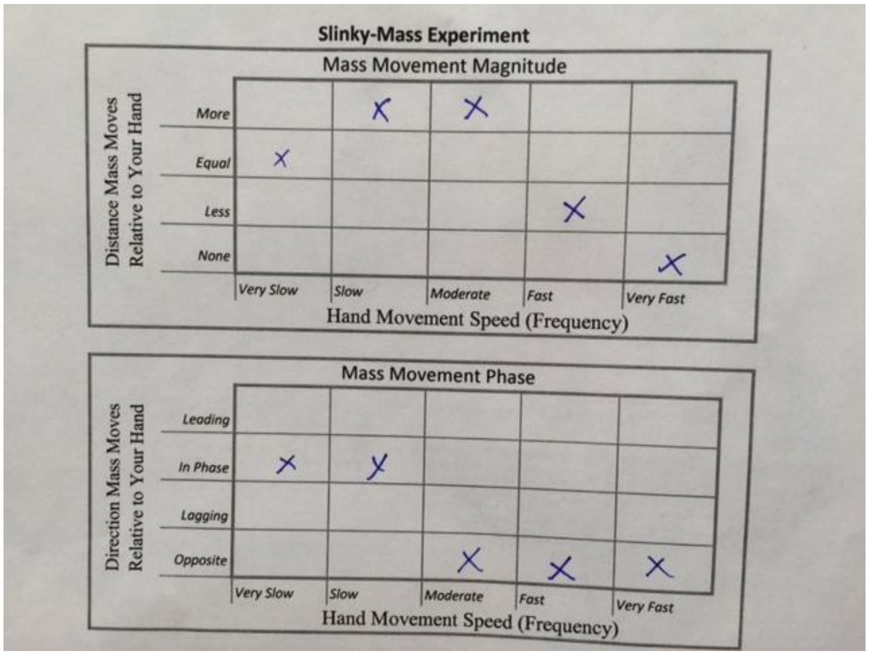


Figure 19: Sample of a student's qualitative data for slinky-mass experiment

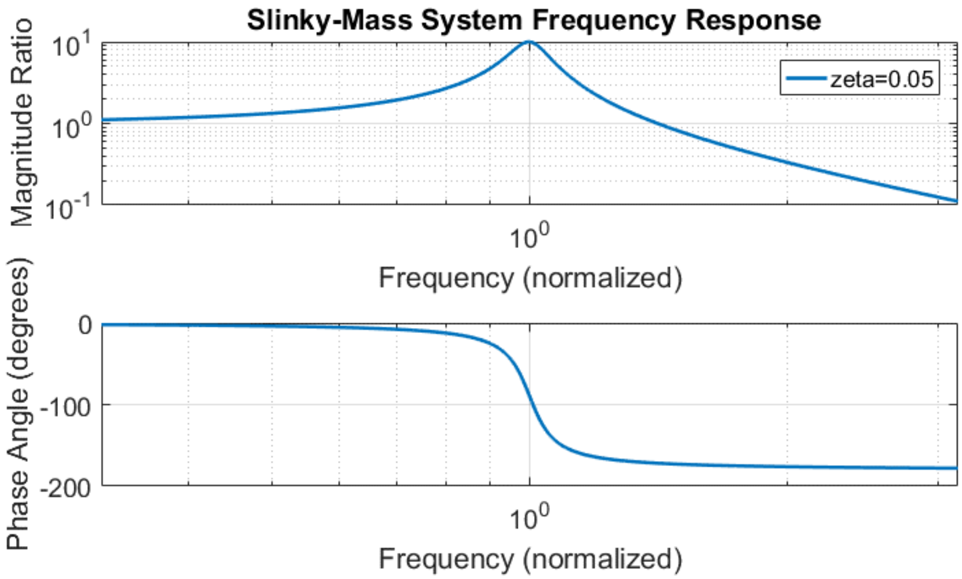


Figure 20: Frequency Response Plot for an Underdamped second order system

<u>Extension</u>

When discussing mechanical model elements, specifically natural frequency and resonance, demonstrate the resonant point and then compare to classic resonant failures like the Tacoma Narrows Bridge, fluttering electrical lines, etc. Also, this is a good opportunity to discuss how to modify the system to prevent those failures by either damping the fluctuations or making changes to get the resonant frequency outside the range of environmental/operational input frequencies.

Discuss different types of inputs (step, impulse, ramp) and explore the system behavior for each.

Collect experimental data to calculate the natural frequency, and even the (very low) damping ratio. Model the system, measure element parameters (mass & spring rate), calculate predicted natural frequency, and compare to experimental data.

Summary and Future Work

The exercises presented in this paper are intended to present the key concepts of an introductory system modeling course at the beginning of the term using a "constructivist" approach, by emphasizing observation and qualitative descriptions of behaviors for a variety of systems and then introducing students to the correct terminology to describe those behaviors. These activities also provide an introduction to the skill of model formation and system identification. Adding activities like the ones presented in this paper to a course syllabus has the potential to enhance a student's ability to retain and apply this knowledge to engineering design decisions long after the course has ended, achieving a broader goal for engineering educators.

These experiments have just recently been implemented into an existing lumped systems modeling course. The efficacy of these activities will be measured and monitored over several years, with the outcomes presented in a follow-on paper.

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Appendix: Bill of Materials

Written for a course with 40 students assuming all experiments are done in pairs in a lab room so some equipment can be shared.

Description	Vendor(Part Number)	Qty x Unit Price	Cost
Experiment 1: Canoe Coast-Down			
No supplies needed for video observation			
Experiment 2: Playdough Hot Potato			
Play-Doh, pack of 24, 3 oz cans	Amazon (20383F01)	1 x $17.75	$17.75
Experiment 3: Fluid-in-a-Tube			
Clear PVC tubing 3/8" ID, ½"OD	McMaster-Carr (5233K63)	100 ft x $0.35	$35.00
Plastic funnel, 3/8" spout	McMaster-Carr (4383T2)	5 x $1.31	$ 6.55
Pitcher, plastic, 2.25 qt	Amazon (B000BQO932)	5 x $6.95	$34.73
Food coloring	Amazon (B016F7D46S)	1 x $3.08	$ 3.08
*Experiment 4: Slinky and mass**			
Slinky "Magic Spring", 12 pack	Oriental Trading (IN-9/610)	2 x $3.99	$ 7.98
Aluminum rod, 1¼ ", 3ft	McMaster-Carr (8974K16)	1 x $27.60	$27.60
Duct tape, 1"	McMaster-Carr (76135A49)	1 x $6.09	$ 6.09
Total			**$138.78**

* Alternative equipment set-up for the Slinky and Mass Experiment