

# Calculation of Electrical Quantities in Three Phase Circuits using MATLAB

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## Abstract

This paper presents a MATLAB program that utilizes a Graphical User Interface (GUI) to calculate the electrical quantities of three phase circuits. The GUI is designed in a way to allow the user to enter the resistances and reactances of the source, the transmission line and that of the load respectively. It also requires a voltage value of one of the phases at the generator end. This program performs the calculations with respect to the specified connection type. There are four types: wye-wye, wye-delta, delta-wye and delta-delta. This allows the students to compare the obtained results and can check the validity of their solutions of homework problems involving three phase circuits in electric circuits, energy conversion and power systems courses.

## Introduction

When considering ac steady-state circuits and their relation to electric power generation and transmission, we are dealing with what is known as polyphase circuits, or more specifically three-phase circuits. It is not only more economical to transmit electric power through three-phase circuits, but also power transmission is more efficiently accomplished at such high voltages. In addition, it is possible to obtain much more power from fewer components with a three-phase circuit as well as having the ability to deliver a constant amount of power to the load at all times.

Students usually have difficulties in applying the concepts in three phase circuits due to the different configurations of such circuits. There are four different configurations, Y-Y, Y-Delta, Delta-Y and Delta-Delta. The difficulty results from the fact that three phase circuits are alternating current circuits which their calculations deal with complex numbers. When dealing with complex numbers, there are two formats, rectangular and polar. Usually it is easier to deal with polar form since it gives the magnitude and the phase angle of voltages and currents involved in such systems. In addition, there are many terms involved regarding current, voltage and power. These are phase current, line current, phase voltage, line voltage, apparent power per phase, real power per phase, reactive power per phase, total apparent power, total real power and total reactive power.

There are many software packages available to analyze electric and electronic circuits. The most popular packages are PSPICE and WORKBENCH. These packages require students to draw the circuit to be analyzed then it will be simulated and give the students the results based on the desired analysis and set up. Using such software packages is an excellent practice, but the problem with those they don't show students the intermediate steps or how they went about to solve for the requirements. When students compare their solutions with the obtained results

from these packages and have some errors in their solutions, they will correct their solutions through trial and error procedure which is a time consuming process.

In the learning process the students should not be passive recipients of knowledge watching things happen, they should be participants. This paper is a result of one of the assigned projects to students in the power systems course (a junior level) at Ohio Northern University where they have demonstrated that they are active participants. They have written many MATLAB programs to calculate various unknowns for different systems. They have coded these programs so students in lower level classes such as those in electric circuits courses who need the programs can be made aware of why a certain thing should be done and what value it has in the overall picture. One program is to calculate electrical quantities in balanced three phase circuits.

The program is written in MATLAB that utilizes a GUI to calculate currents, voltages, real and reactive powers. In addition, it calculates the power factor and the value of the required parallel capacitance in order to correct the power factor to a desired value. The program also allows the users to see the intermediate steps of the calculation which help them to verify their hand calculated values (solutions) and use the program to check every single step of the calculation. The following sections describe three phase circuits, the developed program, some examples and finally a conclusion.

### Overview of Three Phase Circuits

The three-phase circuit is composed of a three-phase system of voltages connected to a three-phase load configuration. If the voltages have the same magnitude and frequency and each voltage is  $120^\circ$  out of phase with the other voltages, the voltages are said to be balanced. In addition, if the loads are configured in a way that the resulting currents are balanced, then the result is a balanced three-phase circuit [1-5].

In a three-phase circuit, the voltages are connected to the corresponding loads by lines a, b and c. This results in three voltages (Y-connection),  $\tilde{V}_{an}$ ,  $\tilde{V}_{bn}$ , and  $\tilde{V}_{cn}$ . The notations a, b, and c represent the three phases and n represents the neutral. Equation 1 shows the formula for the three voltages.

$$\begin{aligned}\tilde{V}_{an} &= V_p \angle 0^\circ \\ \tilde{V}_{bn} &= V_p \angle -120^\circ \\ \tilde{V}_{cn} &= V_p \angle +120^\circ\end{aligned}\tag{1}$$

where  $V_p$  is the phase voltage. With balanced three-phase systems it is important to note that

$$\tilde{V}_{an} + \tilde{V}_{bn} + \tilde{V}_{cn} = 0\tag{2}$$

When connecting the source of the three phase generator and the load, there are two types of connections, a *wye* (Y) and a *delta* ( $\Delta$ ) connection. Because the source and the load can each be

connected in two different ways, there exists four different configurations for balanced three-phase circuits, Y-Y, Y-Δ, Δ-Y, and Δ-Δ. These various circuits are discussed in the following sections.

### Y-Y Connection

A balanced Y-Y connection is shown in Figure 1.

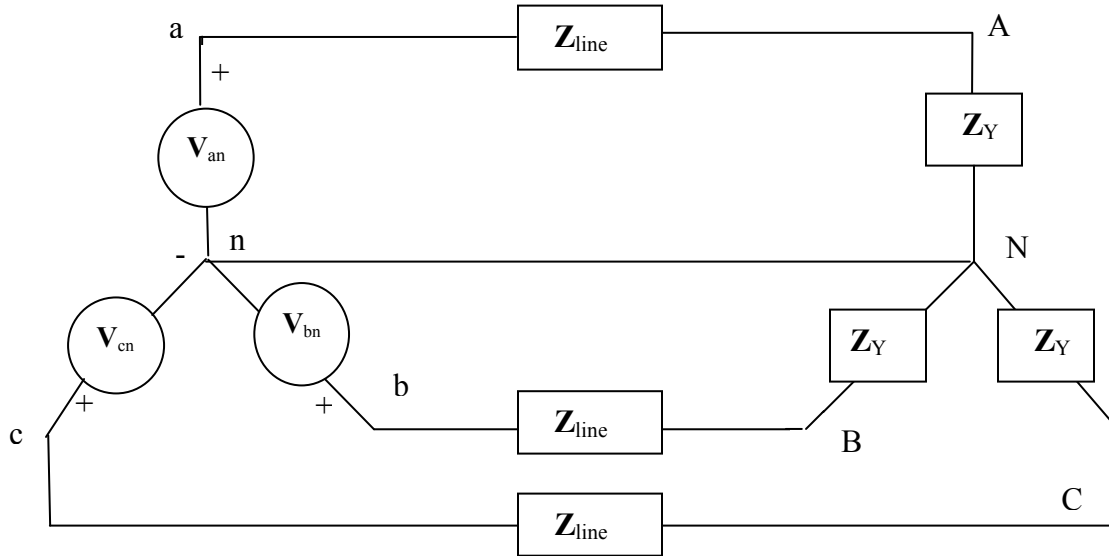


Fig. 1 Y-Y Connection

Using (1) for phase voltages, the line voltages can be calculated using KVL to obtain the following equations.

$$\begin{aligned}\tilde{V}_{ab} &= \sqrt{3} * V_p \angle 30^\circ \\ \tilde{V}_{bc} &= \sqrt{3} * V_p \angle -90^\circ \\ \tilde{V}_{ca} &= \sqrt{3} * V_p \angle -210^\circ\end{aligned}\tag{3}$$

The line currents can be calculated by

$$\tilde{I}_a = \frac{\tilde{V}_{an}}{\tilde{Z}_Y}\tag{4}$$

The neutral current can then be calculated by

$$\tilde{I}_n = \tilde{I}_a + \tilde{I}_b + \tilde{I}_c = 0\tag{5}$$

It is also important to note that in a Y-Y connection, the current in the line connecting the source to the load is the same as the phase current that is flowing through  $Z_Y$ .

### **$\Delta$ - $\Delta$ Connection**

For a delta connected source, the sources are connected line-to-line, which means that the delta sources are

$$\begin{aligned}\tilde{V}_{ab} &= V_L \angle 0^\circ \\ \tilde{V}_{bc} &= V_L \angle -120^\circ \\ \tilde{V}_{ca} &= V_L \angle +120^\circ\end{aligned}\tag{6}$$

Where  $V_L$  is the magnitude of the line voltage. Figure 2 shows a balanced  $\Delta$ - $\Delta$  connection.

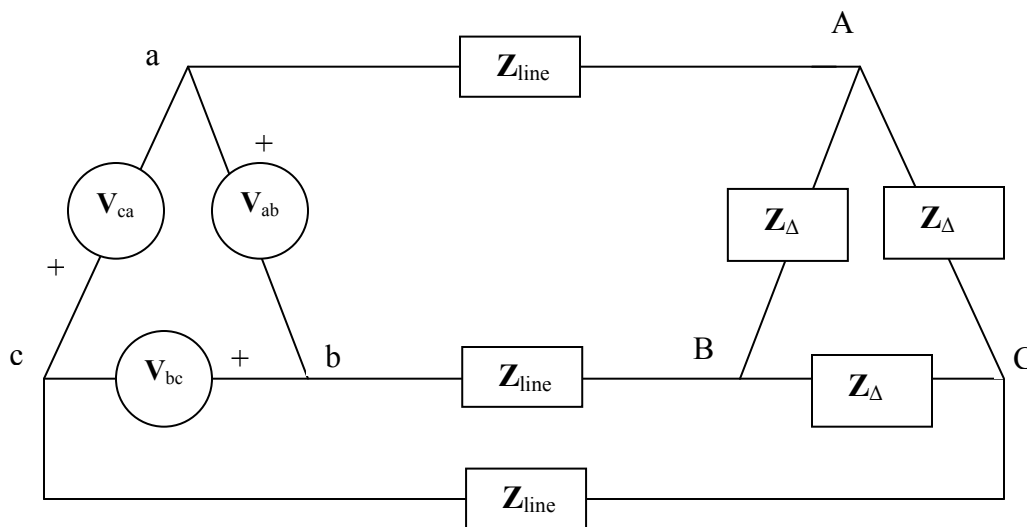


Fig. 2  $\Delta$ - $\Delta$  Connection

It is easy to convert three-phase delta-connected voltage sources to its *wye*-connected equivalent. The following equations show how this is done.

$$\begin{aligned}\tilde{V}_{an} &= \frac{V_L}{\sqrt{3}} \angle -30^\circ = V_p \angle -30^\circ \\ \tilde{V}_{bn} &= \frac{V_L}{\sqrt{3}} \angle -150^\circ = V_p \angle -150^\circ \\ \tilde{V}_{cn} &= \frac{V_L}{\sqrt{3}} \angle -270^\circ = V_p \angle +90^\circ\end{aligned}\tag{7}$$

In a delta-connected load, the line-to-line voltage is the voltage across each load impedance which means that the line voltage is equal to the phase voltage. The line voltages of the source are

$$\begin{aligned}
\tilde{V}_{ab} &= \sqrt{3} * V_p \angle 30^\circ = V_L \angle 30^\circ = \tilde{V}_{AB} \\
\tilde{V}_{bc} &= \sqrt{3} * V_p \angle -90^\circ = V_L \angle -90^\circ = \tilde{V}_{BC} \\
\tilde{V}_{ca} &= \sqrt{3} * V_p \angle -210^\circ = V_L \angle -210^\circ = \tilde{V}_{CA}
\end{aligned}
\tag{8}$$

where  $V_L$  is the magnitude of the line voltage at the delta-connected load and at the source. This occurs if there is no line impedance present in the system.

The phase currents are

$$\tilde{I}_{AB} = \frac{\tilde{V}_{AB}}{\tilde{Z}_\Delta}
\tag{9}$$

$\tilde{I}_{BC}$  and  $\tilde{I}_{CA}$  have the same magnitude but lag  $\tilde{I}_{AB}$  by  $120^\circ$  and  $240^\circ$ , respectively. To determine the line currents, KCL can be used to produce

$$\tilde{I}_{aA} = \tilde{I}_{AB} + \tilde{I}_{AC} = \tilde{I}_{AB} - \tilde{I}_{CA}
\tag{10}$$

As was mentioned with delta-connected voltage sources, it is easier to convert the delta-connected load to a wye connected load and then solve. In this case

$$3 * \tilde{Z}_Y = \tilde{Z}_\Delta
\tag{11}$$

and the line current  $\tilde{I}_{aA}$  is

$$\tilde{I}_{aA} = \frac{\tilde{V}_{an}}{\tilde{Z}_Y}
\tag{12}$$

The relationship between the magnitudes of the phase currents in the delta-connected load and the line currents is

$$I_L = \sqrt{3} * I_\Delta
\tag{13}$$

### **Y-Δ Connection**

Figure 3 shows a balanced Y-Δ Connection. For the wye-delta connection we can again use the positive sequence, and assume that the phase voltages are

$$\begin{aligned}
\tilde{V}_{an} &= V_p \angle 0^\circ \\
\tilde{V}_{bn} &= V_p \angle -120^\circ \\
\tilde{V}_{cn} &= V_p \angle +120^\circ
\end{aligned}
\tag{14}$$

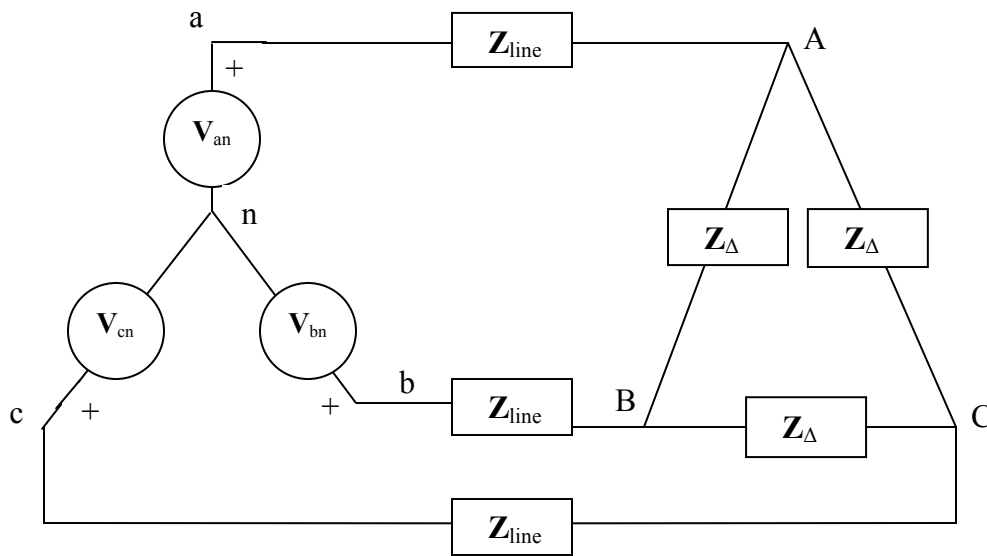


Fig. 3 Y-Δ Connection

Knowing this, the line voltages can be represented as

$$\begin{aligned}
 \tilde{V}_{ab} &= \sqrt{3} * V_p \angle 30^\circ = \tilde{V}_{AB} \\
 \tilde{V}_{bc} &= \sqrt{3} * V_p \angle -90^\circ = \tilde{V}_{BC} \\
 \tilde{V}_{ca} &= \sqrt{3} * V_p = \tilde{V}_{CA}
 \end{aligned}
 \tag{15}$$

If the load impedances are given and we know the line voltages, the phase currents can be calculated from the following set of equations

$$\tilde{I}_{AB} = \frac{\tilde{V}_{AB}}{\tilde{Z}_{\Delta}} \quad \tilde{I}_{BC} = \frac{\tilde{V}_{BC}}{\tilde{Z}_{\Delta}} \quad \tilde{I}_{CA} = \frac{\tilde{V}_{CA}}{\tilde{Z}_{\Delta}}
 \tag{16}$$

In order to find the line currents for this type of connection we can apply KCL at nodes A, B, C on the delta-connected load. This results in the following equation.

$$\begin{aligned}
 \tilde{I}_{aA} &= \tilde{I}_{AB} - \tilde{I}_{CA} \\
 \tilde{I}_{bB} &= \tilde{I}_{BC} - \tilde{I}_{AB} \\
 \tilde{I}_{cC} &= \tilde{I}_{CA} - \tilde{I}_{BC}
 \end{aligned}
 \tag{17}$$

In addition, the line current can be related to the phase current by the following equation

$$\tilde{I}_L = \sqrt{3} * \tilde{I}_p \angle -30^\circ
 \tag{18}$$

The connections for this network are made similarly to the connections for the other two configurations. The simplest way to solve for this circuit is to establish what type of connection is being made, and then convert any delta-connections to wye-connections.

### ***Δ-Y Connection***

Figure 4 shows a balanced Δ-Y Connection.

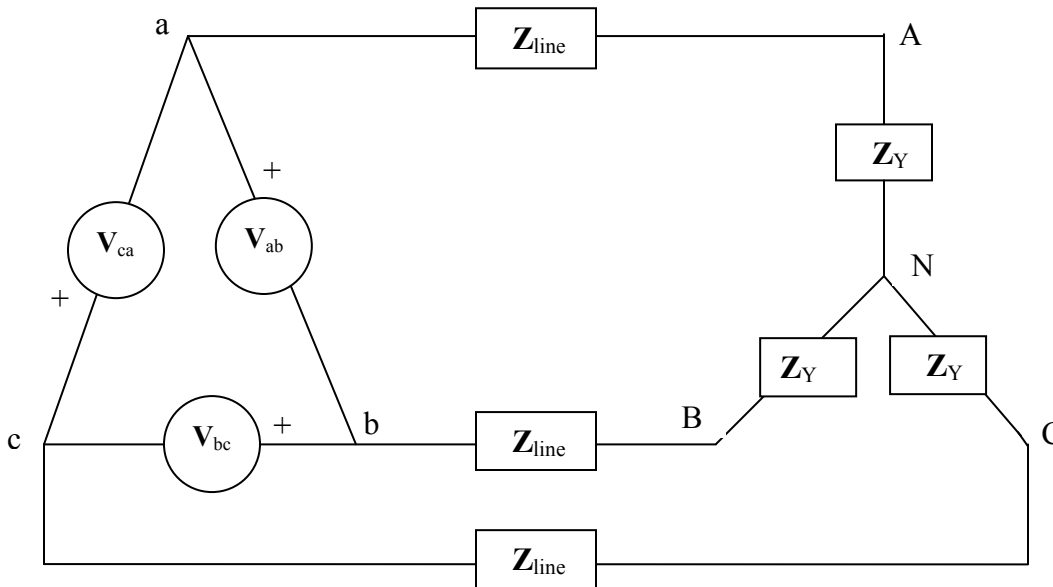


Fig. 4 Δ-Y Connection

A positive sequence is assumed, the phase voltages are

$$\begin{aligned}
 \tilde{V}_{ab} &= V_L \angle 0^\circ \\
 \tilde{V}_{bc} &= V_L \angle -120^\circ \\
 \tilde{V}_{ca} &= V_L \angle +120^\circ
 \end{aligned}
 \tag{19}$$

For the case of the delta-wye connection, the phase voltages are equivalent to the line voltages at the source end. With this being said, the line currents can be calculated using the following equations.

$$\begin{aligned}
 \tilde{I}_a &= \frac{\tilde{V}_{an}}{\tilde{Z}_Y} \\
 \tilde{I}_b &= \tilde{I}_a \angle -120^\circ \\
 \tilde{I}_c &= \tilde{I}_a \angle 120^\circ
 \end{aligned}
 \tag{20}$$

When solving problems in balanced three-phase networks, some general rules apply. The phase of the voltages and currents in a delta connection is  $30^\circ$  ahead of those in a Y connection. The magnitude of the delta-connection phase voltage is  $\sqrt{3}$  times that of the wye-connection phase voltage. The magnitude of the line current, or the wye-connection phase current is  $\sqrt{3}$  times that of the delta-connection phase current. Finally, the load impedance of the delta-connection is three times the load impedance of the Y-connection and the phases are identical.

### **Power Calculations**

The single phase power equations can be applied to each phase of a Y- or a  $\Delta$ -connected three phase load. The real, reactive, and apparent powers supplied to a balanced three phase load in phase quantities are given by:

$$\begin{aligned} P &= 3 V_p I_p \cos \theta \\ Q &= 3 V_p I_p \sin \theta \\ S &= 3 V_p I_p \end{aligned} \quad (21)$$

Also, it is possible to express the power in a balanced three phase load in terms of line quantities. The following equations are applicable regardless the connection:

$$\begin{aligned} P &= \sqrt{3} V_L I_L \cos \theta \\ Q &= \sqrt{3} V_L I_L \sin \theta \\ S &= \sqrt{3} V_L I_L \end{aligned} \quad (22)$$

All the above equations cause confusion for students. To overcome this problem, and allow students to check their solutions of the assigned homework problems, the presented MATLAB program was created.

### **Program Requirements**

The program requires the MATLAB software package to be executed. At the command window, the user types the file name. In addition, it requires the user to input the following data:

- Source Reference Voltage.
- Line Impedance.
- Load Impedance.
- Desired Power Factor. The choice of leading or lagging.
- Circuit Type (Y-Y, Y-  $\Delta$ ,  $\Delta$ -Y or  $\Delta$ -  $\Delta$ ).
- Phase Sequence.
- Show step. The choice of yes or no.



To perform the simulation, the user needs to click on *calculate* button and *Finish* button. If there are any additional problems to be solved, the user can click on *Reset* button to restart the calculation for a different case.

### Example

Figure 5 shows the window that will be displayed when the user runs the program. It shows all the elements mentioned above. It is shown for a Y-Y connection and no calculation steps to be shown. The problem to be solved is stated as:

A balanced *abc*-sequence Y-connected source with  $\tilde{V}_{an} = 110\angle 0^\circ$  V is connected to a Y-connected balanced load with an impedance of  $(10 + j8)$   $\Omega$  per phase. The line joining the generator and the load has an impedance of  $(5 - j2)$   $\Omega$  per phase. Find:

- Source voltages.
- Line currents.
- Voltage drop across each line impedance.
- Voltage drop across each load impedance (Phase voltages).
- Complex power of combined line impedances.
- Complex power of combined load impedances.
- Total complex power.
- Complex power supplied by source.
- Power factor angle of the load.
- Power factor of the load.
- The value of required capacitance to correct PF to 0.80 Lagging.

The screenshot shows a software window titled "Figure No. 1: Power Systems Lab 2". It contains the following input fields and controls:

- Source Ref. Voltage: 110 < 0°
- Line Impedance: 5 +j -2 ohm
- Load Impedance: 10 +j 8 ohm
- Desired Power Factor: 0.8, lagging (dropdown menu)
- Circuit Type: Y-Y (dropdown menu)
- Phase Sequence: abc (dropdown menu)
- Show steps: No (dropdown menu)
- Buttons: Calculate, Reset, Finish

Figure 5 Program input window

The program output based on the input data shown in Figure 5 is shown in Table 1. The output shows respectively, the source voltages, phase currents, voltage drop across each line impedance, voltage drop across each load impedance, complex power of combined line impedances, complex power of combined load impedances, total complex power, complex power supplied by source, power factor angle of the load, power factor of the load, and the value of the reactive element to correct the power factor to a desired value. The choice for the “Show Steps” was set to “No”, so the output shown in Table 1 is without showing any intermediate steps.

Table 1 Program output without calculation steps

Project 2 - Three Phase Circuits ECCS-336 Power Systems	
Source reference voltage:	(110.000 @ +0.000 deg) V (Y Connection)
Line Impedance:	(5.000 , -2.000) ohm
Load Impedance:	(10.000 , 8.000) ohm (Y Connection)
Desired Power Factor:	0.80 Lagging
Phase Sequence:	abc
Calculating Y-Y System Connection...	
Done.	
Calculation Results (Note: All angles are in degrees)	
Source voltages:	
	Van = (110.000 @ +0.000 deg) V
	Vbn = (110.000 @ -120.000 deg) V
	Vcn = (110.000 @ +120.000 deg) V
Line currents:	
	IaA = (6.809 @ -21.801 deg) V
	IbB = (6.809 @ -141.801 deg) V
	IcC = (6.809 @ +98.199 deg) V
Voltage drop across each line impedance:	
	VlineA = (36.667 @ -43.603 deg) V
	VlineB = (36.667 @ -163.603 deg) V
	VlineC = (36.667 @ +76.397 deg) V
Voltage drop across each load impedance:	
	VloadA = (87.196 @ +16.858 deg) V
	VloadB = (87.196 @ -103.142 deg) V
	VloadC = (87.196 @ +136.858 deg) V
Complex power of combined line impedances:	
	Sline = (748.971 @ -21.801 deg) VA
Complex power of combined load impedances:	
	Sload = (1781.099 @ +38.660 deg) VA
Total complex power:	
	Stotal = (2246.914 @ +21.801 deg) VA
Complex power supplied by source:	
	Ssource = (2246.914 @ +21.801 deg) VA
Power factor angle of the load:	

```

        angLoad = 38.660 deg

Power factor of the load:
        Ppload = 0.78 Lagging

To correct PF to 0.80 Lagging, add a 15.245 uF
Capacitor

Done.

```

The output of the program when the intermediate steps are desired is shown in Table 2.

Table 2 Program output with calculation steps

```

Project 2 - Three Phase Circuits
ECCS-336 Power Systems

DEBUG:
> VsMag = 110.000000
> VsArg = 0.000000
> Rline = 5.000000
> Xline = -2.000000
> Rload = 10.000000
> Xload = 8.000000
> PFdesired = 0.800000
> LeadLag = 1.000000
> CircuitType = 1.000000
> PhaseSequence = 1.000000

Source reference voltage: (110.000 @ +0.000 deg) V (Y Connection)
Line Impedance: (5.000 , -2.000) ohm
Load Impedance: (10.000 , 8.000) ohm (Y Connection)
Desired Power Factor: 0.80 Lagging
Phase Sequence: abc

DEBUG: Vs = (110.000 , 0.000)
DEBUG: Zline = (5.000 , -2.000)
DEBUG: Zload = (10.000 , 8.000)
Calculating Y-Y System Connection...
DEBUG: Van = (110.000 @ +0.000 deg)
DEBUG: Vbn = (110.000 @ -120.000 deg)
DEBUG: Vcn = (110.000 @ +120.000 deg)
DEBUG: Ian = (6.809 @ -21.801 deg)
DEBUG: Ibn = (6.809 @ -141.801 deg)
DEBUG: Icn = (6.809 @ +98.199 deg)
DEBUG: VlineA = (36.667 @ -43.603 deg)
DEBUG: VlineB = (36.667 @ -163.603 deg)
DEBUG: VlineC = (36.667 @ +76.397 deg)
DEBUG: VloadA = (87.196 @ +16.858 deg)
DEBUG: VloadB = (87.196 @ -103.142 deg)
DEBUG: VloadC = (87.196 @ +136.858 deg)
DEBUG: Sline = (695.402 , -278.161)
DEBUG: Sline = (748.971 @ -21.801 deg)
DEBUG: Sload = (1390.805 , 1112.644)
DEBUG: Sload = (1781.099 @ +38.660 deg)
DEBUG: Stotal = (2086.207 , 834.483)
DEBUG: Stotal = (2246.914 @ +21.801 deg)
DEBUG: Ssource = (2086.207 , 834.483)
DEBUG: Ssource = (2246.914 @ +21.801 deg)
DEBUG: argLoad = (0.675 @ +0.000 deg)

```

```
DEBUG: PFlload = (0.781 @ +0.000 deg)
DEBUG: Xc = (174.000 @ +0.000 deg)
DEBUG: C = (0.000 @ +0.000 deg)
Done.
```

Calculation Results  
(Note: All angles are in degrees)

Source voltages:

```
Van = (110.000 @ +0.000 deg) V
Vbn = (110.000 @ -120.000 deg) V
Vcn = (110.000 @ +120.000 deg) V
```

Line currents:

```
IaA = (6.809 @ -21.801 deg) V
IbB = (6.809 @ -141.801 deg) V
IcC = (6.809 @ +98.199 deg) V
```

Voltage drop across each line impedance:

```
VlineA = (36.667 @ -43.603 deg) V
VlineB = (36.667 @ -163.603 deg) V
VlineC = (36.667 @ +76.397 deg) V
```

Voltage drop across each load impedance:

```
VloadA = (87.196 @ +16.858 deg) V
VloadB = (87.196 @ -103.142 deg) V
VloadC = (87.196 @ +136.858 deg) V
```

Complex power of combined line impedances:

```
Sline = (748.971 @ -21.801 deg) VA
```

Complex power of combined load impedances:

```
Sload = (1781.099 @ +38.660 deg) VA
```

Total complex power:

```
Stotal = (2246.914 @ +21.801 deg) VA
```

Complex power supplied by source:

```
Ssource = (2246.914 @ +21.801 deg) VA
```

Power factor angle of the load:

```
angLoad = 38.660 deg
```

Power factor of the load:

```
PFlload = 0.78 Lagging
```

To correct PF to 0.80 Lagging, add a 15.245 uF  
Capacitor

Done.

As mentioned the outputs in tables 1 and 2 are for the case of Y-Y connection. All other connections run cases will be shown during the conference presentation. Also, the program will be running during the presentation for demonstration.

## Conclusion

An overview of a developed MATLAB program to solve for any given balanced three phase circuits was presented. The program allows users to choose the desired type of connection from a menu besides it has the capability to show intermediate steps so students can follow the calculation and correct their homework solutions if necessary. In addition, it is a graphical user interface which makes data insertion an easy task. Students in Circuits 2 course who have used this program found it to be a very useful tool.

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## Author Biographies

**Dr. Khalid S. Al-Olimat** is an associate professor in the Department of Electrical & Computer Engineering and Computer Science at Ohio Northern University. He obtained his BS in Electrical Engineering from Far Eastern University on 1990, the MS in Manufacturing Engineering from Bradley University on 1994 and his PhD in Electrical Engineering from the University of Toledo on 1999. Dr. Al-Olimat has many publications in the area of adaptive control, fuzzy control, machine drives and engineering education. His areas of interest are power engineering, adaptive, fuzzy and intelligent control. He is a member of ASEE and IEEE where he is serving as the Vice-Chair of IEEE-Lima section.

**Pete Jankovsky** is a senior electrical engineering student in the Department of Electrical & Computer Engineering and Computer Science at Ohio Northern University. He is in the honors program. He has received NASA Undergraduate Research Scholarship, IEEE Future Engineer Award and Tau Beta Pi Outstanding Award. He has three publications. He has co-opped with American Electric Power (AEP), British Petroleum Chemical (BP) and Applied Innovation. Mr. Jankovsky is going to attend a graduate school starting August 2005.

**Matt Valerio** is a Senior pursuing a B.S.E.E. at Ohio Northern University, Ada, OH, with minors in computer science and applied mathematics. Mr. Valerio is a member of the Institute of Electrical and Electronic Engineers (IEEE), the Mathematical Association of America, Tau Beta Pi (Engineering honorary), Kappa Mu Epsilon (Mathematics honorary), Phi Kappa Phi (Junior honorary), and Phi Eta Sigma (Freshman honorary). He was awarded the 2004 IEEE Lima Section Future Engineer Award and the 2004 James L. Klingenberg Award.

**Jack Skinner** is a junior electrical engineering student in the Department of Electrical & Computer Engineering and Computer Science at Ohio Northern University.