AC 2009-1225: CALCULUS AT A DISTANCE: BRINGING ADVANCED MATHEMATICS TO HIGH-SCHOOL STUDENTS THROUGH DISTANCE LEARNING

Nelson Baker, Georgia Institute of Technology
Dr. Nelson C. Baker is the Vice Provost for Distance Learning and Professional Education (DLPE) and a faculty member in Civil and Environmental Engineering at the Georgia Institute of Technology. Nelson received his B.S. in civil engineering from Georgia Tech, and his M.S. and Ph.D. in civil engineering from Carnegie Mellon. His work focuses on educational learning tools and student learning, faculty usage, and assessment of these techniques for instruction and learning.

Marion Usselman, Georgia Institute of Technology
Dr. Marion C. Usselman is a Senior Research Scientist at the Center for Education Integrating Science, Mathematics and Computing (CEISMC) at the Georgia Institute of Technology. Marion received her B.A. in biophysics from the University of California, San Diego, and her Ph.D. in biophysics from Johns Hopkins University. She focuses on K-12 educational reform, university-K-12 partnerships, and equity issues in education.

Thomas Morley, Georgia Institute of Technology
Dr. Morley, is a CEISMC Professor of Mathematics at Georgia Tech, dividing his time between the School of Mathematics, and the Center for Education, Integrating Science, Mathematics and Computing. He is the author of two books, and over 50 papers on pure and applied mathematics and education. He was recently involved in writing the new standards for Georgia's high school's Mathematics curriculum.

Rick Clark, Georgia Institute of Technology
Rick Clark is the Director of Undergraduate Admission at Georgia Tech. In this role, he directs the Institute’s recruitment and outreach efforts, directs the review of all undergraduate applications, and manages the admissions team. Rick works closely with Georgia Tech’s Athletic Association in review of all prospective student-athletes, as well as with the Alumni Association on national recruitment initiatives. A native of Atlanta, he earned a B.A. from the University of North Carolina at Chapel Hill and a M.Ed. from Georgia State University. He, his wife, Amy, and their son AJ (who proudly wears GT apparel to assist in recruiting the nation’s finest students) live in Decatur. Prior to coming to Tech, Rick was on the admissions staff at Georgia State, The McCallie School and Wake Forest University.
Calculus at a Distance: Bringing Advanced Mathematics to High School Students through Distance Learning

Abstract

The Georgia Institute of Technology, in collaboration with four local school systems, is teaching sophomore level calculus via distance learning to students who have exhausted the math offerings in their high school. Students enrolled in the program are highly successful in the course compared to their on-campus college peers. They also matriculate in large numbers to Georgia Tech, making it a highly effective recruitment tool for attracting the most academically advanced students in the state. This paper addresses the many benefits to the students and university in such a program. Further, it highlights some of the potential problem areas for both K-12 public school systems and universities when trying to forge similar partnerships.

Key Words: Distance Learning, university calculus, student recruiting, student retention

1. Program Overview and Objectives

Introductory calculus has become a staple in most high schools, primarily through the College Board’s Advance Placement (AP) Calculus program. Long considered the final high school mathematics class for advanced seniors, there is a growing trend for K-12 school systems to accelerate their mathematics course sequence so that the top academic students take AP Calculus during their junior, rather than their senior, year. While the pedagogical wisdom of this tendency to accelerate mathematics rather than to explore the earlier concepts in more depth can be debated, the reality is that mathematics curriculum reform movements have often promoted a math sequence in which the standard 8th grade math course is Algebra 1. If “on-grade-level” students are taking algebra in the 8th grade, then “advanced” and “gifted” students typically start the course a year earlier, namely in 7th grade, and sometimes even earlier. By their senior year in high school, these students have exhausted the mathematics course offerings of the high school. Few school systems can reliably guarantee that they will have a teacher capable of teaching Advanced Calculus, and even if they do, colleges and universities have no way to judge the caliber of the course.

In the fall of 2004, a conversation between a staff member from Georgia Tech’s Center for Education Integrating Science, Mathematics and Computing (CEISMC--Georgia Tech’s K-12 outreach center), and a Fulton County School System curriculum coordinator revealed that the school system, with its 82,000 students, had a growing number of students taking AP Calculus during their junior or even sophomore year, and that the system was struggling to provide additional mathematics education in the senior year. While there were existing mechanisms for joint enrollment between the high schools and Georgia Tech, the transportation issues were too costly and time consuming to allow the students to physically attend Georgia Tech while still in high school, and the local junior colleges did not offer appropriate courses for this level of student. Providing instruction through the use of distance learning seemed to be a viable alternative.
Working together, a team partnering people from both Georgia Tech and the Fulton County School System investigated the many challenges inherent in such projects, and proposed potential solutions. Team members from the university included representatives from the School of Mathematics, the Office of Undergraduate Admission, the Office of Financial Aid, CEISMC, and the Distance Learning and Professional Education (DLPE) office, and from the school system included the school system curriculum coordinator, technology staff, and high school-level teachers and administrators.

The objectives of the program were to:
- Identify students with both the interest and academic qualifications to pursue advanced calculus while still in high school;
- Provide a pedagogically sound distance delivered program to these high school students; and,
- Track and support the students in the program to be sure that they were successful.

The planning team identified a variety of issues that needed to be addressed, including:
- Admissions requirements, and student status for high school distance education students;
- Pedagogical requirements for effective mathematics instruction and student support, including help sessions;
- Technology infrastructure requirements and related costs;
- Available communication infrastructures including telephone and internet bandwidth;
- Personnel requirements for both the high schools and university that would be necessary to allow the students to receive the education via distance education rather than physically coming to campus for classes; and
- Funding issues at both the university and school system level.

Fortunately, Georgia has an educational scholarship and grant program for academically talented high school students that covers course tuition and a percentage of necessary books for students who enroll in university system courses during their junior or senior year in high school. This Accel Program eliminated the most obvious student funding issue, though there are still costs associated with the distance transmission. Beginning in the fall of 2005, the partnership began providing advanced calculus education to students in five schools within the Fulton County School System via distance learning as a pilot program. The costs to the school systems, since the program’s inception, have been a modest $3,000/semester per school location. This paper outlines the challenges and the solutions utilized for creating such a program, the academic success of the students while in the program, and the unforeseen results related to recruiting and retaining the students within Georgia Tech.

2. Program Details

The Distance Calculus program has expanded since 2005, and now serves 21 high schools in four school systems. Many standard processes have been established, ranging from technological support for school systems when purchasing and installing equipment, to information sessions for prospective students and their parents, to procedures for notifying students and school counselors of grades during, and at the end of, the semester. A brief description of these processes, including the curricular content follows.
2.1 Course Content

The curriculum for the program consists of two semester-long courses in college calculus, designed to pick up following successful completion of high school level AP Calculus. These courses were originally designed by the School of Mathematics in consultation with many departments of science and engineering. Since these calculus courses are required for many majors at Georgia Tech, they have a very large enrollment of on-campus freshman and sophomores.

High school students entering the Distance Calculus program are required to have completed the AB and/or BC Advanced Placement (AP) Calculus course at their high school, and earned a 4 or higher on the AB Calculus exam or a 3 or higher on the BC exam. Their AP instruction should cover topics on limits, differentiation, integration, complex numbers and the applications of these mathematical techniques to real world problems. They also need to score at least a 600 on the math section of the SAT, or a 26 on the ACT, and have at least a 3.0 cumulative GPA in their core academic and math specific courses. The students pay the normal $50 university application fee to apply, and earn 4 credit hours of Georgia Tech credit for each semester long class.

Beginning in the fall semester, high school students enroll remotely in the same Calculus II course that university students are simultaneously taking on campus. All required homework, quizzes, and exams are identical for on-campus and distance students. From a recruitment perspective, this delivery provides the high school students with authentic and invaluable first-hand insight and interaction with university students, creates opportunities for peer mentoring, and gives all students experience with distance education. Since the high school students are receiving full college credit, it is also crucial that they participate directly with the college students to ensure that academic requirements and standards are the same for both groups.

The Georgia Tech Calculus II course is primarily a course on linear algebra and series methods in calculus. The course covers topics such as Taylor polynomials and approximation, infinite series, power series, numerical integration and ordinary differential equations, vectors and matrices, systems of linear equations, determinants and cross products and finally, eigenvalues and eigenvectors. There is a significant coverage of numerical methods in linear algebra, including the LU and QR decompositions of matrices. (The strong coverage of linear algebra was chosen for two reasons: as a foundation for multivariate calculus, and because of the extensive use of linear algebra in such early engineering courses as Digital Signal Processing.) The text is at the level of Calculus, One and Several Variables by Salas, Hille, and Etgen and Beginning with Linear Algebra by Carlen and Carvalho for linear algebra.

Spring semester, the students enroll in Calculus III to study multivariable calculus. Course topics include linear approximation and Taylor’s theorems, Lagrange multiplices, and vector analysis including the theorems of Green, Gauss and Stokes. The text utilized for the spring course is the same as for the fall semester course.
2.2 Technology and Delivery Mechanism

Both the fall semester and spring semester class meets five days per week for one hour. When trying to coordinate schedules from multiple high schools and the university, the best time is early in the morning, from 8:00-9:00. Three of the class sessions are held as lecture sessions where all students participate together with the instructor to cover new material. On the other two days each week the class is subdivided into much smaller groups for recitation time enabling problem solving, response to questions and group problem solving. In each of these two settings the high schools are connected via live video-teleconferencing equipment which is also captured for later playback. Both the university and high school students have the ability to replay the class captures which are saved and posted in a course management system (see Figure 1).

Figure 1: Sample Screen capture of Distance Calculus Delivery

The instructor and teaching assistants all utilize Tablet PCs in their instruction. This enables both free form handwriting as well as software-based problem solving. The course regularly includes instruction using the software program Mathematica as well as internet sites that provide real world problems requiring the application of mathematical concepts being taught and examples of solutions. Through the use of video-teleconferencing and computer application sharing, both local and distant students can ask questions to the instructor and all students can see and hear the answer. The remote sites where students are actively speaking are shown on large screens at the rear of the lecture hall. Because of the large number of remote locations, not
all are shown at one time. However as a question is posed by a different site, the video is
switched to that location.

On days when there is an exam, the instructor sends the exam to the high school proctors for
administration of the test. The individual proctors are sometimes a mathematics teacher or
sometimes another staff employee at the high school. The proctors are responsible for
administering the exam under the conditions outlined by the instructor. The video-
teleconferencing system is available so that students can ask questions during the exam to the
instructor. After the exam, the proctor collects the papers and submits them to Georgia Tech
using a customized faxserver technology. Each student is provided with a unique barcoded
cover sheet to be used when submitting any homework or exam. The faxserver files the
barcoded work into the student’s electronic folder and allows the student to monitor where the
homework or exam is in the grading process, as well as see and download the final graded
solution in a confidential manner. Homework, which is provided to the students via electronic
methods, can also be returned through electronic drop-boxes in the course management system.

2.3 Recruiting, Preparation, and Communication

Effective communication with potential students about the Distance Calculus program is an
essential component to a successful experience. The communication must start early, continue
throughout the program, and provide the students with accurate and timely information. Students
should hear as early as possible in their high school career about the course requirements in order
to ensure that they have the proper qualifications for eligibility. Ideally, this communication
should begin in the 9th grade so that students can plot a path that allows them to complete AP
Calculus by their junior year. Potential students also need to make sure they have taken the SAT
or ACT and the AP Exam in time for admission into the Distance Calculus program. Teachers,
counselors, prospective students and their parents all need to be fully informed about the
program, to ensure that they understand that Distance Calculus students are considered to be
“real” college students, and that course grades will become part of their permanent college
record. Each January, a virtual open house using the course distance learning technology is held
for the students and their parents. At this event they meet the professor and the Distance
Learning staff, experience the distance technology first-hand, and learn more about the
admissions requirements.

University registration procedures and application for Georgia’s Accel Scholarship program
must also be carefully explained and supported. The scholarship program pays the student’s
tuition directly to the university and provides a book allowance to the student. This process
requires approval from both high school and university officials, and enables the students to
attend the program by paying only a small student fee. These processes and the registration
process are all new to the students and to most high school counselors, so good communication
is critical.

Good communication is also crucial between the technical staff at Georgia Tech’s Distance
Learning and Professional Education office and their counterparts in the school systems. Careful
testing of each school’s information technology system is required to ensure there is sufficient
bandwidth and support to provide the live video-teleconferencing required by the course without
interfering with other communication needs within the school. The school system and school-level staff must also understand the necessity of maintenance and infrastructure support. Keeping the IT system operational, with backup strategies for when key employees are on vacation or sick, is critical to the success and viability of the program.

2.4. Course Operation

At the start of each fall semester, a student-parent orientation is mandatory for all students, as well as facilitators of the program. At this program students learn more about course registration and logistics of tuition payment, and have the opportunity to purchase books and, most importantly, to meet the instructors and other support personnel. This is an excellent opportunity for the high school students to also visit campus, to hear from other students who have completed the program, and to hear about the other high schools that will be participating in the program. The students leave with a level of excitement and inclusion that would be difficult to achieve otherwise. Further, the parents learn about the process and see through demonstrations what their children will be experiencing for a learning environment, one that is also available at home and/or in public libraries when not in school. This greatly helps the students to study for the course.

During the registration process, another important component is explained to all students and their parents. In a university-level classroom, the communication protocols with parents are absent. This contrasts with high schools, which have mechanisms in place to report student progress to parents. In order to provide grades to the high schools, students must complete a FERPA release. This document allows the university to release student grades to the high schools for reporting, but still parents must understand that there will not be weekly updates sent to them from the instructors. Much of what takes place during this student-parent orientation is expectation setting and providing contact information for problem resolution and question answering that will come throughout the program.

3. Program History

This program began in the fall of 2005 with five high schools from one school system. From these five high schools 33 students enrolled in the program (Table 1). During the current (2008-2009) academic year, 21 high schools in four school systems are participating, with 205 students enrolled in the program.

<table>
<thead>
<tr>
<th>Year</th>
<th># School Systems</th>
<th># Schools</th>
<th># Applicants</th>
<th># Students Enrolled</th>
<th>Seniors</th>
<th>Juniors</th>
<th>Soph.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005-06</td>
<td>1</td>
<td>5</td>
<td>42</td>
<td>34</td>
<td>33</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2006-07</td>
<td>2</td>
<td>9</td>
<td>87</td>
<td>79</td>
<td>76</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>2007-08</td>
<td>2</td>
<td>12</td>
<td>107</td>
<td>98</td>
<td>92</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>2008-09</td>
<td>4</td>
<td>21</td>
<td>232</td>
<td>205</td>
<td>197</td>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>

Typically each year there are a small group of students who initially indicate an interest in participating in the program, but who do not take the required entrance exams and/or who do not
complete the application process. There are also some students who are not accepted due to deficiencies in their academic record. Most students complete both semesters of the program, but some students at one high school drop it in the spring because of a conflicting senior internship that is required for math/science magnet students at the school.

The students who have participated in the program have performed extremely well (Table 2). The grades for the on-campus, full-time registered students are included for the spring 2008 semester as a comparison. Analysis of how students with different incoming academic qualifications perform reveals that the current requirements for the Distance Calculus program are sufficient, and that the incoming scores for the small number of students receiving a C or below are not distinguishable from students receiving an A or B (data not shown).

<table>
<thead>
<tr>
<th>Table 2—Calculus Grades for Distance and On-Campus Students, 2005-2009</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semester</td>
</tr>
<tr>
<td>-------------------</td>
</tr>
<tr>
<td>Fall 2005-06</td>
</tr>
<tr>
<td>Spring 2005-06</td>
</tr>
<tr>
<td>Fall 2006-07</td>
</tr>
<tr>
<td>Spring 2006-07</td>
</tr>
<tr>
<td>Fall 2007-08</td>
</tr>
<tr>
<td>Spring 2007-08</td>
</tr>
<tr>
<td>On-campus</td>
</tr>
<tr>
<td>Fall 2008-09</td>
</tr>
</tbody>
</table>

4. Recruiting Results

The Distance Calculus program has grown tremendously since 2005, and has the potential to take another leap in 2010. Georgia’s new mathematics curriculum promises to funnel increasing numbers of advanced students into calculus during their junior year in high school, and the currently participating school systems, which together enroll over 375,000 students, are planning to utilize the Distance Calculus program as the senior year mathematics course for their most advanced students. In addition, there are numerous other school systems in the state, as well as private and home schools, who are likely going to be interested as distance learning becomes more common. Through this program, a large fraction of the most academically gifted students in Georgia may one day start their college career with Distance Calculus. As a recruiting tool, this provides the Office of Admissions with unprecedented access to Georgia’s top students.

Distance Calculus students are encouraged to submit applications to Georgia Tech during their senior year in high school. The normal application fee is waived, since they paid it to apply to the Distance Calculus program, and a large majority of the students apply for freshman admission (Table 3). To date, essentially all have been accepted because the entrance criteria for the high school program mirrors closely the freshman entrance criteria, and the percent of those students who have deposited, indicating intention to attend, has been much higher than normal for those schools and that student population. In addition, a large fraction of the incoming
students designated as Presidential Scholarship Semi-finalists have participated in the Distance Calculus program.

<table>
<thead>
<tr>
<th>Year in HS Program</th>
<th>% of Eligible Students who Applied</th>
<th>% Accepted</th>
<th>% Yield of Eligible Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005-06</td>
<td>97%</td>
<td>100%</td>
<td>57%</td>
</tr>
<tr>
<td>2006-07</td>
<td>86%</td>
<td>100%</td>
<td>48%</td>
</tr>
<tr>
<td>2007-08</td>
<td>88%</td>
<td>100%</td>
<td>61%</td>
</tr>
</tbody>
</table>

5. Retention and Progress Results

Preliminary results indicate that Distance Calculus students both come to Georgia Tech in large numbers, and do well. Below are some results for students who completed the Distance Calculus program in 2005-6 and 2006-7:

<table>
<thead>
<tr>
<th>Year in Program</th>
<th># Completed Program</th>
<th># Entered this University</th>
<th>% Had 4.0 GPA Fall Term</th>
<th>% Had 3.5 or Better GPA Fall Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005-06</td>
<td>32</td>
<td>17</td>
<td>47.1%</td>
<td>76.5%</td>
</tr>
<tr>
<td>2006-07</td>
<td>71</td>
<td>32</td>
<td>46.9%</td>
<td>78.1%</td>
</tr>
</tbody>
</table>

6. Lessons Learned

Grading
The students in the Distance Calculus project receive two grades for their performance--a grade from the university (and thus on a college transcript), and a grade on their high school transcript. The policy of Georgia Tech is to only give letter grades: A = 4, B = 3, etc. This conflicts with the policy of many of the school systems, where students receive numerical grades 0=100. The details of the translation from the letter grade to a numerical grade are based on the policies of the individual school systems. One system, for instance, uses the translation A = 93, together with a 7 point addition because the course is “Advanced”. Because of the high achievement level of the students involved, this is actually a cause of some anxiety by the students and their parents, as in many cases these students are in competition to be school valedictorian. These issues should be communicated to parents and students before the program starts. Ultimately, how this is resolved is a school system-based decision.

Confidentiality
Confidentiality of high school grades and college grades is subject to different policies and federal laws and regulations. The applicable federal law is the Federal Educational Records and Privacy Act (FERPA), which substantially prohibits discussion of student grades and class performance with third parties, including the parents of students. This is, of course, different
from high school policy, where grades and performance are often discussed with parents. Grades and performance may be legally released to third parties that have a “right to know”, such as the high schools, since they are educational partners. However, we also request that high school students sign forms giving permission for university staff to disclose their grades to their respective school systems. This allows us to flag school counselors if a student is not doing well in the course.

Calendar
The Distance Calculus policy is that the courses are run on the university calendar. There are some discrepancies between the Georgia Tech calendar and those of the various school systems. Ordinarily we try to avoid tests or quizzes on dates where the school systems are not in session. But nonetheless, the high school students are responsible for all course material that is presented, even for dates when the K-12 school systems are not in session. The students can easily go to the class web site and view any previous classes. They can also sign into the course from anywhere in the world, and view classes live, even when they are on spring break.

Technology
A major challenge for distance learning programs between universities and high schools is the issue of technological support. Each school system, and even each high school, is typically organized and operated differently, with different teleconferencing equipment and a networking infrastructure that includes a diverse array of people. These organizational structures present challenges to the university since it is often not clear who the right person is to fix problems that arise, including issues of access, authorization to computer accounts, understanding of school and system network infrastructure and firewalls, as well as policy level discussions about audio and video file transmission on the school network. Further, the 8:00 a.m. class time is earlier than the start time at some of the high schools, so support staff, and back-up staff, must be designated to ensure that the students have access to the classrooms and equipment. The university has provided training to high school staff regarding equipment operation and connection instructions, but understanding the policy and system-level infrastructure requires a different set of resources for discussion and problem resolution.

Another factor to consider in operating remote classes, is that high schools and the school systems often upgrade equipment over the summer, often without much thought to the distance applications since they are not taught locally. Therefore, a new round of problem solving is often required each fall semester after renovations take place. Good communication and assistance for identifying these impacts between all parties is crucial for a sustainable partnership.

Relations with High Schools
There has been some concerns expressed by teachers at the local high schools about being “replaced”; that is, that instruction may come from outside the school for subjects that they are capable of teaching. Some of these schools understandably pride themselves in having successfully built up advanced mathematics and science programs, and are not anxious to outsource to the local university. For this reason, the decision about whether to include a particular high school or school system in the program has been left primarily to the school systems with the understanding that the partnerships are between the university and the school
system, not between the university and particular schools or teachers. After four years of operation, we find that the best situation is when a math teacher at the school actively participates in the program, even sitting in on the daily instruction; a wonderful way for the teachers to also refresh their knowledge for subsequent teaching. We are also starting to see examples of the technology use whereby teachers in the high school system are sharing classes between high schools and also seeking to use the equipment for their own educational advancement without having to leave the high schools.

**Student, High School and University Benefits**

Having the teachers utilize the equipment for their own professional development and for sharing instructional expertise across the school systems is certainly a significant benefit to high school teachers and their students. All involved gain substantial experience with the uses of distance learning technologies, positioning them better for the future of education.

Further benefits for students and their parents include the ability to earn up to 8 semester credit hours of calculus from Georgia Tech for essentially no cost; credit that can be applied to other universities via transfer credit. The Georgia Tech School of Mathematics provides letters of explanation for any student applying to colleges out of state that might question the acceptability of high school distance learning courses. No college has thus far denied Distance Calculus students credit for this advanced calculus course.

From Georgia Tech’s perspective, this program is a terrific recruiting tool. We have found that students who have experienced this program stay at this university in higher numbers than would have typically enrolled from the participating high schools.

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