Can Flipped Classrooms Be Utilized to Effectively Produce Successful, Engaged Engineering Students? A Comparison of an On-Line vs. Inverted Classroom through a Junior-Level Transportation Engineering Course

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Roxann is currently Civil Engineering Faculty in the College of Engineering and Applied Science (CEAS) at CU-Denver (UCD). She has been teaching both graduate and undergraduate classes at UCD since 2011, including Advanced Highway Design, Highway Capacity Analysis, Transportation Impact Analysis, and Introduction to Transportation Engineering. She also teaches the Transportation Depth Professional Engineer (PE) Examination Refresher Review Course for Continuing Engineering Education Program. Ms. Hayes additionally serves as the undergraduate student advisor for Civil Engineering and the Dean’s Office.

Roxann received her M.Sc. in 2011 in Civil Engineering (Specialty in Transportation Engineering) in May of 2011. She also received her B.Sc. in 1995 from the Colorado School of Mines (CSM) in Engineering with a Civil Specialty and two minors in Environmental Engineering and Public Affairs.

Upon graduating from CSM, Ms. Hayes worked for over fifteen years in the private and government sector as a professional civil engineer. This included such companies as Larimer and Archuleta Counties, the City of La Porte (TX), Bayer Chemical and Exxon Chemical. She continues to run her own small consulting engineering business, conducting traffic impact studies. Roxann has a professional engineering license in the State of Colorado.

In 2005, she "retired" from full-time engineering to take care of her children and started working part-time as a lecturer and faculty advisor for CSM, teaching Mechanics of Materials and Senior Design. She has since left teaching at CSM to work full-time at UCD.

Roxann has served as the President of the American Society of Civil Engineers (ASCE) Colorado Section, and on the Board of Directors for the CSM Alumni Association.

Roxann was also recently named the 2015 College of Engineering and Applied Science Outstanding Faculty in Teaching. The award—a cash prize and commemorative plaque—will be presented to Roxann at the 2015 Year-End Celebration on May 15.
Can Flipped Classrooms Be Utilized to Effectively Produce Successful, Engaged Engineering Students? A Comparison of an On-Line vs. Inverted Classroom through a Junior-Level Transportation Engineering Course

Abstract
One of the latest buzz words in engineering pedagogy is the concept of an inverted or “flipped” classroom. Students complete the lecture portion of the class on their own time by using video lessons prepared by the professor, and utilize the textbook and other materials as a study guide. Then, classroom time is dedicated to a more “hands-on” approach. Flipped classroom activities include guided, independent practice or lab work, and group-based interactive learning activities or inquiries. Is a flipped classroom a better instructional technique to enhance student learning? This research project will help solve this question by comparing two sections of a class in a statistically valid manner during the same semester.

In the Fall of 2014, two sections of junior-level CVEN 3602 – Transportation Engineering were taught at the University of Colorado-Denver in the Civil Engineering Department. One of these sections is a traditional classroom, assigned to meet two days a week, at 1.25 hours each day. For a portion of the semester, this section was “flipped”. The second section is an entirely on-line course.

Student performance was measured throughout the semester. The two sections were compared with the exact same test material given on the same date. The research concentrates on the topic of geometric design (both vertical and horizontal curves). Both sections accessed the same video lectures, and were given the same materials for studying.

During the non-flipped portion of the class, the on-line students performed better on the first exam, but worse on a second exam. The on-line section also performed better on Question Eight material on the comprehensive final (which covered non-flipped classroom material).

Once the class was flipped, on-line quizzes were given before classroom time to better ensure students watched the video lectures. The quiz scores of the flipped classroom exceeded the on-line classroom in every topic.

A majority of the comprehensive final tested the topics covered in the flipped classroom. Overall, the thirty students in the flipped classroom performed significantly better than the on-line classroom. For every question except Question Eight, the flipped class exceeded the mean of the on-line class. However, since these same thirty students didn’t always score higher on previous exams in the course, it shows these students are not necessarily better test takers or higher performing students in general.

From the results of this study, it can be concluded that the flipped classroom was more effective at conveying junior-level transportation engineering-related material.
**Introduction**

Typically, a traditional classroom at the University level consists of lectures during class time, delivered at a student population by a professor. Homework is completed after these lectures by the students on their own time, at an individual level. Some students may attempt this work as a team, and may meet with the teaching assistant or professor during office hours for limited assistance. Tests are given periodically during the semester to gauge student learning.

A “flipped” classroom attempts a completely different delivery. Students complete the “lecture” portion of the class on their own time. This may consist of studying the topics by themselves, using video lessons prepared by the professor, and utilizing the textbook and other materials as a study guide. Then, classroom time is dedicated to a more “hands-on” approach. Students apply the knowledge they have gathered by working the homework or solving other assigned problems during the scheduled meeting time. The professor and teaching assistant are available to work with the individual student, or teams, as they complete the problems. The classroom time may also be used for completing practical work (for transportation engineering, this may include field time to conduct parking or traffic studies).

“We define the flipped classroom as an educational technique that consists of two parts: interactive group learning activities inside the classroom, and direct computer-based individual instruction outside the classroom.”¹

**Literature Review**

A literature review on flipped classrooms has shown that existing studies have two problems:

- They compare a flipped vs. traditional classroom on a semester-by-semester basis.
- The existing studies do not examine student performance throughout the semester, or continue to evaluate their performance in later activities.¹

Through the literature review, it was determined that many of the existing studies compare a flipped vs. traditional classroom strictly on a semester-by-semester basis. That is, a traditional classroom from the first semester is compared to a “flipped” classroom the second semester. This introduces a wide variety of irregularities between the two semesters: test structure and content, professor interaction, lecture inconsistencies, and other variables.

Bishop conducted a literature review on the topic of flipped classrooms, and found 24 relevant studies.¹ Very few of these studies examined a flipped and traditional course the same semester. In addition, only one study, from Day, examined student performance throughout the semester.² Bishop came to the conclusion: “Additional research is needed to examine the influence of flipped classroom instruction on objective learning outcomes.”¹

**The Question**

Is a flipped classroom a better instructional technique to enhance student learning? This study will help solve this question by comparing two sections in a statistically valid manner during the same semester.

**Implementation**

In the Fall of 2014, two sections of junior-level CVEN 3602 – Transportation Engineering were taught by the principal investigator at the University of Colorado-Denver. This course is required
for all undergraduate Civil Engineering students at this university, consisting of mostly junior-level students. One of these sections is a traditional classroom, assigned to meet two days a week, at 1.25 hours each day. For approximately one month of the semester, this section was “flipped.” The second section is an entirely on-line course. Both courses utilized Canvas, the Learning Management System (LMS) currently utilized by this university. Canvas was first introduced in Fall of 2013.

This is the first time within the College of this university that a flipped classroom technique has ever been used. The results from this study may lead the College to implement the flipped classroom in other departments and courses.

Within this controlled environment, the flipped classroom was compared to an on-line section. Both sections accessed the same video lectures, and were given the same materials for studying. Students were free to choose their desired section of CVEN 3602 (whether the on-line or flipped classroom). Table 1 (below) shows the similarities and differences between the two sections:

<table>
<thead>
<tr>
<th><strong>“Flipped Classroom” Section</strong></th>
<th><strong>On-Line Section</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Section 001</strong></td>
<td><strong>Section E01</strong></td>
</tr>
<tr>
<td>• Watch a 1.25 hour, twice weekly video lecture. Typical lecture: PowerPoint Slides, videos on transportation engineering topics, and problems worked on a board.</td>
<td>• Watch the same 1.25 hour, twice weekly video lecture.</td>
</tr>
<tr>
<td>On-line quiz after each video lecture to ensure students have watched the video. Usually two to three short questions (multiple-choice, true or false, and fill-in-the blank). See Appendix A for quiz content.</td>
<td>• On-line quizzes.</td>
</tr>
<tr>
<td>• Office hours (available 2 hours a week each by professor and TA).</td>
<td>• Office hours.</td>
</tr>
<tr>
<td>• Homework submitted via Canvas, and graded by TA.</td>
<td>• Homework (same teacher’s assistant graded both sections).</td>
</tr>
<tr>
<td>• “Flipped Classroom” activities during assigned class time.</td>
<td>• No flipped classroom activities. • Additional discussion module for the on-line course, resulting in invigorating and varied dialogues in a variety of transportation engineering topics.</td>
</tr>
<tr>
<td>• Tests identical to on-line section.</td>
<td>• Tests.</td>
</tr>
</tbody>
</table>
The flipped classroom concept was incorporated only during a portion of the semester, focusing on highway geometric design (horizontal and vertical curves). The two sections were compared with the exact same test material given on the same date. The students in both sections also completed two separate knowledge self-assessments: a pre-topic and post-topic assessment.

The students will continue to be evaluated (results not published as part of this study) in CVEN 4602: Advanced Highway Engineering and the Fundamentals of Engineering (FE) Exam.

On-line quizzes were given to both sections during the “flipped” portion of the semester to better assure that students were watching the videos and watching the notes before the classroom work. One previous study on flipped classrooms in engineering showed that students were not watching the videos prior to the in-class work. This was due to a lack of accountability – they were not tested on content of those videos until the actual exams.

Exam One and Exam Two Results
Two exams were given to the classes prior to the start of the flipped classroom work. Figure 1 (following) shows that the flipped classroom performed better in Exam One, yet the on-line classroom performed better in Exam Two.

Table 2 (following) shows that the flipped classroom performed over five percentage points higher on Exam One, yet was over two percentage points lower on Exam Two. These “control” exams show, before there were any flipped class meetings, the students in the flipped class did not always perform better than the students in the on-line classroom.
Table 2 – Comparison of Exam One and Exam Two Means

<table>
<thead>
<tr>
<th></th>
<th>No. of Students</th>
<th>Section Mean</th>
<th>Std. Deviation</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Flipped</td>
<td>On-Line</td>
<td>Flipped</td>
<td>On-Line</td>
</tr>
<tr>
<td>Exam One</td>
<td>30</td>
<td>30</td>
<td>79.6</td>
<td>74.4</td>
</tr>
<tr>
<td>Exam Two</td>
<td>8</td>
<td>8</td>
<td>85.9</td>
<td>88.1</td>
</tr>
</tbody>
</table>

Quiz Results

Four on-line short quizzes were given (due before the flipped class met) to encourage students to observe the videos. As a side effect of these quizzes, student performance can be measured through the flipped portion of the class.

Figure 2 (below) shows that the results of the before-class on-line quizzes between the two sections.

Statistical analysis was utilized (t-Test: Two-Sample Assuming Unequal Variances) in order to effectively evaluate these results. The comparison of the two sections is found in Table 3, below. The quiz scores of the flipped classroom exceeded the on-line classroom in every topic, but only the final quiz (Quiz 23 – Spiral and Compound Curves) found the results to be significant.

Table 3 – Comparison of Statistics for Quiz Results

<table>
<thead>
<tr>
<th>Quiz</th>
<th>Total No. of Points</th>
<th>No. of Observations</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>p-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quiz 20 - Horizontal Design</td>
<td>30</td>
<td>29</td>
<td>8</td>
<td>28.3</td>
<td>23.8</td>
</tr>
<tr>
<td>Quiz 21 - Crest Vertical Design</td>
<td>30</td>
<td>29</td>
<td>8</td>
<td>25.2</td>
<td>21.3</td>
</tr>
<tr>
<td>Quiz 22 - Sag Vertical Design</td>
<td>40</td>
<td>28</td>
<td>8</td>
<td>33.6</td>
<td>30.0</td>
</tr>
<tr>
<td>Quiz 23 - Spiral and Compound Curves*</td>
<td>40</td>
<td>26</td>
<td>8</td>
<td>33.1</td>
<td>22.5</td>
</tr>
</tbody>
</table>

*Comparison of means t-test significant for p<0.05
Overall Comprehensive Final Results
Each section was given the exact same in-class final comprehensive test with eight questions (see Appendix B) on the same date. Figure 3 (below) shows a comparison of the mean of each question from the on-line classroom to the flipped classroom.

Once again, a t-Test: Two-Sample Assuming Unequal Variances was used to evaluate the differences in the means of the two sections. For every question except question eight, the flipped class exceeded the mean of the on-line class. However, only the results regarding questions one, two, three and eight were found to be significant at the 95% level of confidence. This may be due to the relatively low number of observations for the on-line class (eight students signed the consent form to agree to be in this study, while thirty students participated in the flipped classroom experience).

The final was comprehensive, which tested class material from all portions of the class (flipped and traditional). However, this study specifically examined the flipped classroom portion, which only utilized horizontal and vertical curve design. Question Eight contained information that was not provided in the flipped classroom portion of the semester. Figure 4 (following) shows the problem statement for Question Eight.
At this point, the principal investigator then removed Question Eight from the data set. In evaluating the revised data, it was determined that the means were significant for the overall total.

The comparison of the two sections is found in Table 4, below. The mean score of the final for the flipped class was 69.5 points, while the on-line mean was 57.8 points. With Question Eight removed from the data, the difference in means t-test of the overall total is significant.

<table>
<thead>
<tr>
<th>Question</th>
<th>Total No. of Points</th>
<th>Flipped Mean</th>
<th>On-Line Mean</th>
<th>Flipped Std. Dev.</th>
<th>On-Line Std. Dev.</th>
<th>p-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1*</td>
<td>14</td>
<td>12.5</td>
<td>11.4</td>
<td>2.0</td>
<td>1.8</td>
<td>0.078</td>
</tr>
<tr>
<td>Q2*</td>
<td>6</td>
<td>5.0</td>
<td>3.9</td>
<td>1.2</td>
<td>1.5</td>
<td>0.041</td>
</tr>
<tr>
<td>Q3*</td>
<td>15</td>
<td>11.5</td>
<td>7.9</td>
<td>4.7</td>
<td>3.6</td>
<td>0.016</td>
</tr>
<tr>
<td>Q4</td>
<td>18</td>
<td>14.8</td>
<td>12.3</td>
<td>4.4</td>
<td>5.6</td>
<td>0.134</td>
</tr>
<tr>
<td>Q5</td>
<td>18</td>
<td>13.5</td>
<td>11.0</td>
<td>5.4</td>
<td>5.8</td>
<td>0.152</td>
</tr>
<tr>
<td>Q6</td>
<td>8</td>
<td>6.7</td>
<td>5.9</td>
<td>1.9</td>
<td>1.7</td>
<td>0.140</td>
</tr>
<tr>
<td>Q7</td>
<td>6</td>
<td>5.6</td>
<td>5.5</td>
<td>1.4</td>
<td>0.9</td>
<td>0.406</td>
</tr>
<tr>
<td>Q8*</td>
<td>15</td>
<td>10.0</td>
<td>12.5</td>
<td>5.3</td>
<td>4.5</td>
<td>0.099</td>
</tr>
</tbody>
</table>

*Sample size for flipped classroom = 30, sample size for on-line classroom = 8
*Comparison of means t-test significant for p<0.05

**Knowledge Pre- and Post-Assessments**

In addition, knowledge pre- and post-assessments were given to the students in both sections (see Appendix C for examples of these assessments). These assessments were given before the flipped classroom portion of the semester began, and again given immediately after. Each assessment asked the students to attempt a problem, and then to grade themselves. Figure 5 (following) shows the grading scale each student used for the four problems. Once the students had attempted the problems and provided a grade, the instructor then evaluated the attempt and provided an adjusted “instructor score.”
Table 5 (below) shows the adjusted pre- and post-assessment scores. The flipped section improved 1.49 points from the pre-assessment scores, while the on-line class only improved 1.22 points.

<table>
<thead>
<tr>
<th>Consent Form Signed</th>
<th>Flipped</th>
<th>On-Line</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowledge Pre-Assessment # of Observations</td>
<td>29</td>
<td>8</td>
</tr>
<tr>
<td>Knowledge Post-Assessment # of Observations</td>
<td>27</td>
<td>8</td>
</tr>
<tr>
<td>Knowledge Pre-Assessment - Mean</td>
<td>2.94</td>
<td>2.75</td>
</tr>
<tr>
<td>Knowledge Post-Assessment - Mean</td>
<td>1.45</td>
<td>1.53</td>
</tr>
<tr>
<td>Change from Pre to Post</td>
<td>+1.49</td>
<td>+1.22</td>
</tr>
</tbody>
</table>

**Conclusion**

This study followed the same set of students through a flipped and non-flipped classroom portion of a semester. During the non-flipped portion, the eight on-line students performed better on one exam given earlier in the semester, as well as the Question Eight material on the comprehensive final. For the comprehensive final, the thirty students in the flipped classroom performed significantly better than the on-line classroom. However, since these same thirty students didn’t always score higher on previous exams in the course, it shows these students are not necessarily better test takers or higher performing students in general.

Therefore, it can be concluded that the flipped classroom was more effective at conveying junior-level transportation engineering-related material.

**Continuation of Tracking**

The class has since concluded for Fall 2014. The principal investigator will continue tracking the students in this course. Success in education can be measured not only by the grade received in a class, but also how a student thrives in advanced classes on the same topic. The two sections can be compared with their success in CVEN 4602: Advanced Highway Design through the evaluation of test scores.

The students will also be tracked as they enter their senior year and take the Fundamentals of Engineering (FE) exam. The FE exam is the first step in the process leading to a Professional Engineering license. This is a national, six-hour test administered by the National Council of Examiners for Engineering and Surveyors (NCEES), and is required of all graduating seniors at CU-Denver. Transportation engineering, however, is only a portion of the 110 multiple-choice questions on the FE exam.
Bibliography


Appendix A: Quiz Content
Appendix B: Final Comprehensive Exam
Appendix C: Pre- and Post- Assessment Examples
Appendix A: Quiz Content
Students have either already taken or started taking this quiz, so be careful about editing it. If you change any quiz questions in a significant way, you may want to consider regrading students who took the old version of the quiz.

**Lecture 20 Quiz**

### Question

Horizontal curves are:

- A simple curve
- A parallelogram
- A parabola
- No answer text provided.

### Question

Choose the reverse curve:
The two yellow lines found on the photo of the horizontal curve, then later in the lecture, represent tangents.

- True
- False
Quiz Summary

Average Score: 94%
High Score: 100%
Low Score: 67%
Standard Deviation: 3.78
Average Time: 02:23

Question Breakdown

Attempts: 29 out of 29

Horizontal curves are:

- **Correct answer**: 86% of your students correctly answered this question.
  - 86% of your students correctly answered this question.

- **A parallelogram**: 0%
  - 0% of your students answered this option.

- **A parabola**: 14%
  - 14% of your students answered this option.

- **A simple curve**: 86%
  - 86% of your students answered this option.

Attempts: 29 out of 29

Choose the reverse curve:

- **Correct answer**: 100% of your students correctly answered this question.
  - 100% of your students correctly answered this question.

- **0%**: 0%
  - 0% of your students answered this option.

- **100%**: 0%
  - 0% of your students answered this option.
The two yellow lines found on the photo of the horizontal curve, then later in the lecture, represent tangents.

**Correct answer**
97% of your students correctly answered this question.

**+0.41 Discrimination Index**

True

False

97% 3%
Quiz Summary

- Average Score: 93%
- High Score: 100%
- Low Score: 67%
- Standard Deviation: 4.00
- Average Time: 02:34

Question Breakdown

Attempts: 10 out of 10

**Horizontal curves are:**

- **Correct answer**
  - 80% of your students correctly answered this question.

- A parabola: 0%
- A simple curve: 80%

**Discrimination Index:** +1.00

Attempts: 10 out of 10

**Choose the reverse curve:**

- **Correct answer**
  - 100% of your students correctly answered this question.

- No answer text provided: 100%

**Discrimination Index:** -0.00
The two yellow lines found on the photo of the horizontal curve, then later in the lecture, represent tangents.

Correct answer

100% of your students correctly answered this question.

-0.00 Discrimination Index

100% True

0% False
Lecture 21 Quiz

Questions

1. If I have a crest vertical curve with grades of +4% and -5%, what is A?
   - 9
   - 0.09
   - 1
   - 0.1

2. I have a crest vertical curve with grades of +4% and -5%. For the SSD equation, I will choose a value of G that will give me the worst case scenario, since grade constantly changes on vertical curve.
   - True
   - False

3. In making an assumption for design, \( h_2 \) can be used for the height of a tail light on the front car. What else did I say that \( h_2 \) can be?
   - An object in the road.
   - The height of the tail light on the car in back.
   - The height of the driver in the car up ahead.

Notify users this quiz has changed

Points 30
Published

https://ucdenver.instructure.com/courses/31809/quizze/35393/edit
Quiz Summary

Average Score: 84%
High Score: 100%
Low Score: 33%
Standard Deviation: 6.23
Average Time: 07:13

Question Breakdown

If I have a crest vertical curve with grades of +4% and -5%, what is A?

Correct answer: 83%
83% of your students correctly answered this question.

Discrimination Index: +0.53

I have a crest vertical curve with grades of +4% and -5%. For the SSD equation, I will choose a value of G that will give me the worst case scenario, since grade constantly changes on vertical curve.

Correct answer: 90%
90% of your students correctly answered this question.

Discrimination Index: +0.65

True: 90%
False: 10%

Attempts: 29 out of 29
In making an assumption for design, $h_2$ can be used for the height of a tail light on the front car. What else did I say that $h_2$ can be?

- Correct answer: 79% of your students correctly answered this question.
- The height of the driver in the car up ahead.
- The height of the tail light on the car in back.
- An object in the road.

Discrimination Index: +0.56
Quiz Summary

Average Score: 85%
High Score: 100%
Low Score: 33%
Standard Deviation: 6.56
Average Time: 06:37

Question Breakdown

If I have a crest vertical curve with grades of +4% and -5%, what is A?

- Correct answer: 82%
  - 82% of your students correctly answered this question.

I have a crest vertical curve with grades of +4% and -5%. For the SSD equation, I will choose a value of G that will give me the worst case scenario, since grade constantly changes on vertical curve.

- Correct answer: 100%
  - 100% of your students correctly answered this question.

https://ucdenver.instructure.com/courses/318009/quizzes/35393/statistics
In making an assumption for design, $h_2$ can be used for the height of a tail light on the front car. What else did I say that $h_2$ can be?

<table>
<thead>
<tr>
<th>Option</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correct answer: 73% of your students correctly answered this question.</td>
<td>73%</td>
</tr>
<tr>
<td>An object in the road.</td>
<td>73%</td>
</tr>
<tr>
<td>The height of the driver in the car up ahead.</td>
<td>18%</td>
</tr>
<tr>
<td>The height of the tail light on the car in back.</td>
<td>9%</td>
</tr>
</tbody>
</table>

Discrimination Index: +0.82
Students have either already taken or started taking this quiz, so be careful about editing it. If you change any quiz questions in a significant way, you may want to consider regrading students who took the old version of the quiz.

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**Question**

The PVC of a sag vertical curve is located at station 41+20.60. What is location of the first full 100 ft station?

- 42+00.00
- 42+10.00
- 41+00.00
- 50+00.00

**Question**

Convert 43550.00 ft to stations.

- 435+50.00
- 43+550.00
- 4+3550.00
- 4355+00.00

**Question**

If the curve is symmetrical, the absolute value of $g_1$ cannot be equal to the absolute value of $g_2$.

- True
- False

**Question**

Which equation should you use to determine the location of the high point from the PVC?

$$\frac{g_1 L}{g_1 - g_2}$$
\[ Y_{PVC} + g_1 x + \frac{(g_2 - g_1)}{(2L)} x^2 \]
\[ Y_{PVI} + g_2 (x - L/2) \]
Quiz Summary

Average Score: 84%
High Score: 100%
Low Score: 25%
Standard Deviation: 7.18
Average Time: 05:15

Question Breakdown

The PVC of a sag vertical curve is located at station 41+20.60. What is location of the first full 100 ft station?

- Correct answer: 89%
- Discrimination Index: +0.65
- Students' responses:
  - 89%: 42+00.00
  - 7%: 41+00.00

Convert 43550.00 ft to stations.

- Correct answer: 96%
- Discrimination Index: -0.10
- Students' responses:
  - 96%: 435+50.00
  - 0%: 4+3550.00
If the curve is symmetrical, the absolute value of $g_1$ cannot be equal to the absolute value of $g_2$.

- **Correct answer**
- 96% of your students correctly answered this question.
- **Discrimination Index**: +0.63

- True
- False

Which equation should you use to determine the location of the high point from the PVC?

- **Correct answer**
- 54% of your students correctly answered this question.
- **Discrimination Index**: +0.76
Quiz Summary

- Average Score: 81%
- High Score: 100%
- Low Score: 50%
- Standard Deviation: 8.29
- Average Time: 05:17

Question Breakdown

The PVC of a sag vertical curve is located at station 41+20.60. What is location of the first full 100 ft station?

Correct answer: 75% of your students correctly answered this question.

Attempts: 12 out of 12

- 75% 42+00.00
- 0% 42+10.00

Discrimination Index: +0.64

Convert 43550.00 ft to stations.

Correct answer: 83% of your students correctly answered this question.

Attempts: 12 out of 12

- 83% 435+50.00
- 0% 4355+00.00

Discrimination Index: +0.40
If the curve is symmetrical, the absolute value of \( g_1 \) cannot be equal to the absolute value of \( g_2 \).

- **Correct answer**: 100% of your students correctly answered this question.

- **Discrimination Index**: -0.00

- **False**

Which equation should you use to determine the location of the high point from the PVC?

- **Correct answer**: 67% of your students correctly answered this question.

- **Discrimination Index**: +0.85

No answer text provided.
What does TS mean?
- Point of change from tangent to spiral curve.
- Tangent Spiral
- Trouble Space
- Point of change from spiral to tangent curve

CS = PT for a spiral curve.
- True
- False

In a compound curve, PC + L = PCC.
- True
- False

In a compound curve, L + PCC = TS
- True
Correct Answer: False

Notify users this quiz has changed

New Question
New Question Group
Find Questions

Cancel  Save
Quiz Summary

- **Average Score**: 83%
- **High Score**: 100%
- **Low Score**: 50%
- **Standard Deviation**: 6.21
- **Average Time**: 06:08

Question Breakdown

1. **Attempts**: 23 out of 23
   - **Question**: CS = PT for a spiral curve.
   - **Correct Answer**: True (87%)
   - **Discrimination Index**: +0.40

2. **Attempts**: 23 out of 23
   - **Question**: What does TS mean?
   - **Correct Answer**: Point of change from spiral to tangent curve
   - **Discrimination Index**: +0.40

3. **Attempts**: 23 out of 23
   - **Question**: In a compound curve, L + PCC = TS
     - **Point of change from tangent to spiral curve**: Tangent Spiral
     - **Point of change from spiral to tangent curve**: Trouble Space

---

https://ucdenver.instructure.com/courses/12355/quizzes/35581/statistics
In a compound curve, $PC + L = PCC$. 

Correct answer
70% of your students correctly answered this question.

70° True
30° False

Attempts: 23 out of 23

$+0.48$ Discrimination Index

70% 30%
Quiz Summary

- Average Score: 68%
- High Score: 100%
- Low Score: 25%
- Standard Deviation: 9.00
- Average Time: 07:31

Question Breakdown

1. What does TS mean?
   - Correct answer: 90% of your students correctly answered this question.
   - Discrimination Index: +0.26
   - Trouble Space: 0°
   - Point of change from tangent to spiral curve: 90°
   - Point of change from spiral to tangent curve: 0°
   - Tangent Spiral: 10°

2. CS = PT for a spiral curve.
   - Correct answer: 50% of your students correctly answered this question.
   - Discrimination Index: +0.56
   - True: 50°
   - False: 50°

3. In a compound curve, L + PCC = TS
   - Discrimination Index: +0.75
   - True: 50°
   - False: 50°
In a compound curve, PC + L = PCC.

Correct answer
60% of your students correctly answered this question.

Attempts: 10 out of 10

60% True
40% False

Discrimination Index: +0.41
Appendix B: Final Comprehensive Exam
From NCEES FE Reference Handbook

\[ D = \text{Density (veh/mi/lane)} \]
\[ O = \text{occupancy (decimal)} \]
\[ L_v = \text{length of average vehicle (ft)} \]
\[ L_d = \text{length of detector (ft)} \]
\[ SSD = \text{stopping sight distance (ft)} \]
\[ t = \text{driver reaction time (sec)} \]
\[ V = \text{design speed (mph)} \]
\[ W = \text{width of intersection, curb-to-curb (ft)} \]
\[ l = \text{length of vehicle (ft)} \]
\[ y = \text{length of yellow interval to nearest 0.1 sec (sec)} \]
\[ r = \text{length of red clearance interval to nearest 0.1 sec (sec)} \]

Traffic Safety Equations

\[ RMEV = 4 \times 1,000,000 \frac{V}{V} \]
where
\[ RMEV = \text{crash rate per million entering vehicles} \]
\[ A = \text{number of crashes, total or by type occurring in a single year at the location} \]
\[ V = \text{average daily traffic entering intersection} \]

\[ RMVM = A \times 100,000,000 \frac{VMT}{VMT} \]
where
\[ RMVM = \text{crash rate per million vehicle miles} \]
\[ A = \text{number of crashes, total or by type at the study location, during a given period} \]
\[ VMT = \text{vehicle miles of travel during the given period} \]

Stopping Sight Distance

\[ SSD = 1.47V + \frac{V^2}{30(\frac{9}{32.2}) \pm G} \]

Where:

\[ D = \frac{5.280 \times O}{L_v + L_d} \]

where:

\[ D = \text{Density (veh/mi/lane)} \]
\[ O = \text{occupancy (decimal)} \]
\[ L_v = \text{length of average vehicle (ft)} \]
\[ L_d = \text{length of detector (ft)} \]

\[ v = \frac{3,600}{h_a} \]

where:

\[ v = \text{rate of flow, veh/h/ln} \]
\[ h_a = \text{average headway in the lane, s} \]

If 15-min periods are used, the PHF may be computed by Equation 4-2:

\[ PHF = \frac{V}{4 \times V_{15}} \]

where

\[ PHF = \text{peak hour factor,} \]
\[ V = \text{hourly volume (veh/h), and} \]
\[ V_{15} = \text{volume during the peak 15 min of the analysis hour (veh/15 min).} \]

Equation 4-2 HCM 2010

\[ FFS = 75.4 - f_{LW} - f_{LC} - 3.22TRD^{0.84} \]

where

\[ FFS = \text{free-flow speed} \]
\[ f_{LW} = \text{adjustment for lane width (mi/h) (Table 9.1)} \]
\[ f_{LC} = \text{adjustment for right side lateral clearance (mi/h) (Table 9.2)} \]
\[ TRD = \text{total ramp density (ramps per mile)} \]

Equation 4-3 HCM 2010

\[ d = \frac{v_r}{S} \]

where

\[ v_r = \text{flow rate (pc/hln)} \]
\[ S = \text{Average passenger car speed (mi/h)} \]
\[ D = \text{Density (pc/mi/h)} \]
\[ V_p = \frac{V}{(PHF)(N)(f_{p})(f_{ae})} \]  
(9.4)

where

\( v_c \) = demand flow rate under equivalent base conditions (pc/h/ln)
\( v' \) = demand volume under prevailing conditions (veh/h)
\( PHF \) = peak hour factor
\( N \) = number of lanes in the analysis direction
\( f_p \) = adjustment factor for unsaturated driver populations. Range is 0.85-1.00 (In general, use 1.00 for commuters or other acclimated drivers unless evidence exists to the contrary.)
\( f_{ae} \) = adjustment factor for presence of heavy vehicles in traffic stream (Eq. 9.5)

### Average Lane Width (ft) Reduction in FFS, \( f_{ae} \) (mi/h)

<table>
<thead>
<tr>
<th>Lane Width</th>
<th>Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>≥12</td>
<td>0.0</td>
</tr>
<tr>
<td>11-12</td>
<td>1.9</td>
</tr>
<tr>
<td>10-11</td>
<td>6.6</td>
</tr>
</tbody>
</table>

**Exhibit 11-8**
Adjustment to FFS for Average Lane Width

The federal regulations also stipulate that the overall maximum gross weight for a group of two or more consecutive axles should be determined from Eq. 3.2:

\[ W = 500 \left( \frac{LN}{N-1} + 12N + 36 \right) \]  
(3.2)

where

\( W \) = overall gross weight (calculated to the nearest 500 lb)
\( L \) = the extreme of any group of two or more consecutive axles (ft)
\( N \) = number of axles in the group under consideration

A general equation for the braking distance can therefore be written as

\[ D_B = \frac{u^2}{360(f_e G)} \]  
(3.4)

where

\( D_B \) = horizontal component of distance traveled during braking (that is, from time brakes are applied to time the vehicle comes to rest)
\( u \) = speed when brakes applied
\( f_e \) = coefficient of friction between the tires and the road pavement
\( G = \tan \gamma \) (% grade/100)

\[ S = \sqrt{\sum (u_i - \bar{u})^2 / (N-1)} \]  
(4.2)

where

\( S \) = standard deviation
\( \bar{u} \) = arithmetic mean
\( u_i \) = jth observation
\( N \) = number of observations

However, speed data are frequently presented in classes where each class consists of a range of speeds. The standard deviation is computed for such cases as

\[ S = \sqrt{\sum (f_i (u_i - \bar{u})^2) / (N-1)} \]  
(4.3)

where

\( u_i \) = midpoint of speed class \( i \)
\( f_i \) = frequency of speed class \( i \)

\[ n = \frac{\sum ((S_i / d_i)^2)}{1 + (1/N) \sum (S_i / d_i)^2} \]  
(4.7)

where

\( n \) = minimum number of count locations required
\( t \) = value of the student's distribution with \( (1 - \alpha/2) \) confidence level (\( N - 1 \) degrees of freedom)
\( N \) = total number of links (population) from which a sample is to be selected
\( \alpha \) = significance level
\( S \) = estimate of the spatial standard deviation of the link volumes
\( d \) = allowable range of error

### LOS Criteria for Freeway Segments

<table>
<thead>
<tr>
<th>LOS</th>
<th>Density (pc/mln)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>≤11</td>
</tr>
<tr>
<td>B</td>
<td>&gt;11-18</td>
</tr>
<tr>
<td>C</td>
<td>&gt;18-26</td>
</tr>
<tr>
<td>D</td>
<td>&gt;26-35</td>
</tr>
<tr>
<td>E</td>
<td>&gt;35-45</td>
</tr>
<tr>
<td>F</td>
<td>Demand exceeds capacity &gt;45</td>
</tr>
</tbody>
</table>

**Exhibit 11-9**
Adjustment to FFS for Right Side Lateral Clearence, \( f_{ae} \) (%)

### Table 4.1 Constant Corresponding to Level of Confidence

<table>
<thead>
<tr>
<th>Confidence Level (%)</th>
<th>Constant Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>68.3</td>
<td>1.00</td>
</tr>
<tr>
<td>86.6</td>
<td>1.64</td>
</tr>
<tr>
<td>90.0</td>
<td>1.96</td>
</tr>
<tr>
<td>95.5</td>
<td>2.00</td>
</tr>
<tr>
<td>98.8</td>
<td>2.50</td>
</tr>
<tr>
<td>99.0</td>
<td>2.58</td>
</tr>
<tr>
<td>99.7</td>
<td>3.00</td>
</tr>
</tbody>
</table>

where

\( N \) = minimum sample size
\( Z \) = number of standard deviations corresponding to the required confidence level (1.96 for 95 percent confidence level (Table 4.1))
\( \sigma \) = standard deviation (mi/h)
\( d \) = limit of acceptable error in the average speed estimate (mi/h)

### Comparison of Mean Speeds

- **Z-test**
  - Conduct to compare Absolute Difference Between:
    - Sample mean speeds vs. Product of standard deviation of the difference in means and the factor Z for a given confidence level.
    - If Absolute Difference between sample mean speeds is greater:
      - There is a significant difference in sample means at that specific confidence level.

\[ S_d = \sqrt{\frac{\sum S_1^2}{n_1} + \frac{\sum S_2^2}{n_2}} \]  
(4.6)

where

\( n_1 \) = sample size for study 1
\( n_2 \) = sample size for study 2
\( S_1 \) = square root of the variance of the difference in means
\( S_1^2 \) = variance about the mean for study 1
\( S_2^2 \) = variance about the mean for study 2
Table 9.3  PCEs for Heavy Vehicles in General Terrain Segments

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>Level</th>
<th>Rolling</th>
<th>Mountainous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trucks and buses, $E_T$</td>
<td>1.5</td>
<td>2.5</td>
<td>4.5</td>
</tr>
<tr>
<td>RVs, $E_r$</td>
<td>1.2</td>
<td>2.0</td>
<td>4.0</td>
</tr>
</tbody>
</table>


Horizontal Curve Formulas

- $D$ = Degree of Curve, Arc Definition
- $PC$ = Point of Curve (also called BC)
- $PT$ = Point of Tangent (also called EC)
- $PI$ = Point of Intersection
- $I$ = Intersection Angle (also called $\Delta$)
  Angle Between Two Tangents
- $L$ = Length of Curve, from $PC$ to $PT$
- $T$ = Tangent Distance
- $E$ = External Distance
- $R$ = Radius
- $LC$ = Length of Long Chord
- $M$ = Length of Middle Ordinate
- $c$ = Length of Sub-Chord
- $d$ = Angle of Sub-Chord
- $l$ = Curve Length for Sub-Chord

\[
R = \frac{5729.58}{D}
\]

\[
R = \frac{LC}{2 \sin(I/2)}
\]

\[
T = R \tan(I/2) = \frac{LC}{2 \cos(I/2)}
\]

\[
L = \frac{RY}{180} = \frac{L}{100}
\]

\[
M = R \left[1 - \cos(I/2)\right]
\]

\[
\frac{R}{E + R} = \cos(I/2)
\]

\[
\frac{R - M}{R} = \cos(I/2)
\]

\[
c = 2R \sin(d/2)
\]

\[
l = Rd \left(\frac{\pi}{180}\right)
\]

\[
E = R \left[\frac{1}{\cos(I/2)} - 1\right]
\]

Deflection angle per 100 feet of arc length equals $D/2$

Horizontal Curves

- Side friction factor (based on superelevation)
  \[
  0.01e + f = \frac{V^2}{15R}
  \]

- Spiral Transition Length
  \[
  L_s = \frac{3.15V^3}{RC}
  \]
  \[C = \text{rate of increase of lateral acceleration}\]
  \[\text{use 1 ft/sec}^3 \text{ unless otherwise stated}\]

- Sight Distance (to see around obstruction)
  \[
  HSO = R \left[1 - \cos \left(\frac{28.65S}{R}\right)\right]
  \]
  \[HSO = \text{Horizontal sight line offset}\]
### Vertical Curves: Sight Distance Related to Curve Length

<table>
<thead>
<tr>
<th></th>
<th>( S \leq L )</th>
<th>( S &gt; L )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Crest Vertical Curve</strong></td>
<td>( L = \frac{AS^2}{100(\sqrt{2}h_1 + \sqrt{2}h_2)^2} )</td>
<td>( L = 2S - \frac{200\left(\sqrt{h_1} + \sqrt{h_2}\right)^2}{A} )</td>
</tr>
<tr>
<td><strong>Standard Criteria:</strong></td>
<td>( h_1 = 3.50 \text{ ft and } h_2 = 2.0 \text{ ft:} )</td>
<td>( L = 2S - \frac{2.158}{A} )</td>
</tr>
<tr>
<td><strong>Sag Vertical Curve</strong> (based on standard headlight criteria)</td>
<td>( L = \frac{AS^2}{400 + 3.5S} )</td>
<td>( L = 2S - \frac{400 + 3.5S}{A} )</td>
</tr>
<tr>
<td><strong>Sag Vertical Curve</strong> (based on riding comfort)</td>
<td>( L = \frac{AT^2}{46.5} )</td>
<td></td>
</tr>
<tr>
<td><strong>Sag Vertical Curve</strong> (based on adequate sight distance under an overhead structure to see an object beyond a sag vertical curve)</td>
<td>( L = \frac{AS^2}{800\left(C - \frac{h_1 + h_2}{2}\right)} )</td>
<td>( L = 2S - \frac{800}{A}\left(C - \frac{h_1 + h_2}{2}\right) )</td>
</tr>
</tbody>
</table>

\( C \) = vertical clearance for overhead structure (overpass) located within 200 feet of the midpoint of the curve.

---

\( L \) = Length of curve  
\( PVC \) = Point of vertical curvature  
\( PVI \) = Point of vertical intersection  
\( PVT \) = Point of vertical tangency  
\( g_1 \) = Grade of back tangent  
\( x \) = Horizontal distance from \( PVC \) to point on curve  
\( g_3 \) = Grade of forward tangent  
\( a \) = Parabola constant  
\( y \) = Tangent offset  
\( E \) = Tangent offset at \( PVT = AL/800 \)  
\( r \) = Rate of change of grade  
\( K \) = Rate of vertical curvature

\( x_m \) = Horizontal distance to min/max elevation on curve = \(-\frac{g_1}{2a} = \frac{g_3L}{g_1 - g_2} \)

Tangent elevation = \( Y_{PVC} + g_1x \) and = \( Y_{PVI} + g_2(x - L/2) \)

Curve elevation = \( Y_{PVC} + g_1x + ax^2 = Y_{PVC} + g_1x + [(g_2 - g_1)/(2L)]x^2 \)

\[ y = ax^2 \quad a = \frac{g_2 - g_1}{2L} \quad E = a\left(\frac{L}{2}\right)^2 \quad r = \frac{g_2 - g_1}{L} \quad K = \frac{L}{A} \]
Name: ________________________________

Total Points = 100

This exam is closed book and closed notes.

FE Approved Calculators Only. The calculator I used: ________________________________

Be sure to show all work for maximum credit. Final answers should be shown in the space provided for worked problems. Show correct units.

You are not allowed to talk to or ask any question of the proctor. Write any assumptions on your test.

<table>
<thead>
<tr>
<th>Question #</th>
<th>Subject</th>
<th># Points Worth</th>
<th>Your Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Quick Problems</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Stopping Distance</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Horizontal Curve</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Crest Vertical Curve</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Vertical Clearance</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Sag Vertical Curve</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Compound Curve</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Traffic Flow Relationships</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>Exam No. 1</td>
<td>100 max</td>
<td></td>
</tr>
</tbody>
</table>
(1) Quick Problems
   a. Equivalent Volumes: A freeway operating in mountainous terrain has a traffic composition of 7% trucks, 3% buses, and 5% RV’s. What is the heavy vehicle factor? (4 pts)

   ANSWER: ____________________

   b. Spiral Curve: Given a horizontal curve with a 1360 ft radius, estimate the minimum length of spiral necessary for a smooth transition from tangent alignment to the circular curve. The design speed is 65 mph. (4 pts)

   ANSWER: ________________

   c. True or false (Circle One): Passenger-car equivalent for RVs on downgrade sections is taken to be the same as that for level terrain sections ($E_R = 1.2$) (2 pts)

   TRUE  FALSE

   d. Vertical Curve: A vertical curve connects two tangents. The approach tangent has a slope of +3.00%. The slope of the departure tangent is -2.00%. These two tangents intersect at STA 26+00.00 and elevation of 231.00 feet. If the length of the vertical curve is 1648 feet, what is most nearly the elevation of the PVT? (4 pts)

   ANSWER: ____________________
(2) Stopping Sight Distance
A vertical curve has a +4.33% grade intersecting with a -2.45% grade. What is the stopping sight distance of the curve with a 70 mph design speed? Assume $\frac{a}{g} = 0.35$ and a perception reaction time of 2.5 seconds.

(3) Horizontal Curve
The centerline of a circular curve in a two-lane roadway is shown. Each lane is 12 feet wide. There is no shoulder. The PI Station is at 112+40.00. The curve radius is 2,080 feet. The interior angle is 60 degrees. What is the PT station?

SSD: ________________

PT at STA: ________________
(4) Crest Vertical Curve

A four-lane highway runs north-south and passes through suburban areas. The preliminary design of the north-bound lanes includes a vertical curve with a length of 2200 feet. The curve connects a +3.00% grade with a -5.00% grade. The two vertical tangents intersect at STA 91+70 and elevation of 1453.61 feet. The design speed is 65 mph.

What is the station and elevation of the highest point on the curve?

Station ________________
Elevation = ______________
(5) **Vertical Clearance**

The vertical alignment of a section of proposed highway is shown in the figure.

What is the vertical clearance (ft) between the bridge structure at STA 73+00.00 and the vertical curve?

Vertical Clearance = ________________
(6) Sag Vertical Curve Problem
A sag vertical curve is being built on a roadway in an unlit suburban area. The two grades for the curve are -6.00% and +3.50%. The SSD required for 35 mph is 250 feet. What is the required length of the curve for SSD criteria only? State all assumptions.

\[ L_{\text{min}} = \text{____________} \]

(7) Compound Curve Problem
Given a compound circular curve with radii of 400 ft and 550 feet designed to connect two tangents that intersect at an angle of 75 degrees, determine the second curve central angle. The central angle of the first curve is 35 degrees, and the PCC is at station (45+22.25).

Second curve angle = \text{__________}
(8) Traffic Flow Relationships

A traffic flow relationship is given by \( q = k \cdot v \), where \( q \) is the traffic volume in veh/hr, \( k \) is the traffic density in veh/mi, and \( v \) is the mean speed in mi/hr. The mean speed on a road in mi/hr is given by the relationship:

\[
v = 40 \frac{mi}{hr} - \left( 0.4 \frac{mi^2}{veh - hr} \right) k
\]

What is the maximum capacity of overall traffic volume for this road?

Capacity = 

\[
\text{Capacity} = \text{__________________}
\]
Appendix C: Pre- and Post- Assessment Examples
Knowledge Assessment Survey

Course: Intro to Transportation  Course #: CVEN 3602  Instructor: Roxann Hayes  Date: ______

Name: ____________________________________________

INSTRUCTIONS: This is a knowledge survey and NOT a graded test. The purpose is to evaluate the effect this course has had on your knowledge. Please try to answer the questions provided. Also, rate (on a three-point scale) your confidence to answer the questions based upon your present level of knowledge. Do your best to provide a totally honest assessment of your present knowledge. Your answers will not affect your course grade in any manner. This survey will be given again near the end of the semester.

Excerpts and equations from the Fundamentals of Engineering (FE) Exam reference handbook can be found at the end of this survey.

Please indicate your ability to answer or solve the following questions:
1 = I can solve this problem.
2 = I am 50% confident that I can answer this problem.
3 = I cannot presently answer this question (or hardly at all).

1) Design of a Horizontal Circular Curve

A horizontal circular curve has the following data:

I = 40°50'
R = 800.00
Station of PI = 20+00.00

The station of the PI is most nearly:

A. 22+00.76
B. 22+04.27
C. 22+23.34
D. 22+32.34

Student (circle one)

1  2  3

Instructor

1  2  3
2) **Design of a Horizontal Compound Curve**

A compound curve is to be set out at a highway intersection. The point of the compound curve (PCC) is located at station 365+35.

What is the station of PC?
What is the station of PT?
3) **Design of a Crest Vertical Curve**

A crest vertical curve joining a +3 percent and a -4 percent grade is to be designed for 75 mi/hr. The tangents intersect at station (445+60.00) at an elevation of 250 feet. The station of BVC is (434+68.00), while the elevation of BVC is 217.24 ft. The length of the curve (L) is 2184 feet. Determine the elevation of the curve at station (436+00) and station (456+52).
4) **Vertical Curve Design**
A vertical curve is shown at bottom. Answer all the questions regarding this curve in the space provided.

At what station is the low point located? __________

What is the elevation of the low point? __________

What is the elevation of STA 21+25.04? __________

Based on comfort, what is the highest design speed for the curve? __________
Horizontal Curve Formulas

- $D$ = Degree of Curve, Arc Definition
- $PC$ = Point of Curve (also called BC)
- $PT$ = Point of Tangent (also called EC)
- $PI$ = Point of Intersection
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- $M$ = Length of Middle Ordinate
- $c$ = Length of Sub-Chord
- $d$ = Angle of Sub-Chord
- $l$ = Curve Length for Sub-Chord

$$R = \frac{5729.58}{D}$$

$$R = \frac{LC}{2 \sin(I/2)}$$

$$T = R \tan(I/2) = \frac{LC}{2 \cos(I/2)}$$

$$L = R \left[ \frac{\pi}{180} \right] = \frac{L}{D} \times 100$$

$$M = R \left[ 1 - \cos(I/2) \right]$$

$$\frac{R}{E + R} = \cos(I/2)$$

$$\frac{R - M}{R} = \cos(I/2)$$

$$c = 2R \sin(d/2)$$

$$l = Rd \left( \frac{\pi}{180} \right)$$

$$E = R \left[ \frac{1}{\cos(I/2)} - 1 \right]$$

Deflection angle per 100 feet of arc length equals $D/2$

Vertical Curve Formulas

\[ L = \text{Length of curve} \]
\[ 
\begin{align*}
PV &= \text{Point of vertical curvature} \\
PI &= \text{Point of vertical intersection} \\
PT &= \text{Point of vertical tangency} \\
g_1 &= \text{Grade of back tangent} \\
x &= \text{Horizontal distance from PVC to point on curve} \\
\end{align*}
\]
\[ g_2 = \text{Grade of forward tangent} \]
\[ a = \text{Parabola constant} \]
\[ y = \text{Tangent offset} \]
\[ E = \text{Tangent offset at PI = } \frac{AL}{800} \]
\[ r = \text{Rate of change of grade} \]
\[ K = \text{Rate of vertical curvature} \]

\[ x_m = \text{Horizontal distance to min/max elevation on curve} = -\frac{g_1}{2a} \frac{gL}{g_0 - g_3} \]

Tangent elevation = \( Y_{PVC} + g_1 x \) and = \( Y_{PT} + g_2 (x - L/2) \)

Curve elevation = \( Y_{PVC} + g_1 x + ax^2 = Y_{PVC} + g_1 x + [(g_2 - g_1)/(2L)]x^2 \)

\[ y = ax^2 \quad a = \frac{g_0 - g_1}{2L} \quad E = a \left( \frac{L}{2} \right)^2 \quad r = \frac{g_2 - g_1}{L} \quad K = \frac{L}{A} \]

EARTHWORK FORMULAS

Average End Area Formula, \( V = \frac{L(A_1 + A_2)}{2} \)

Prismoidal Formula, \( V = \frac{L(A_1 - 4A_m + A_2)}{6} \)

where \( A_m = \text{area of mid-section} \)
\[ L = \text{distance between } A_1 \text{ and } A_2 \]

Pyramid or Cone, \( V = \frac{h}{(A \text{ of Base})/3} \)

AREA FORMULAS

Area by Coordinates: \( \text{Area} = \left[ X_A(Y_B - Y_1) + X_B(Y_C - Y_A) + X_C(Y_D - Y_B) + \ldots + X_N(Y_A - Y_{N-1}) \right] / 2 \)

Trapezoidal Rule: \( \text{Area} = w \left( \frac{h + h_1}{2} + h_2 + h_3 + h_4 + \ldots + h_{n-1} \right) \)

\( w = \text{common interval} \)

Simpson’s 1/3 Rule: \( \text{Area} = w \left[ h_1 + 2 \left( \sum_{k=3,5,\ldots} h_k \right) + 4 \left( \sum_{k=1,4,\ldots} h_k \right) + h_n \right] / 3 \)

\( n \) must be odd number of measurements

(only for Simpson’s 1/3 Rule)