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Cantilever Beam Experiment

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Abstract

Electromechanical system course offered at Wentworth Institute of Technology (WIT) focuses on modeling mechanical & electrical dynamic systems. It is a is 4-credits and consists of 3 hours lecture and two hours lab each week. The objective of this course is to model and analyze the dynamic behavior of electromechanical systems. Software application programs are used to facilitate how electromechanical systems are analyzed, simulated and designed. In the past, the lab experiments were highly simulation based. Students used MATLAB and Simulink, SimScape Multibody to obtain the system performance. Most of the simulations represented ideal cases, students are lack of exposure to real systems. In order to expose students to real dynamic systems, physical lab experiments are needed.

This paper describes the experimental design of a cantilever beam system, which illustrates two different cases: free vibration and vibration due to rotating unbalance. Those concepts were taught in lecture and were reinforced with the experiments. For a series of two experiments, students analyzed the real beam system by characterizing the damping coefficient of the beam. They observed and measured the frequency changes of the beam with various loads applied. Students also observed and measured resonant frequency of the beam due to rotating unbalance. At the end, the experimental results were compared to the theoretical results. The newly developed experiments have received positive feedback from students, as they have expressed that these labs have helped them better understand course concepts.

1. Introduction

Educators have developed various ways to teach the difficult topics of the dynamics behavior of mechanical systems. Today, simulation software programs are available that accurately emulate many technical and physical processes. These software programs play an important role in engineering education [1]. Lazaro [2] described in their paper how to use the symbolic software Wolfram Mathematica to create a simple graphical model of a single degree of freedom (SDF) vibrating system, which allows students to visualize concepts like damping, resonance or forced vibrations. Danish-Yazdi et. al [3] developed eleven interactive simulation modules (ISMs) in MATLAB which depict the motion of the system under free/forced vibration and allow the students to control many of the parameters of vibration to see the effect of each of them on the response. Scotts et.al [4] developed some animations to illustrate important concepts of SDF systems. The animations provide parameter variation/control and self-test questions with diagnostic feedback. In addition, these webpages also provide conventional notes with access to theoretical derivations, which help students link the theoretic calculation with simulation results.

Simulation is a powerful tool and has been widely used to provide illustrations of phenomena that are not easily visualized or to compensate for a lack of equipment for demonstration.

However, there are some concerns about realistic of the simulation results [1]. Because simulation is purely based on the mathematical model of the system, there are many assumptions are made during this process. Several researchers have developed some physical labs for dynamics/vibration course [5] - [9] to help students observe the real vibration systems. Glean et. al. [5] developed some laboratory experiments, which not only to foster a better understanding of the principles of the system dynamics course, but also expose students to the various tools used in making engineering measurements. Sridhara and White [6] developed five different labs with donated equipment to measure the frequency of the vibration system, as well as to teach students how to use accelerometer. Ruhala [7], [8] developed four free vibration and five forced vibration experiments with commercially available translational system and one rotational lumped mass system.

Electromechanical system I course is a required undergraduate course for 5th year students in Electromechanical program at Wentworth Institute of Technology. This course analyzes the dynamic behavior of mechanical, electrical, fluid and thermal systems using modeling and simulation techniques. Steady state and transient conditions are examined in both free and forced modes. This course is 4-credits and consists of 3 hours lecture and two hours lab each week. In the laboratory, various simulation software packages are used to analyze electromechanical systems. The lab experiments are highly simulation based. Students use MATLAB and Simulink, SimScape Multibody to obtain the system performance. However, there is a lack of physical experiments to help students understand the concepts, especially forced vibration. To fill this gap, some new labs were developed to introduce experimental methods to explore a SDF system free vibration and forced vibration due to rotating unbalance. Commercialized vibration systems are usually very expensive, and we do not have funding to purchase them Therefore, the only option for us to develop new experiments is to use available equipment. In this paper, the development of a cantilever beam system to help students understand free and forced vibration is presented.

2. Cantilever Beam Vibration

A system is said to be a cantilever beam system if one end of the system is rigidly fixed to a support and the other end is free to move. Examples of real life systems that can be approximated to cantilever beam are shown in Figure 1 and Figure 2.

In real systems, due to manufacturing inaccuracies there is a high chance of existence of imbalance in the finished rotating component (i.e. center of gravity is not on the axis of rotation). This can cause an external force on the body where the rotating component is installed.

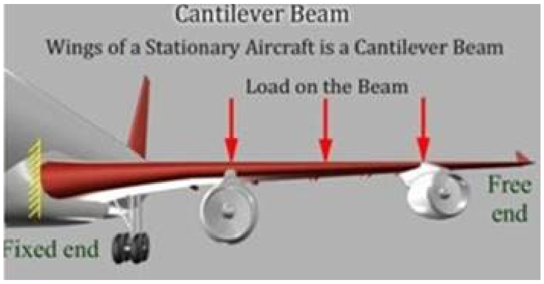


Figure 1. Aircraft wing as a cantilever beam

Figure 2. Tower crane overhang as a cantilever beam

The derivation of cantilever beam vibration:

For the free vibration of a cantilever beam with mass ***m***, shown in Figure 3, classical Euler-Bernoulli beam dynamics theory is used to analyze the problem.

$$y(x,t) = \bar{y}(x) sin\omega t \tag{1}$$

where $\bar{y}(x)$ is the associated mode shape of the vibration, ω is the natural frequency.

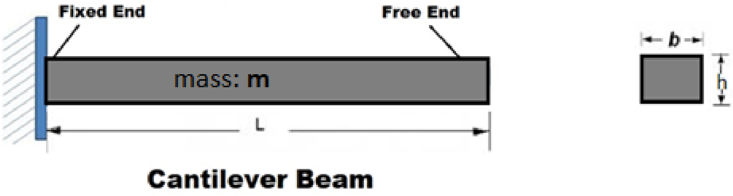


Figure 3. Cantilever beam

a. Equation of motion of a differential element

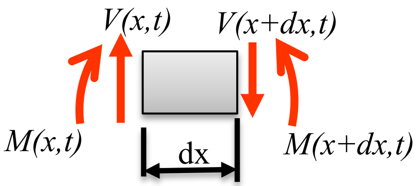


Figure 4. Differential element

$$\sum F = V(x+dx,t) - V(x) \approx \frac{\partial V(x,t)}{\partial x} dx$$

$$y(x,t) = \bar{y}(x) sin\omega t$$
$$\dot{y}(x,t) = \frac{\partial y(x,t)}{\partial t} = \omega \bar{y}(x) cos\omega t$$
$$\ddot{y}(x,t) = \frac{\partial^2 y(x,t)}{\partial t^2} = -\omega^2 \bar{y}(x) sin\omega t$$

The mass for the differential element: $dm = \rho A dx$

Apply Newton's 2nd law:

$$\sum F = dm * \ddot{y}(x,t)$$
$$\sum F = \frac{\partial V(x,t)}{\partial x} dx = dm * \ddot{y}(x,t) = \rho A dx(-\omega^2 \bar{y}(x) sin\omega t)$$

$$\frac{\partial V(x,t)}{\partial x} = -\omega^2 \bar{y}(x) \rho A sin\omega t \qquad (2)$$

b. The z-component of the moment is balanced for the differential element:

$$\sum M = M(x+dx,t) - M(x,t) + V(x,t) * dx = 0$$
$$M(x+dx,t) - M(x,t) \approx \frac{\partial M(x,t)}{\partial x} dx$$
$$\rightarrow \qquad \frac{\partial M(x,t)}{\partial x} + V(x,t) = 0$$

Take the partial derivative with respect to x

$$\frac{\partial^2 M(x,t)}{\partial x^2} + \frac{\partial V(x,t)}{\partial x} = 0 \qquad (3)$$

Plug equation 2 to equation 3:

$$\frac{\partial^2 M(x,t)}{\partial x^2} - \omega^2 \bar{y}(x) \rho A sin\omega t = 0 \qquad (4)$$

c. Using the elastic beam curvature/bending moment theorem:

$$M(x,t) = EI \frac{\partial^2 y(x,t)}{\partial x^2}$$

Take partial derivative with respect to x twice.

$$\frac{\partial^2 M(x,t)}{\partial x^2} = EI \frac{\partial^4 y(x,t)}{\partial x^4} \qquad (5)$$

From equation 1, take partial derivative with respect to x:

$$\frac{\partial y(x,t)}{\partial x} = \bar{y}'(x) sin\omega t$$

$$\frac{\partial^4 y(x,t)}{\partial x^4} = \bar{y}''''(x) sin\omega t \tag{6}$$

Plug equation 6 into equation 5:

$$\frac{\partial^2 M(x,t)}{\partial x^2} = EI\frac{\partial^4 y(x,t)}{\partial x^4} = EI\bar{y}''''(x) sin\omega t \tag{7}$$

Plug equation 7 into equation 4:

$$sin\omega t(EI\bar{y}''''(x) - \omega^2\bar{y}(x)\rho A) = 0$$

Then $$EI\bar{y}''''(x) - \omega^2\bar{y}(x)\rho A) = 0$$

$$\bar{y}''''(x) - \frac{\omega^2\bar{y}(x)\rho A}{EI} = 0 \tag{8}$$

d. <u>Solve for the 4th order ODEs</u>:

Set $\beta^4 = \frac{\omega^2 \rho A}{EI}$, the differential equation 8 becomes:

$$\bar{y}''''(x) - \beta^4\bar{y}(x) = 0$$

The homogeneous solution is:

$$\bar{y}(x) = C_1 cosh\beta x + C_2 sinh\beta x + C_3 cos\beta x + C_4 sin\beta x \tag{9}$$

Apply geometric boundary conditions: $at\ x = 0, \bar{y}(0) = 0, \bar{y}'(0) = 0$ then plug in to equation 9, it yields:

$$C_1 + C_3 = 0 \ \rightarrow C_3 = -C_1$$
$$C_2 + C_4 = 0 \ \rightarrow C_4 = -C_2$$

Hence

$$\bar{y}(x) = C_1(cosh\beta x - cos\beta x) + C_2(sinh\beta x - sin\beta x)$$

Apply the force boundary condition: $at\ x = L,\ M(L,t) = 0,\ V(L,t) = 0$. These in turn require

$$at\ x = L, \qquad \bar{y}''(L) = 0, \qquad \bar{y}'''\,(L) = 0$$

$$\bar{y}''(x) = C_1(\beta^2 cosh\beta x + \beta^2 cos\beta x) + C_2(\beta^2 sinh\beta x + \beta^2 sin\beta x)$$
$$\bar{y}'''(x) = C_1(\beta^3 sinh\beta x - \beta^3 sin\beta x) + C_2(\beta^3 cosh\beta x + \beta^3 cos\beta x)$$

Plug in *x* = *L*, and represent them in matrix form:

$$\begin{bmatrix} \beta^2 cosh\beta L + \beta^2 cos\beta L & \beta^2 sinh\beta L + \beta^2 sin\beta L \\ \beta^3 sinh\beta L - \beta^3 sin\beta L & \beta^3 cosh\beta L + \beta^3 cos\beta L \end{bmatrix} \begin{Bmatrix} C_1 \\ C_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\rightarrow \begin{bmatrix} cosh\beta L + cos\beta L & sinh\beta L + sin\beta L \\ sinh\beta L - sin\beta L & cosh\beta L + cos\beta L \end{bmatrix} \begin{Bmatrix} C_1 \\ C_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

For non-trivial solution of the above matrix, it requires:

$$(cosh\beta L + cos\beta L)^2 - (sinh\beta L + sin\beta L)(sinh\beta L - sin\beta L) = 0$$

Which yields $2(1 + cosh\beta L * cos\beta L) = 0$

Hence $cosh\beta L * cos\beta L = -1$

There are an infinite number of βL values satisfying this relation; the smallest one is $\beta L = 1.875$, which is corresponding the lowest natural frequency.

$$\omega^2 = \frac{EI\beta^4}{\rho A} = \frac{EI(\beta L)^4}{L^4 \rho A} = \frac{EI(1.875)^4}{L^4 \rho A}$$

$$\boldsymbol{\omega = \frac{(1.875)^2}{L^2}\sqrt{\frac{EI}{\rho A}}} \tag{10}$$

Equation 10 represents the relationship between natural frequency of the beam and the property of the beam.

3. Development of the labs

There are two laboratory experiments developed for the cantilever beam: Free vibration and forced vibration due to rotating unbalance.

3.1 Free vibration

The objectives of the free vibration of the cantilever beam are twofold: a) Introduce experimental methods for quantifying elastic properties of materials and elastic response of cantilever beams. b) Use the natural frequencies of the vibrating cantilever beams measured in the lab, along with the specimen dimensions, and the appropriate mass values, to estimate modulus of elasticity E and compare with the book value.

Experiment apparatus includes Flextor, Aluminum beams, strain gauge, Omega BCM-1 strain gauge bridge, Load Cells and Transducers and Omega DMD 465-WD strain amplifier/Signal Conditioners Modules for Strain Gauges. The Flextor and experiment setting are shown in Figure 5.

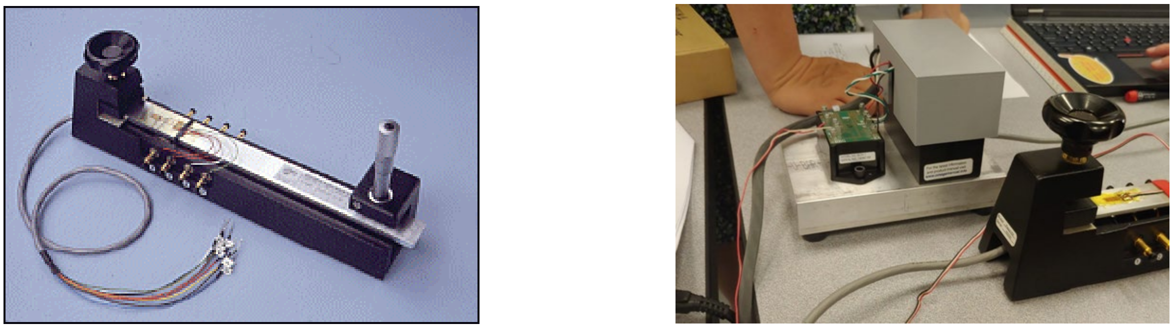
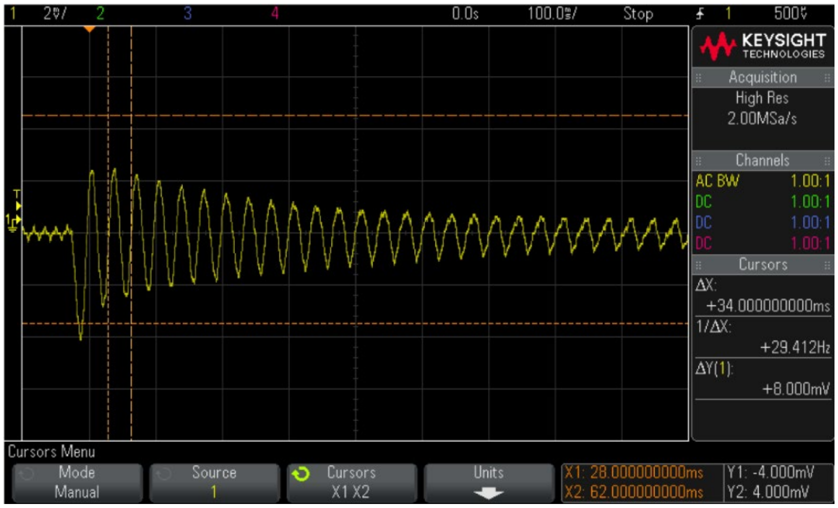


Figure 5. Flexor Cantilever beam and experiment setting



Acquisition: High Res 2.00MSa/s

Channels:
AC BW 1.00:1
DC 1.00:1
DC 1.00:1
DC 1.00:1

Cursors:
ΔX: +34.000000000ms
1/ΔX: +29.412Hz
ΔY(1): +8.000mV

Mode: Manual
Source: 1
Cursors: X1 X2
X1: 28.000000000ms
X2: 62.000000000ms
Y1: -4.000mV
Y2: 4.000mV

Figure 6. Measurement screen shot from Oscilloscope.

Given an initial displacement, the beam will vibrate. A sample measurement from the oscilloscope is shown in Figure 6, from which the damped natural frequency of the beam can be obtained. The damping ratio ξ can be calculated using logarithmic decrement method as defined in equation 11 as described in the book System Dynamics [10]. Several consecutive cycles from the oscilloscope screenshot may be chosen for a better result. Then the natural frequency of the system can be calculated as $\omega_n = \frac{\omega_d}{\sqrt{1-\xi^2}}$. Equation 10 is then used to determine the modules of elasticity of the beam. The expected results are shown in table 1.

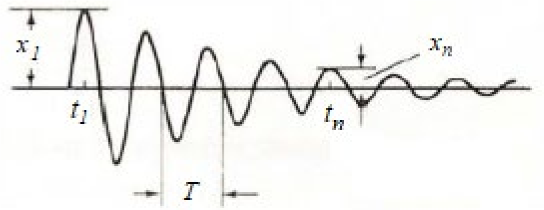


Figure 7. Logarithmic Decrement Method

$$\xi = \frac{\frac{1}{n-1}\left(ln\frac{x_1}{x_n}\right)}{\sqrt{4\pi^2+\left[\frac{1}{n-1}\left(ln\frac{x_1}{x_n}\right)\right]^2}} \tag{11}$$

Table 1: Beam Free vibration – Estimated Modules of Elasticity

Given Information			**Results**		
Material	Aluminum 6061-T6		**f**: experiment obtained frequency:	Hz	29.300
ρ: density	lb/in^3	0.098	**ω$_d$**: damped frequency	rad/s	184.097
L: length	in	11.500	**ξ:** damping ratio		0.010
b: width	in	1.000	**ω$_n$:** natural frequency	rad/s	184.107
h: thickness	in	0.125	**f:** natural frequency	Hz	29.301
A: area	in^2	0.125	**E: Estimated Modules of Elasticity**	psi	**9295086**
I: area moment of inertia of area	in^4	1.63E-04	Modules of Elasticity book value	psi	10000000
M: mass	lb	0.140	**Percentage error**		**7.05%**

3.2 Forced vibration

The objectives of this lab are four-folds. a) introduce experimental methods to explore forced vibration under rotating unbalance; b) observe/measure the natural frequency of the beam with added concentrated mass; c) observe the behavior of the cantilever beam under different frequency force applied; and d) observe resonance behavior of the system and measure the resonance frequency.

In this experiment, a small mass is connected at a distance to the rotor of a motor. This mass on the motor acts as an imbalance. When the motor is running, the cantilever beam experiences an external harmonic excitation. The experimental set up of the system is shown in Figure 8. The magnitude of the steady state displacement *X,* as a function of rotating speed (frequency) can be shown as [10]:

$$X = \frac{m_0 e}{m}\frac{r^2}{\sqrt{(1-r^2)^2+(2\xi r)^2}} \tag{12}$$

$$\phi = tan^{-1}\frac{2\xi r}{1-r^2},$$

Where r is frequency ratio which is defined as: $r = \omega/\omega_n$, e is the eccentric distance.

These last two expressions yield the magnitude and phase of the motion of the mass, *m*, and due to the rotating unbalance of mass m_0.

Procedure:

- Model the given real system to an equivalent simplified model of a cantilever beam with motor and eccentric mass with suitable assumptions / idealizations.
- Calculate the natural frequency and damped natural frequency of the whole system
- Find the stiffness of the system.
- Calculate damping coefficient of the system.
- Experimentally obtain the natural frequency and damping ratio of the cantilever beam undergoing small amplitude flexural vibrations when a static load is applied.
- Observe the vibration of the beam due to rotating unbalance, by varying the speed of the motor, for three different scenarios: *r>1, r=1, and r<1.*

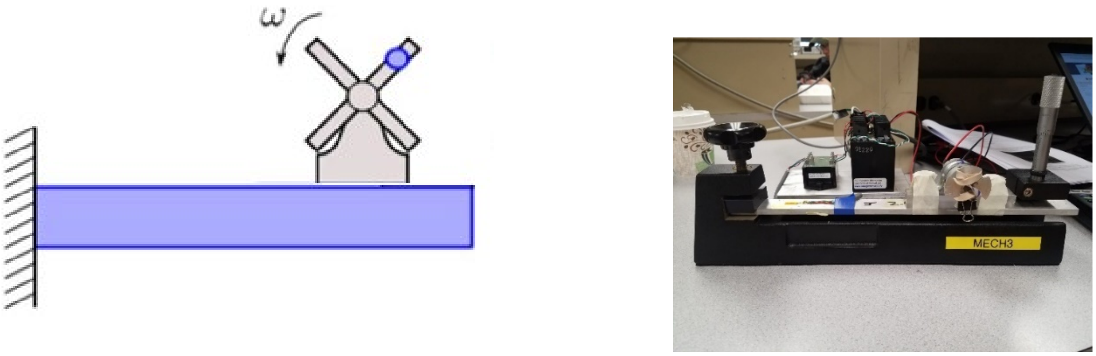


Figure 8: Illustration and experiment setting for forced vibration

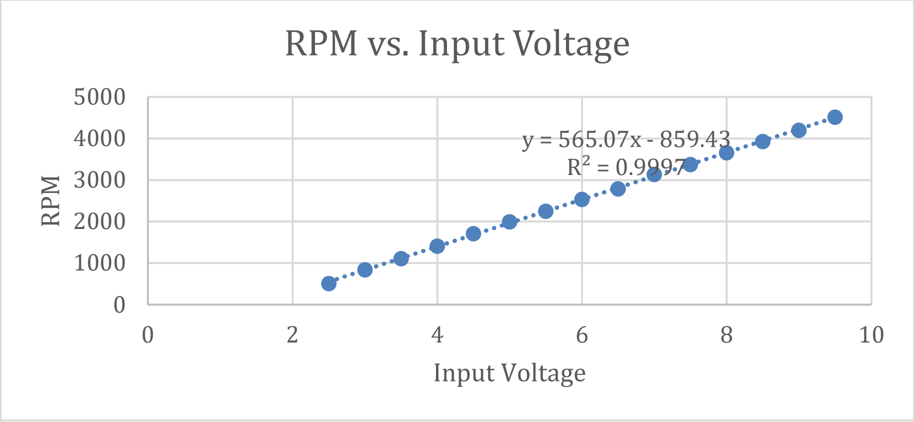


Figure 9. RPM vs. input voltage

Adjusting the voltage input of the DC motor by increasing it gradually, the behavior of the beam including resonance can be observed. The forcing frequency is obtained based on the input voltage of the motor shown in Figure 9. The expected experiment results are shown in Table 2.

Table 2: Forced vibration – rotating unbalance

Material	Aluminum 6061-T6			
ρ: density	0.10	lb/in^3	2700	kg/m^3
L: length	11.50	In	0.2921	m
b: width	1.00	In	0.0254	m
h: thickness	0.125	In	0.003175	m
A: area	0.125	in^2	8.06E-05	m^2
I: area moment of inertia of area	1.63E-04	in^4	6.77E-11	m^4
m: mass	1.4016E-10	Lb	6.3602E-11	Kg
E: Modules of Elasticity	10000000.00	Psi	7.00E+10	Pa
m_0: unbalanced mass			1.5	g
e: eccentricity			15	mm
voltage input – motor			3	V
forcing frequency			14.5	Hz
total mass (motor + unbalanced mass)			178.5	g
position of the motor	7.50	In	0.1905	m
m_{eq}: equivalent mass			0.1785	Kg
k: spring constant			2.06E+03	N/m
ω_n: natural frequency of the beam			107.371717	rad/s
f_n: natural frequency of the beam			17.0887395	Hz
Percentage Error	**15.15%**			

3.3 Discussion of the experiment results

The free vibration of the beam yielded 7% error of the modules of elasticity, which is reasonable. However, the forced vibration experiment yielded 15% error of the natural frequency between the calculated and experiment result. The following are some possible reasons:

i) The precision of the data being measured, including the eccentricity *e* and the location of the motor. Estimating the measurements based on a ruler causes variation in the accuracy of the distance.

ii) The RPM vs. voltage relationship of the motor was not accurate. The chart is provided to students based on the average data, while each motor is different, and the rotor attached to it varies too.

iii) By controlling the speed of the rotating unbalanced motor, it can be expected to have a resonant frequency of the system. However, obtaining the maximum deflection of the beam by observing yields some errors.

4. Data Collection and Analysis

A survey was conducted after students completed these experiments and the overall feedback was positive. There were 21 students in the class and 14 students participated in the anonymous survey. Figure 10 shows the average scores of the survey questions in a scale of 5: 1- strongly disagree, 2 – disagree, 3- neutral, 4 – agree, 5 – Strongly agree. The detailed survey results are shown in Figure 11.

It can be seen that 100% of students agreed that the experiment helped them to better understand the subject, the average score is 4.43. 100% of students agreed that the experiment was designed closely related to the topics discussed in lectures, with an average score 4.57. More than 85.7% of students agreed that the experiment improved their learning experience, and the average score is 4.

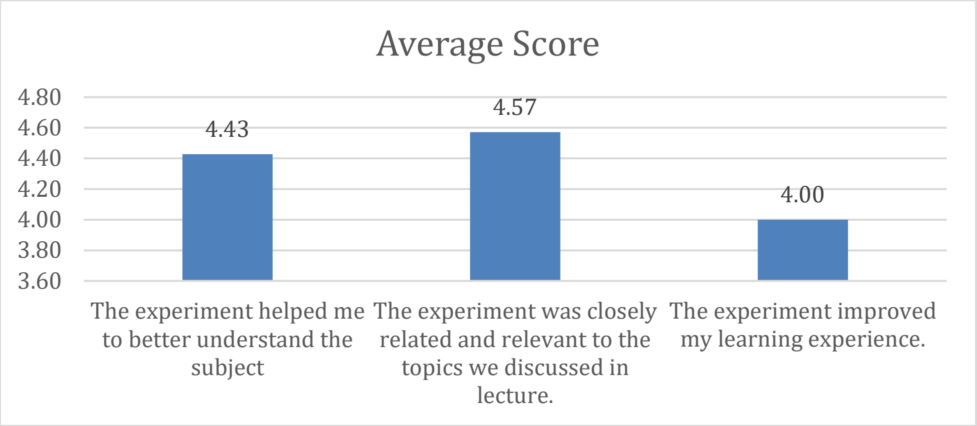


Figure 10. Average score of survey questions

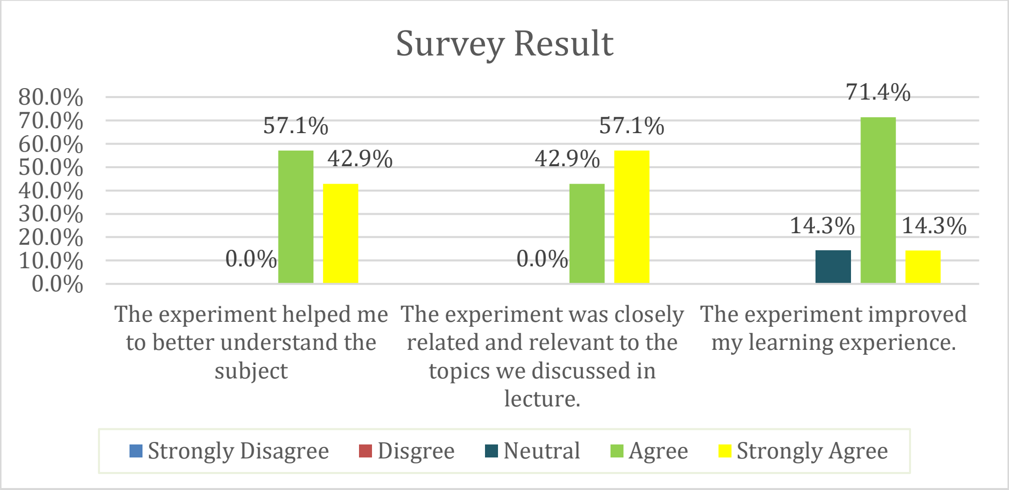


Figure 11. Detailed survey results

An open-ended question is also asked in the survey: “Other demonstration or experiments that you want to see/do in this course?” The feedback showed that students were eager to conduct more experiments to handle real problems. Some students commented: “more”, “Basically what we did in class, but on a larger scale”. One student commented: “something used in the real world and not just a contraption”. We will take this comment into consideration when developing new experiments in the future. When working on real problems, students will have the chance to learn how to make reasonable assumption, and then model the system. We are thinking to develop a project to ask students, possibly by groups, to choose a real world problem that is interested to them and present their findings to the whole class. Doing this way students will be more motivated and participating.

About 12 students were interviewed about their experience with these new labs. They liked the fact that they can observe the behavior of the system under different loads, observe the resonance of the system, and take the measurement to analyze the system.

5. Conclusion and future work

Electromechanical system I course is a strong application oriented course, but the concepts are highly theory based. This causes some problems, as students have difficulties to establish the connection between the theory and application. Simulation has been integrated into the lab to help students visualize the dynamic behavior of systems. However, students still prefer physical labs. Therefore, new labs are developed to help students better understand the free and forced vibration. These labs introduce experimental methods to explore a SDF system. They help students better understand the behavior of the beam with different loads applied and the resonance of the system. The feedback from students is positive. 100% of students agreed that the experiment helped them to better understand the subject, and it was designed closely related to the topics discussed in lectures. More than 85.7% of students agreed that the experiment improved their learning experience. Students also indicated that more physical experiments were desired. Overall, these newly developed physical experiments help in bridging the gap between the theory and experimental work.

The presented system successfully illustrates free vibration and forced vibration due to rotating unbalance. However, the precision of the data collected in the experiment is not high. Some improvement should be made to get more accurate measurement data. For example, each motor should be individually characterized; currently the RPM vs. voltage chart was given to students. The unbalanced mass should also be redesigned for safety considerations.

In addition, as a future work, an experiment related to absorber will be development based on the currently beam experiment. Students will have the opportunity to better understand the concept of absorber.

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