



CAREER: Mathematics as a Gatekeeper to Engineering: The Interplay between Mathematical Thinking and Design Thinking – Using Video Data

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Abstract

There is a need to better understand how students gain accurate engineering conceptual understandings and how they apply them in practice. There are varied approaches to study the development of engineering knowledge and thinking skills. This paper focuses on the use of video data to uncover and document students' thinking and development and presents lessons our team has learned as we use video data to support our investigation. Video data allows the researcher to review and re-immense him or herself back in to the original context and explore points of interest that could not be captured fully in the field notes, observations and existing artifacts ^[1].

In this study we explore the cognitive discord, which can occur when engineering students, who have been previously taught convergent mathematical thinking strategies, are exposed to the divergent manner in which design problems are solved in educational engineering environments and in the real world. First-year engineers and senior design, engineering and mathematics students are recruited to work, in isolation, on a common design task. This study uses the think aloud protocol to capture student thinking processes. The data includes a collection of artifacts (e.g. drawings, audio transcriptions, screen capture, interview transcriptions, and video data). We anticipate that using video as data will help us to identify critical incidences of transition between divergent thinking and convergent thinking and the moments that lead to and follow these events.

A freshman engineering student (yet undecided about a specific engineering major) and a college senior majoring in fashion design will serve as examples of using video data to observe mathematical and design thinking. The analysis of the participants' video data will reveal critical moments of transition between convergent and divergent thinking and will yield insight into similarities and differences in these students' approaches to design problems. Although the use of video to record and observe mathematical and design thinking in traditional education environments is not novel, using video as a primary data may create great opportunities for deeper understanding of students' thinking processes in engineering education.

Research Rationale

Engineering education is concerned with helping students to develop accurate understandings of engineering concepts as well as skills that are foundational to engineering practice. These concepts and skills include the ability: to apply knowledge of mathematics, science and

engineering; an ability to design a system, component, or process to meet desired needs within realistic constraints; and an ability to use techniques, skills, and modern engineering tools necessary for engineering practice^[2]. However, there is still a great need for us to better understand how students gain accurate understandings of engineering concepts as well as how students learn and apply engineering skills such as estimation, design, analysis, problem solving systems thinking and creativity^[3,4]. Approaches that have been used to study the development of engineering knowledge and skills include the use of interviews, surveys, concept inventories and other tests, analysis of student work, and observations of students engaged in engineering activities.

This paper focuses on another approach, the use of video data to uncover students' understandings of engineering concepts as well as to document students' skills and abilities. In this paper, we describe a specific study that is underway where video data is used to address our research questions. We present this specific study to provide a context for a larger discussion of video data as an approach that can be adopted for other studies as well. The bulk of the paper is devoted to presenting video data as an approach that is appropriate for engineering education research and how video data supports our study. While the focus of the paper is on the method, we do, however, present some early observations from the Mathematics as a Gatekeeper Study.

The study examines two foundational aspects of engineering education: engineering mathematics and engineering design, and the respective thought processes that students engage in. We focus on these areas in this study for two reasons: (1) they represent two representative areas for future engineering education research that might adopt the use of video data and (2) they represent two styles of thinking that students might engage in throughout the course of engineering education and engineering practice. Mathematics and mathematical thinking skills, which are taught extensively before college, are typically taught in a manner which leads students toward one best path to obtaining a solution. A function of engineering education is to teach students design and design thinking skills, which are more divergent in nature.

So how do students engage in mathematical and design thinking while solving open-ended problems? This study investigates this question by giving undergraduate first-year engineers and seniors majoring in engineering, mathematics and design a common design task. The playground design task, which is known to elicit varying levels of design thinking and processes, and was designed to be accessible for both freshmen and seniors^[5,6], will allow us to understand the types of cognitive barriers which students may encounter. It is coupled with the think aloud protocol and a follow-up interview protocol. Our specific research questions:

1. How do students respond to open-ended, ambiguous design tasks?
2. How do they respond to different forms of ambiguity/uncertainty?
3. How do mathematical thinking activities support/hinder/coincide with design thinking activities?
4. How do students' thinking processes differ based on differences in mathematics, design and engineering backgrounds?

5. How do students' thinking processes differ based on differences in attitudes towards and beliefs about mathematics, design and engineering?

Through the use of verbal protocol analysis^[7] of the playground design task think aloud transcriptions and video analysis techniques, the research team is able to identify the ways in which participants engage in mathematical and design thinking in the engineering design process. Other artifacts include: audio and video data, drawings, sketches, researcher field notes, internet browsing history, screen capture software video and background information on the students mathematical and design experiences, which was collected prior to the start of the design session via a web survey.

As the study continues, video data will provide the support we need to further explore our hypotheses. In our effort to understand how students apply their mathematical and design skills and knowledge to engineering problems, the use of video data in this study will help us record the practices, thoughts, actions and behaviors of students with varying levels of mathematical and design abilities. Through the use of video we anticipate learning more about how students respond to ambiguity in design, how convergent and divergent thinking behaviors are exhibited in their design processes, and how their thinking activities vary based on their mathematical and design experiences.

Some early observations on the hypothesis that students with “lower” math backgrounds will exhibit more divergent thinking and less modeling than students with a “high” math background will also be discussed in this paper. We anticipate that the data from the Mathematics as a Gatekeeper to Engineering study will provide support for our belief that students with more extensive mathematics background would lead students to become very detailed with a solution, thus converging and becoming fixated on that solution. This hypothesis is built upon four major concepts, which we are considering part of the theoretical framework for this study. Those concepts are: mathematical modeling, fixation, divergent thinking, and convergent thinking.

1. *Mathematical Modeling*: not a single mathematical formula, but rather a procedure (model) for solving the problem the embeds multiple mathematical ideas^[9].
2. *Fixation*: premature commitment to a particular problem solution, blind adherence to one solution or one approach to a problem (mechanized thought), which follows a previously laid-out pattern, and prevents the consideration of all relevant knowledge and experience which should be brought to bear on the given problem^[10-12].
3. *Divergent Thinking*: imagining any possibility, head off in many directions, deliberately diverge from the conventional, explore possibilities while temporarily suspending criticism and judgment^[13].
4. *Convergent thinking*: narrow down the options to one or more choices, deriving the best solution from available information, conventional intelligence^[13-15].

Literature Review

Of the four concepts of our theoretical framework, which are introduced above, this literature review will focus on the use of video data to observe convergent and divergent thinking instances. Mathematical modeling and fixation will be discussed briefly but we anticipate more fully exploring how video data support observing those concepts in future publications. We focus on convergent and divergent thinking because it reflects our desired primary focus during observations where field notes were recorded, where this focus was chosen based on our hypothesis that students with increased mathematical background would engage in more divergent thinking. The researcher looks for instances of convergent and divergent thinking and makes note of the participants' actions and verbalizations just before and after those instances. Video data provides flexibility to engage with the data from diverse frameworks because of the ability to review and share the data. Other benefits of using video data will be further discussed in this literature review.

Divergent and Convergent Thinking

Divergent thinking is characterized by the problem solver imagining any possibility, heading in varied directions, possibly a deliberate intent to differ from conventional thoughts, processes and ideas, and exploring possibilities in a state of temporary suspended criticism and judgment^[13]. This type of thinking is required in the concept generation stage of the engineering design process; its function is to generate many diverse ideas that are void of criticism. Divergent thinking is typically the preferred first step towards designing creative solutions. By allowing oneself to explore concept generation from a divergent thinking lens, this allows for the conception of many ideas which may not come to the forefront if exploring concept generations for a convergent thinking lens. Now that a participant has generated unconventional ideas and imagined unconstrained possibilities, problem solving often involves the transition from thinking broadly to narrowing down ideas.

Convergent thinking is characterized by the act of narrowing many ideas as the problem solver arrives at one best answer by using conventional intelligence or stored knowledge^[15]. There are different methods of employing convergent thinking in concept reduction. Some individuals may opt for a more structured approach where they develop an algorithm to eliminate ideas, while others may have a less structured approach. At this point, the problem solver is not creating new possible procedures, ideas or solutions rather they are now trying to determine of the available answers or methods, which one is a preferred option. During this process, less attractive possibilities are eliminated using cognitive skills such as analysis, criticism, logic, argument and reasoning^[16].

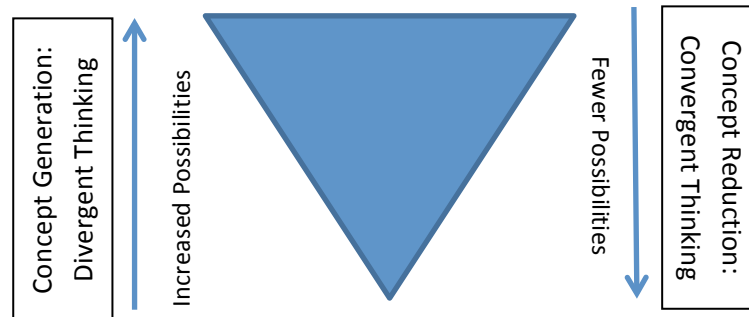


Figure 1: Funnel Illustration of Convergent and Divergent Thinking

It is apparent that divergent thinking and convergent thinking operate on different ends of the problem solving spectrum. Figure 1 illustrates these opposing types of thinking in the form of a funnel. Creative problem solving is best enacted when divergent thinking is followed by convergent thinking and both types of thinking have been explored for an appropriate amount of time. The act of developing an engineering design process, which is a balance of divergent and convergent thinking strategies, can be an obstacle to problem solving and is not intuitive. It is for this reason that teaching the engineering design process and helping students overcome this obstacle is necessary in engineering education.

Using Video as Data

Video recording technology is a data collection instrument which allows researchers to collect both auditory and visual information and re-enter the study environment even after the study is done^[1, 18]. Data can be extracted and carefully selected from the video recording for a more in-depth analysis of specific events which will help them understand the phenomenon under investigation. Unlike audio recorded data, it can capture more information, including body and verbal language, which may be difficult to fully unpack in an audio transcription. Although there are limitations, by collecting information through video recording instruments, researchers have the increased ability to “decompose a complex event and select specific parts to pay future attention to” and to ensure that those events, which may have been deemed important during data collection, still contain critical bits of information and should be further explored. Most importantly, the use of video data helps the investigator overcome obstacles of perception when making observation through field notes. When in the midst of data collection, the researcher can take field notes and has the opportunity to evaluate their notes during video review in order to reveal inconsistencies in observations. These corrections can be noted in time-stamped video transcriptions, which can be used to provide a map of events for any reader, who may not have access to the video.

Video as data create historical records, which can be examined and reviewed repeatedly. It becomes important for the researcher to select aspects of the video which will be transcribed and used as data. Data selection is the “process of focusing on particular information in accordance with the theoretical frameworks, research questions, and an instrument, a researcher chooses ^[19],” and it occurs at various times throughout the research process. So the investigator’s interest drives which aspects of the collected information will be used as data to investigate the phenomenon under study. Data selection is particularly important when a study is designed to collect large amounts of information. In the case of this study we chose to employ the critical incidence method and are specifically looking for instances of convergent and divergent thinking and the individual’s train of thought and actions in the moments leading to and following these critical incidences ^[19, 20].

Video as Data When Observing Design Thinking and Mathematical Thinking

Video has been used to observe and record engineers and engineering students engaged in design thinking activities. Under the premise that “in every design project creativity can be found”, Dorst and Cross ^[21] sought to explore the distinctive event or evolution of a design solution during a 2.5 hour think aloud design task, given to industrial designers with 5 or more years of experience. In this study each participant was recorded with two video cameras; one would capture the general workspace and one would focus on the design to capture drawing behavior.

In previous studies using the Playground Design task, video recordings were also used as an unobtrusive means to enter the design environment. The design thinking skills that have been observed include: sketching or diagramming, creating physical models, gestures to represent designs and models, gathering and using resources. With respect to observing mathematical thinking, collecting information with video is not novel. In pre-college educational settings and in mathematics teaching development, video has been used to observe teachers in action and the ways that the students engage with the materials that are being taught ^[1].

Methods and Rationale for Decisions Made

The design of this study requires that we use a computer with internet access and screen capture software (for the participant to use), a high definition camcorder, memory cards, and a hand held digital voice recorder with built in storage. Figure 2 is a visual representation of the layout of the design environment. The camera is placed on a tripod and angled over the writing shoulder of the participant. For this study we decided to use one camera, which focuses on the workspace of the participant (see Figure 3). An additional laptop was used by the researcher to record field notes.

This camera angle allows us to observe how participants: interact with the computer; access information; create sketches and drawings, and engage with mathematical and design tools. This also allows for the recording of the participant’s body movements as he or she works to solve the design problem. Although there is typically minimal panning and zooming, it is sometimes necessary to zoom in or out in order to capture participant movements or sketches (see figure 4).

Along with video recording, our study uses an audio recorder, screen capture recording software and field notes.

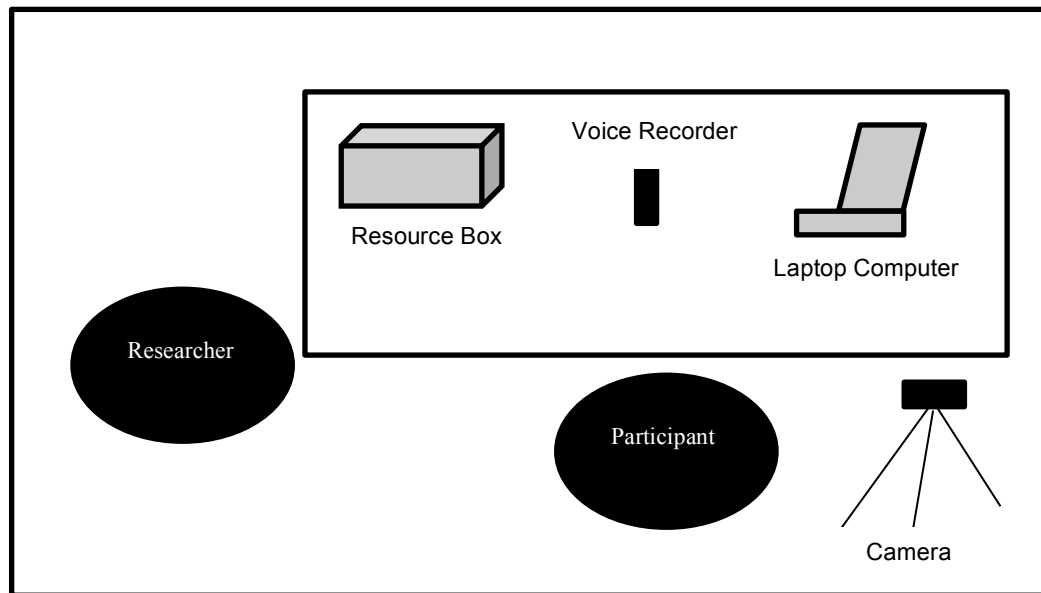


Figure 2: Workspace Layout

Each of these instruments collects or records different aspects of each individuals participation. Having multiple forms of information collection yields vast amounts of information for each participant. It becomes very important for the research team to develop a strategy to organize the data.

Role of the researcher during data collection)

The researcher's involvement is not captured on video but the participant can ask the researcher for additional information about the task, which may be available in the information binder. During data collection the researcher serves multiple roles: 1) serves as the facilitator of the design task (by setting up the workspace and explaining the procedures), 2) serves as an information resource via binder of information sheets and 3) makes observations by recording time stamped field notes. Figure 2 shows that the researcher sits near the participant's workspace. This allows for increased visibility of the workspace, which is particularly helpful when making observations about sketches and drawings.

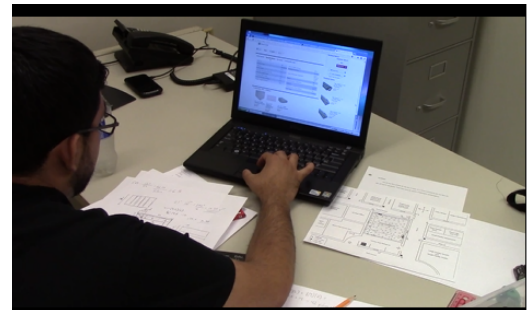


Figure 3: Senior Engineering Participant-Design Environment View

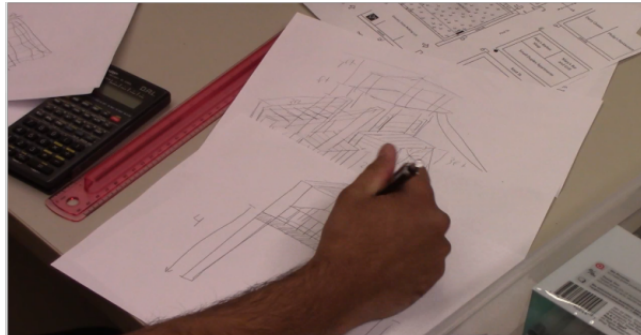


Figure 4: Senior Engineering Participant –Zoom in to Sketch View

Analysis

The video recorder saves data in 1 hour increments. All digital artifacts (i.e. web browsing history, screen capture video, audio recorded sound and field notes) are saved in a folder, using a naming convention that is used across all participant data. This also allows for ease of access to specific pieces of information. It is particularly helpful during data analysis, when particular events need to be retrieved.

During the study we are particularly interested in recording observations on instances of convergent or divergent thinking and the actions or thoughts that lead up to and follow them. We are considering these critical instances. Examples will be shared and discussed later in this paper. Instances that are noted in the field notes can be easily retrieved from the video data. Also when the video is reviewed, other instances of convergent and divergent thinking may surface which were not caught before. The video of the critical instances will be transcribed and the transcript will be analyzed following verbal protocol analysis. We anticipate that themes will emerge which will help us address our research questions.

Mathematics as a Gatekeeper to Engineering- Video Data Examples

To date, 17 students have participated in this study. In this section we provide examples from two participants to provide further evidence of how video data supports this research project.

Example 1: First Year Engineer

This student was a first-year engineering student (specific engineering major not yet determined) who had limited previous experience working on open-ended design activities. Throughout the design activity this student consistently reminded himself to use the design process which was introduced in his introductory engineering course, which he was currently enrolled in. This student shared that the design process which he learned in high-school was very different than that which he was currently learning in college. It was because of that difference in the design

processes that he insisted on using the design process he recently learned in the introduction to engineering course during his participation in the study. Figure 5 shows a representation of the design process this participant used in this study.

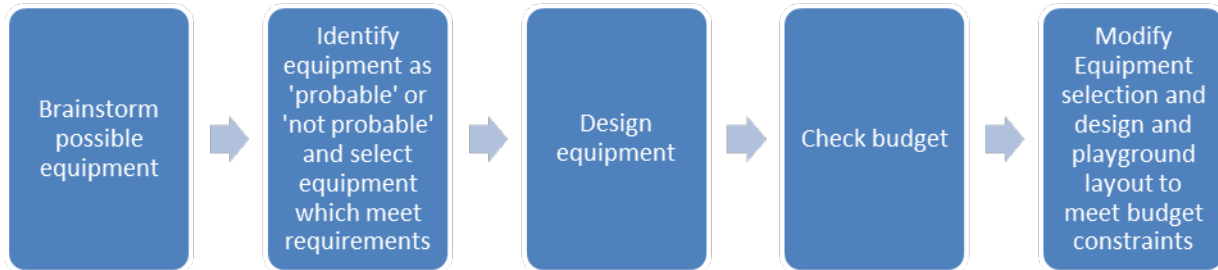


Figure 5 Design process of the First-Year Engineering Participant

The student (see Figure 6) exhibited fixation when using mathematical modeling and when he saw images of pieces of equipment which meet the ADA (Americans with Disabilities Act) requirement. During the process of mathematical modeling, fixation occurred when the student attempted to calculate the distance between monkey bars. Although he reached a final calculation, he did not feel that it was correct and continued to manipulate the numbers until he found the calculation error. With respect to the design fixation exhibited with the equipment, the participant requested an information sheet with information on pieces of equipment for children with disabilities. One of the information sheets has an image of a swing designed specifically to meet the needs of children with physical disabilities. He was determined to use materials to recreate the swing he saw in the image. This represents blind adherence to a design solution (fixation).

On the other hand, this participant exhibited divergent thinking throughout the study. When he realized that his constraints did not allow him to abide by what was provided on the information sheets, he often deliberately decided to go against the best practices recorded on the information sheet. This can be seen in the following statement: “I am reading the merry go round, it says not intended for toddlers, but I am probably going to break that and assume that parents are going to not put their toddler on it, and even if the toddler is on it, it will probably be alright.” He does appear to exhibit premature convergence and fixation in his design process.

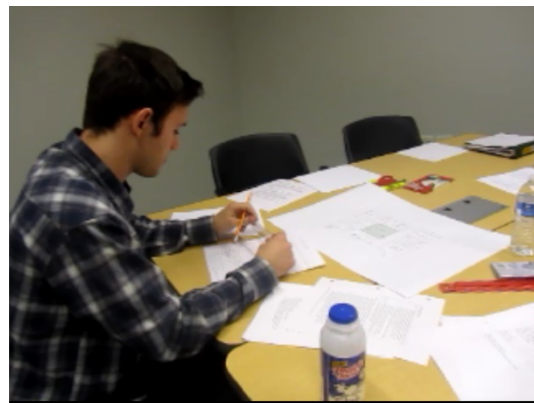


Figure 6 Workspace of First Year Engineering Participant

Example 2: Senior Fashion Design Student

This participant was a senior fashion design student. She had a very apparent and well-developed design cycle. Initially, this participant prepared a list of equipment that one might see on a playground. This is an example of divergent thinking, which we see in both students. They begin with brainstorming and do not initially allow constraints and requirements to limit the equipment that they will design. Once the fashion design participant developed a list of potential equipment, she iterated through the design process for each piece of equipment. Figure 7 is a representation of her design process.

With respect to mathematical modeling, this student often uses a seemingly well-developed model for estimating dimensions of objects and calculating costs. In this participant's think-aloud, mathematical modeling was often followed by either narrowing down the potential solutions or making a decision on a design solution. Here is an example of the design student's mathematical modeling thoughts:

That would include 6 poles, so you have to break that into 944, seven poles...so it would be 4 swing hangers, with the junctions... ok so even if you took the 944 and divided it by the seven poles alone, that's a hundred and thirty five dollars a pole. So let's just say it's going to be like a hundred dollars for the pole. So that's 35 times 7, yea that would be about right.

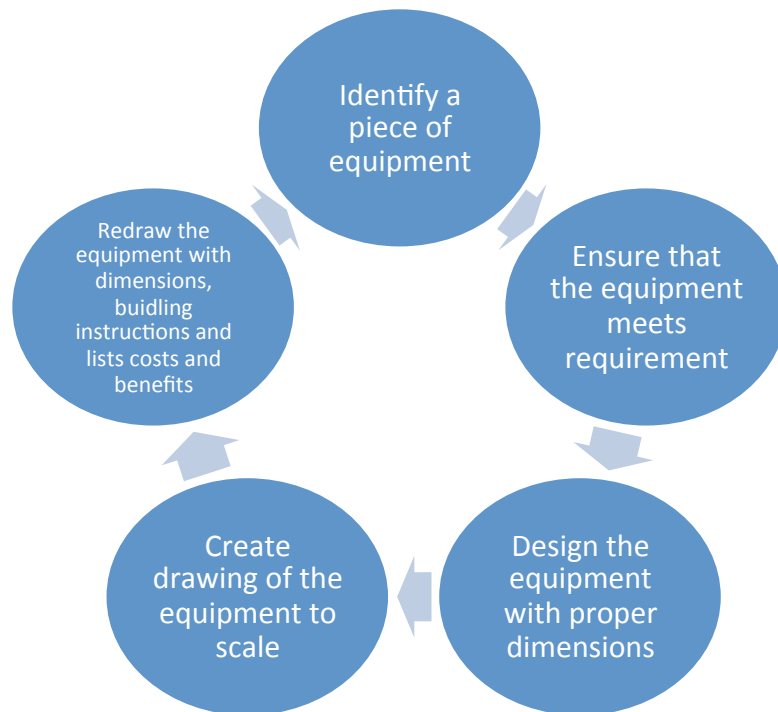


Figure 7: The Iterative Design Process of the Senior Design Student

Here we see that after the participant used her mathematical model to create dimensions for the swing set bar she was able to determine how many bars she could create from the available material, while maintaining a specific budget. Although this participant initially exhibited divergent thinking (e.g. brainstorming), she began to converge on a final solution that will include the pieces of equipment she selects from the list of potential equipment she created. After those pieces of equipment are selected, she displayed divergent thinking once again when she considered the design of the equipment that must meet the Americans for Disabilities Act.

This participant used precise detail when she created the blueprints and instructions for building the playground equipment. She color-coded the dimensions and unique parts of each piece of equipment. She re-drew and re-wrote the dimensions and building instructions to ensure her instructions would be clearly understood and were legible. Overall, this participant has a well-developed design process in which mathematical modeling is built into. There appear to be fewer instances of premature adherence to a particular design solution than the student with a higher mathematics background. Figure 4 shows how using video data can capture that type of detailed work, because of the ability to zoom in to the workspace.

Discussion

Discussion of the Video Data Examples

We see that students with varying level of mathematical ability solve design problems with mathematical components differently. Rather, a more developed design process could be a result of years of refinement and practice just as a lack of fixation might be due to less complex mathematical model or due to the students' experience with quickly developing mathematical models for estimation or creating dimension. Future work on this project can potentially reveal the type of models that students with varying backgrounds use when solving open-ended design problems. We may also discover how mathematical models are used to avoid premature fixation on design solutions.

Reflection on Using Video and Lessons Learned

As we continue with data collection there are some lessons learned about using video to collect data that would be beneficial to discuss. Because audio and video recording devices are being used, it is important to synchronize the equipment and timestamp field notes. This is very helpful during analysis, when trying to find specific incidents in video or audio recording that were recorded in field notes. One method to synchronize recording devices is to begin recording at the same time. In order to synchronize the field notes with the recordings, we have used the Microsoft Word timestamp function. Observations are made every 5-7 minutes. After 3 observations, the time on the audio and video recorders is also recorded in the field notes with the Microsoft Word timestamp.

Field notes are very helpful when using video data. As discussed above, it is very important that the notes and the video are synchronized. It has been very helpful for our team to have a primary focus when making observations during the study. So for each participant, the researcher is looking for the common themes. In our case, we look for instances of convergent and divergent thinking. Because the video is recorded, we have the opportunity to review each recording to ensure that our observations are accurate and then to use a look at the data from a different lens during a second viewing of each case.

We anticipate recruiting 90 students to participate in this study; this will result in much data. Each participant's video recording takes about 4GB of space. Along with the other artifacts that are collected, the folder size for each participant is about 6GB. Our current data management strategy is to use a common naming convention for all the participants' folders. They are saved on the secured password protected university server. We are investigating other data management strategies as this study moves forward.

The use of video data to understand cognitive barriers which may exist when solving design problems seems promising. While focus in the past has been to use video as an auxiliary data instrument, this study has the potential to highlight the value that video as primary data can bring to research on engineering education and engineering education research.

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