Chalkboard vs. Paper: Technique for Improving Collaboration in Active Learning Activities

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Introduction

This study builds on an innovation in the format for collaborative group work in a first-year undergraduate engineering math classroom at Cornell University. The innovation involves directing small groups of students to solve problems together at a chalkboard or whiteboard, rather than directing such groups to collaborate at tables with each solving the problems on their own paper worksheets. While the Engineering Math Workshops have been included as a small-group problem-solving component of this single-variable calculus course for engineering students since 2007 (Schneider, Kelley, Baker, 2007), this particular innovation in the format for group work was first implemented and studied in fall 2015. During the fall 2015 semester, students in two of the twelve total sections were directed to complete the workshop problems in small groups together at the board, resulting in one group solution for each problem, written on the chalkboard or whiteboard. Meanwhile those in the other ten sections completed the workshop problems in the traditional way - seated in small groups at tables, encouraged to discuss and solve the problems as a group while each student recorded the solution to the problems on their own paper worksheet. Observational data and feedback from section instructors from fall 2015 yielded support for the educational value of the chalkboard innovation. However, the student feedback data indicated considerable variation in students’ perceptions of the value of the workshops, and also illuminated the presence of implementation inconsistencies across sections led by different Teaching Assistants (TAs) (Ritz and Schneider-Bentley, 2016, submitted for publication). For the fall 2016 semester, we have worked more closely with the TAs to assure greater consistency in implementation across the workshop sections. We have also designed a more comprehensive comparison whereby students in each section experience the workshops in both formats throughout the term, doing approximately half of the workshops in small groups at tables with individual paper solutions, and the other half in small groups at the board yielding one group solution. In this paper, we will begin by providing a brief background on the Engineering Math Workshops, describing the recent innovation in group work format, and summarizing the previous findings from fall 2015. We will then detail the current implementation effort and data collection in fall 2016, and share our findings and conclusions regarding the comparison of the efficacy of small-group problem-solving at tables on paper versus at the chalkboard in this context.

Engineering Math Workshops and small-group problem-solving format

The Engineering Math Workshops component of the first-semester Calculus for Engineers course at Cornell University was initiated a decade ago, as a joint effort of Engineering and Mathematics faculty, to integrate a series of collaborative, applied, problem-solving workshops into the course. This effort was inspired by multiple studies demonstrating the educational benefits of active and collaborative learning environments for all students, and particularly for engineering undergraduates (Felder, Felder, and Dietz, 1998; Prince, 2004; Hake, 1998; Colbeck, Campbell, and Bjorklund, 2000; Johnson, Johnson, and Smith, 1998a,b; Springer, Stanne, and Donovan, 1999; Terenzini, 2001). The project was further informed by the positive learning and retention outcomes reported following curricular innovations to integrate math, science, and
engineering content in undergraduate engineering programs at other institutions (Carr, 2003; Froyd and Ohland, 2005; Olds and Miller, 2004). The expected benefits for first-year engineering students of the Engineering Math Workshops innovation include: a more thorough understanding of the mathematical concepts in the core course; an enhanced ability, and increased confidence in one’s ability, to apply calculus to engineering or real-world problem-solving; and a stronger sense of engagement with the first-year curriculum and with fellow engineering classmates.

The overall structure of the *Calculus for Engineers* course includes three weekly 50-minute lectures taught by Math department or Engineering college faculty and two weekly 50-minute discussions led by graduate student TAs. There are typically twelve to fourteen discussion sections, enrolling 25-30 students per section. Six to seven TAs staff the course and teach two sections each. The second discussion section each week is generally used for an Engineering Math Workshop, in which students work collaboratively in small groups on solving the applied workshop problems. An example problem is shown in the appendix. The workshops are facilitated by the TA along with an upper-level engineering undergraduate course assistant (CA). The course TAs and CAs attend a 1.5 hour training session at the outset of each semester designed to prepare them to facilitate the Workshops. The training includes an explanation of the motivation for the workshop program and the expected learning gains for students, a description of the optimal workshop environment they are called on to create and tips on how to foster that optimal environment, important information on administrative procedures and record-keeping for program evaluation, and practice guiding group work on the actual workshop problems. In the workshops students are instructed to work in groups on the applied problems. TAs and CAs facilitate the group work and provide guidance where necessary. Students are encouraged to discuss and grapple with the problems together with their group members and to help each other to reach a joint solution. Previous publications describe the development and implementation of the Engineering Math Workshops in more detail (Schneider, Kelley, Baker, 2007), and report findings from efforts to document and characterize student learning gains associated with the workshops (Schneider and Terrell, 2010; Terrell, Terrell, and Schneider, 2010).

Since 2015 the authors have been engaged in a review of the workshop problems, implementation practices, and student feedback, with the aim of updating, improving, and reinforcing the workshops effort where necessary. As part of this review, we recognized an opportunity to attempt to reinvigorate the workshops with a change in group work format. The original workshops effort aims to create a collaborative environment in which students actively engage with each other in small groups, sharing ideas and understandings as they progress through the problem-solving process – constantly engaging with their small group while recording the solutions on their individual worksheets. However, student comments on feedback surveys suggested that the expected benefits of this collaborative group problem-solving set-up were not being fully realized. After conversations with colleagues about this challenge, we decided to experiment with a change in group-work format designed to encourage greater engagement among group members. The use of chalkboards or whiteboards to encourage student participation and collaboration is not a new approach, and benefits have been documented by other educators (Arney et al, 1995; Graham et al, 1999; Khalid et al, 2011; Subramanian et al, 2009). Our intention is to compare the outcomes of small-group, collaborative problem-solving when groups work at the board on a common solution versus when groups work
at tables, with instructions to collaborate on solving the problems together, yet ultimately recording the solutions on individual paper worksheets. Our forthcoming paper details our initial study design and findings from fall semester 2015 (Ritz and Schneider-Bentley, 2016, submitted for publication). In the next sections we briefly summarize the outcomes of that initial effort, and explain how our current study intends to build on and extend those findings.

Summary of findings from fall 2015 semester

As mentioned earlier, in the 2015 fall semester we set up a comparison to detect any observable outcomes between workshops using the chalkboard format versus workshops using the paper format. Since we only had two pairs of sections (i.e., two sections taught in the same room led by the same TA) being held in rooms with sufficient board space, we assigned one of each of those pairs (two sections total) to use the chalkboard format. In order to control for differences in TA effectiveness, each of the TAs leading one of the experimental sections with the chalkboard format was also leading another section using the paper format. This allowed a comparison of the two group work formats in sections led by the same TA.

For that study, we expected that the format of the chalkboard sections, in which students work together with their group members to produce a single written solution, should result in a higher level of group engagement and interaction among group members. In turn, we expected this higher level of engagement and interaction, in the process of attempting to reach consensus for a common solution, should lead to a higher incidence of students helping each other and learning from each other. Finally, we expect that as students are “forced” to talk about their approaches to problem solving, and listen to others’ approaches, because of the need to reach an agreed-upon solution, they will ultimately attain a fuller understanding of the workshop problems. The diagram in Figure 1 illustrates our central hypotheses, with the expectation of positive relationships between each aspect.

Our observational data – from classroom observations, limited videotaping of the paired experimental (chalkboard) and traditional (paper) sections, and instructor perceptions – strongly supported our expectation of increased engagement and interaction among students working together at the chalkboard on a common solution. The noise level, from students discussing and debating problem-solving approaches in their groups, was much higher in the sections using the chalkboard format. The students working in groups at tables spoke to each other occasionally, sharing questions and ideas about the problems, but the interaction was less frequent, as much of the time was spent with each individual working on writing their own solution. The TAs who led one section of each format both indicated a clear perception of greater interaction and engagement among the students participating in the group work at chalkboards.
Student survey responses also supported some aspects of our central hypotheses, as well as revealing some unanticipated patterns and some implementation inconsistencies. Overall, students participating in the group problem-solving at the chalkboards were significantly more likely than those using the traditional group work format at tables on paper to indicate that the students in their group were helping each other understand the problem and solution, and to indicate that they learned something new from participating in the workshop. However, the overall significance of this pattern is complicated by the interaction of TA characteristics – such as teaching and facilitation skill, integrity of the implementation of the workshop, and effort applied to motivating the students’ participation in the group work – with the students’ reported outcomes.

Additional clear patterns that emerged showed that students in the chalkboard sections were significantly more likely than students working at tables on paper to find both the difficulty and the length of the workshop problems to be optimum. Those working in the groups at tables on paper were more likely to indicate that the problems were too long and/or too difficult. These latter findings are quite pertinent, suggesting that the visibly greater engagement and interaction in the chalkboard format facilitates the exchange of ideas and increases the speed of problem-solving, resulting in a successful outcome for the students. Finding the problems to be manageable in the allotted time, allows students to leave the workshop with a feeling of success and competency, rather than frustration.

The main implementation inconsistency that was illuminated through scrutiny of the student feedback was a lack of time dedicated to the workshops in several of the workshop sections. Many student responses, across sections and instructors, emphasized that it was difficult to realize the intended benefits of the workshops when too little time was allotted within the sections to working on the workshop problems. Another less frequent, yet very important, inconsistency that was uncovered was a lack of enforcement of group work by some of the TAs for some of the workshop sections. Some students indicated that their TA did not put them in groups for a particular workshop section, but simply allowed them to work on the problems on their own.

These findings about the importance of the teaching assistants’ role in creating a successful collaborative learning experience for the students echo earlier research on this topic. Webb’s 2009 review of the research on the role of the teacher in fostering successful small group experiences in the classroom focused on four dimensions of the teacher’s role: preparing students to collaborate, forming groups, structuring and guiding group work, and influencing group interaction through teacher discourse (Webb, 2009). Preparing the teaching assistants and course assistants to be effective facilitators of group work on all of these dimensions is challenging, yet critical. With relatively little training, they are called on to create a classroom climate that is encouraging of group work; to guide students in effective group work by instructing, modeling, and motivating; and to intentionally form groups and consistently enforce group interaction in each workshop session. In the next section, we describe our efforts to address the inconsistencies in facilitator implementation that we observed and to better-equip and motivate the TAs to more effectively prepare for and facilitate the small group work. We also describe our efforts to
strengthen the design of our comparison of the group work formats for the fall 2016 round of implementation.

Workshops implementation and data collection for fall 2016

To build on the findings from fall 2015, we took two approaches: improving consistency of workshop implementation between TAs, and designing an expanded experiment. As mentioned above, a clear finding from fall 2015 was a lack of sufficient time being dedicated to the workshops. Some TAs were spending half or more of the discussion section time answering homework questions and getting the room organized for the actual collaborative problem solving portion. To combat that, we made it clear that one full section each week must be dedicated to the workshop, with no homework questions on that day. That removed the individual judgement call about how long to dedicate to the workshop. Additional efforts to improve consistency of implementation focused on clear communication with the TAs about the shortcomings from the previous semester and specific instructions about how to avoid similar pitfalls. While our TA and CA training already contained guidance on successfully fostering an optimal collaborative learning environment, including small group formation, facilitation, and motivation, we especially emphasized the responsibility they held as workshop facilitators to actively create that environment and be proactive in guiding the group work during each session. Also, small changes were made to individual workshop problems based on student and TA feedback. These changes were focused on clarifying confusion and bringing emphasis back to concepts and discussion instead of derivations of formulas, for example.

The success of those improvement is shown by the numbers from 2015 versus the numbers from 2016 for student rating of length, difficulty, and whether they learned something new: in 2015, 58% of the time students rated the workshops length as “just about right,” whereas in 2016, students chose that option 72% of the time; difficulty was rated “just about right” 74% of the time in 2015 and 78% of the time in 2016; 61% of responses indicated they had learned something new in 2015 vs. 72% in 2016. It’s important to note that, in addition to the implementation changes mentioned here, more students were completing the workshops on the board in 2016, as described in the following paragraphs. This may have contributed to the improvements in these results.

In designing the experimental process for fall 2016, we had to weigh several issues. Ideally, the class would be divided into several groups, including a control group of students doing all work on paper, a group doing all work on the board, and a group where students do half of the workshops on paper and half on the board. Student performance on exams would then be tracked and compared, and follow-up tests of student understanding of the calculus topics would be repeated over the next several years. However, there are many confounding issues. For example, some TAs may be generally more or less effective; therefore, each TA should have students in each group. Some sections meet early in the morning and some meet in the afternoon; each of those should have students in each group. When dividing the class into finer and finer classifications, the number of students participating in each population becomes very small, obstructing the emergence of statistically meaningful patterns.
The experimental design for fall 2016 was as follows: for workshops 2-8 (workshop 1 involves filling out missing table entries and as such is ill-suited to work on the board), half of the students completed the workshop on paper and half on the chalkboard. Each TA led two separate sections of students; for each workshop one of their sections worked on paper and one on the board. Each student completed 3–4 workshops on the board and the rest on paper. The main goals of this implementation were to allow students to experience both methods, to allow TAs to experience both methods for each workshop, and to increase the number of students in each category (for example, students completing workshop 2 on paper) with the hope of improving statistical significance of any results.

After each workshop, students were asked to complete a survey with many questions about their experience. The questions (along with the possible answers) we will focus on for the data analysis are:

- What did you think of the difficulty of this workshop?
  - Too difficult.
  - Just about right.
  - Too easy.

- What did you think of the length of this workshop?
  - Too long.
  - Just about right.
  - Too short.

- Did this workshop help you learn something you had not learned before? (e.g. mathematical concepts, applications, different ways of looking at things, etc.)
  - Yes.
  - No.

Instead of attempting to measure actual student learning outcomes, in this study we rely on the extensive literature showing the relationship between collaboration during active learning and eventual learning outcomes. We recognize that student self-perceptions of learning are a proxy for actual learning outcomes, and have designed this experiment using those self-perceptions to allow us to gather an additional important piece of feedback: direct feedback from students who have experience with the workshops using both methodologies.

In addition to student surveys, TAs and CAs also gave feedback after each workshop, specifically about the paper vs. chalkboard comparison. Highlights of their feedback are included along with other findings in the following section. Finally, students were given a survey at the end of the semester asking for retrospective feedback about the workshops in general, instead of any one workshop in particular. On that survey, we asked them to tell us which format they preferred and why (along with many other questions). Examples of student comments from that survey are included below.

**Findings**

Feedback from the TAs was again overwhelmingly in favor of utilizing the chalkboards. Out of 52 responses over the course of the semester, 41 (79%) thought the workshop was better when implemented on the chalkboards. Some example comments in response to, “For this workshop, which went better: board version or paper version? Any idea why?” are:
Board version! The groups actually worked together and helped each other on the board.

I've noticed the board versions are always more engaging because group members talk more with themselves and between groups (can check their method of evaluation with neighboring groups)

Board. It promoted collaboration better. The students working on paper complained that they couldn't use the boards this time

ALWAYS Board. The students collaborate better, rather than just working on their own and not sharing with the group

Some individual responses did indicate that the paper version was better, but even some of those specified that it was only for a particular workshop (due to extensive calculations, for example) and not generally true.

Turning now to the student feedback from surveys following individual workshops, we focused on some of the responses from fall 2015 that showed promising preliminary results. Figures 2-4 show student survey responses to the three questions highlighted above. Those figures include data from surveys after each of the individual workshops 2-8, all combined. The total enrollment in the class was 329, and the average response rate over these seven workshops was approximately 57%, with better participation during the earlier workshops and worse participation for the later workshops. From these data, perception of difficulty does not seem to be influenced by working on the chalkboard vs. on paper, but both perception of length and of whether students learned something new does show a benefit for work on the board, as confirmed with the statistical analysis whose results are shown in Table 1.

![Figure 2: Students' perception of whether the workshop taught them anything new. The number of responses contributing to each category is shown in the legend.](image-url)
Figure 3: Students' perception of the difficulty of the workshop problems. The number of responses contributing to each category is shown in the legend.

Figure 4: Students' perception of the length of the workshop problems. The number of responses contributing to each category is shown in the legend.
Table 1. Correlations between workshop completion format and student survey results.
* indicates statistical significance at p<0.05 level

<table>
<thead>
<tr>
<th>Overall Student Responses, by Format of Group Work, Workshops 2-8</th>
<th>Paper</th>
<th>Chalkboard</th>
<th>Total item responses</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Responses indicating <strong>Optimum Difficulty</strong></td>
<td>524</td>
<td>489</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total responses for each format</td>
<td>664</td>
<td>621</td>
<td>1285</td>
<td></td>
</tr>
<tr>
<td>% indicating <strong>Optimum Difficulty</strong></td>
<td>78.9%</td>
<td>78.7%</td>
<td>.912</td>
<td></td>
</tr>
<tr>
<td>Responses indicating <strong>Optimum Length</strong></td>
<td>459</td>
<td>473</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total responses for each format</td>
<td>662</td>
<td>620</td>
<td>1282</td>
<td>.008</td>
</tr>
<tr>
<td>% indicating <strong>Optimum Length</strong></td>
<td>69.3%*</td>
<td>76.3%*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Responses indicating <strong>Learned Something New</strong></td>
<td>465</td>
<td>469</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total responses for each format</td>
<td>661</td>
<td>622</td>
<td>1283</td>
<td>.042</td>
</tr>
<tr>
<td>% indicating <strong>Learned Something New</strong></td>
<td>70.3%*</td>
<td>75.4%*</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

To determine whether the observed differences in student responses when working on the board versus working on paper are statistically significant, we ran cross-tabulations for each of our three central dependent variables (length, difficulty, and learning something new), and used chi-square analyses to determine the statistical significance of each relationship. From the results summarized in Table 1, we can see that when students were working at the board they were significantly more likely to rate the workshop problem to be of optimum length (76.3% of students working at board versus 69.3% of students working on paper) and significantly more likely to indicate that they learned something new in the workshop (75.4% of students working at board versus 70.3% of students working on paper). In contrast, and contrary to our expectations based on results from 2015, student ratings of the difficulty of the workshop problems did not vary by the group work format used.

These results can be further broken down, for example separating each workshop or separating students in each TA’s sections. Again, the results for length and learning something new show consistency, lending credence to the statistical significance. Figures 5 and 6 show percentages of respondents saying the length is “just about right” and saying they did learn something new, broken down by workshop. These both show benefits for board work for almost every workshop. This is especially true for the two workshops that ran long (workshops 5 and 7). When looking at differences in student responses across TAs, consistent patterns do not emerge. Sometimes, there are quite different patterns of responses across the two different sections led by the same TA, confounding any comparisons among the six TAs.
Finally, one of the goals of the current experimental design was to allow specific feedback from students about their own preferences. On the end-of-term student feedback survey, students were asked to indicate their preferred format for the workshops: Paper, Board, or No Preference. Of the 195 students who responded, 61% (119) indicated that they preferred working at the board, 26% (50) preferred working on paper, and 13% (26) indicated they had no preference. Students were asked to elaborate on the reasons for their preferences, and 64% (124) of the students who responded did so with written comments.
Of those who preferred the board format, 70% (57) of commenters indicated that the work at the board improved collaboration among group members. Most of these simply gave very brief comments such as “more collaborative” or “easier to work together.” The next common response was given by 21% of respondents, who noted that the board work made it easier for all to see the work and follow along. Here are a few of the more articulate responses explaining the board preference, which capture sentiments expressed by many students in this group:

*Working on the board seemed to get everyone more involved as we could all follow along step by step in our solution, rather than work at our own pace on paper.*

*Working on the white board makes it easier to work as a group. Everyone is on the same page and it’s easier to bounce ideas off each other and allow everyone to contribute to the discussion.*

*On the board all members of the group could see what was going on. This prompted more collaboration, and we could all have input.*

*It was easier to talk through the problem with the other group members at the blackboards. When we did the workshops on paper it was almost as if we were all doing the problem individually vs when we did it at the blackboard, everyone contributed and collaborated.*

*Board allowed everyone to understand the solution, whereas when we worked on paper some people would understand something and work ahead of others, who would be left behind and so not want to ask for help.*

The minority of students (26%) who preferred the paper format and commented on this also gave some clear reasons for their preference. Thirty-three percent (11) of this group shared that they simply prefer working out the solution on their own on paper. An additional 24% (8) also indicated a preference for writing out their own solution on paper while discussing with the group as needed during the process, or after they had time to work on it on their own. Smaller subsets (12%, or 4 respondents, for each) noted that they did not like that 1-2 people can easily “take over” the group work at the board, or that they liked having their own solution written out to take home and refer to later. Here are some responses which clearly express the preference for working on paper at tables:

*I have always preferred writing on paper on my own while still working with a group. I can scribble down my thoughts before I really understand how to solve it.*

*I personally am better at understanding things when I write them out in front of me.*

*Paper allows everyone in the group to reach their own conclusion and check each other and avoids one person doing all of the work.*

The small minority of students (13%) who indicated that they had no preference in relation to the different formats sometimes referred to pros and cons for each, such as in the following example:
Working on a board is great so everyone can see what everyone is doing and it forces you to be a lot more collaborative, but then you don't have anything to keep and work off later if you want to.

Discussion

The findings from our fall 2016 round of workshops implementation and data collection provide important insights to inform our efforts to optimize the students’ learning experiences in the Engineering Math Workshops. Most centrally, these findings help provide:

1) Evidence about how successful we have been in our efforts to improve the integrity of workshops implementation;
2) Clarification about how students’ perceptions about individual workshop sessions vary by the format of group work they experience for that session;
3) Information about students’ group work format preferences when asked directly at end-of-term.

For assessing our efforts to improve the consistency of implementation across sections led by different TAs, our findings attest to the success of these efforts. The students’ overall ratings about their perceptions of the workshops in terms of each of our three central variables of interest – length, difficulty, and learning something new - are more positive, on average, in 2016 compared to 2015. This suggests that our emphases in 2016 on ensuring that TAs allot the full section time for the group work on the workshop problems, and focus more attention on enforcing and facilitating the group work, has paid off.

Our more robust experimental design, with each student experiencing roughly half of the workshops at the board and the other half on paper, resulted in a clearer depiction of how students’ perceptions of the workshops vary by the format of group work. The differences obtained were moderate in magnitude. Yet clear and statistically significant differences did emerge for students’ perceptions about the length of the workshop problems being optimum or not, and about whether they learned something new in the workshop. This design also gave each TA the experience of facilitating both types of group work, and their feedback is instructive as well. The preliminary feedback from two TAs in 2015 is echoed by the majority of the TAs and CAs in 2016. From the course staff point of view, supervising workshops is easier when students work at the board. They also observed some important aspects of collaboration more when students worked at the board. For example, some repeated feedback indicates that on the board, students are forced to resolve any disagreements before moving on to later parts of the problem. That gives students the opportunity to defend their own solution and convince others, or to learn how someone else approaches the same problem and see a different solution. Working on paper, students might not bother trying to correct classmates since their own work is unaffected.

Finally, students’ end-of-term responses and comments about their preferences for group work at the board versus at tables on paper are illuminating. These show a strong majority of students preferring to work at the board, and noting the greater opportunity for collaboration that this method afforded.
Conclusion

The value of group work at the board - for promoting collaboration, effectively reaching a solution with limited time, and learning something new – has been demonstrated here. However, it is important to take into account the diversity of student learning styles in a classroom, and promote facilitation methods to engage, inspire, and accommodate the strengths of both active and reflective learners (Felder and Silverman, 1988). While active group engagement is a central goal of the workshops, many students will gain even further benefit when opportunities for individual reflection are also made possible and promoted as part of the process. For example, for the Engineering Math Workshops, students can be encouraged to fully work through the workshop problems on their own after the group exercise in the workshop, or even before the workshop. One possible way to address the complaint about not being able to take the completed workshop problem home would be to encourage students to use their phones to photograph the group work on the board so they can use it for later studying. For many students, significant benefits can be gained from both the individual problem-solving process and the group discussion and collaboration. Providing opportunities to accommodate both within, or in connection to, the workshop is likely to enhance the experience for many students.

When taken together with the results from fall 2015, this study also strongly emphasizes the importance of careful facilitation of group work, regardless of the format used. Our efforts to make full and efficient use of the time available improved the outcomes of the workshops, but further improvements can still be made. While it’s easier to keep track of student work at the board, a special effort should be made to balance participation in each group. Each student should take a turn with the chalk, preventing a single student from taking over. Additionally, the main goal of each problem, and its relation to the course material, must be made clear. Some students continue to see the workshops as an extraneous, instead of integral, aspect of the course. In the future, the group work could be leveraged to reinforce those connections, with specific discussion questions at the end of each workshop where students need to brainstorm and state in their own words the goal and connection for each workshop.
Appendix: Example Workshop Problem

MATH 1910 Workshop

Bloodflow

Introduction: When fluid flows down a pipe not all parts travel at the same velocity. In *laminar* flow, fluid particles move along straight parallel paths in layers, called laminae. The velocities in adjacent layers are not the same. Those in the center of the pipe are faster than those that are closer to the interior surface of the pipe.

We consider the case of *stationary, fully developed* flow in a circular tube. “Stationary” means that the flow is independent of time, while “fully developed” means that the flow velocity is independent of distance in the direction of the flow (i.e., that entrance effects can be neglected). Practically speaking, that means we’re considering regions of pipe far from bends, junctions, pumps, holes, etc. The pressure in the pipe is taken to be a function of distance in the direction of the flow, only. For the flow to be stationary, the forces on a cylindrical tube of fluid of length $L$ and radius $r$ must balance (see figure).

These forces consist of the shear force (shear stress $\tau$ times area $2\pi r L$ of cylindrical surface) and the pressure force (pressure difference $p_1 - p_2$ times the cross-sectional area $\pi r^2$ of the column). In laminar flow, the shear stress $\tau$ is proportional to the velocity gradient

$$\tau = -\mu \frac{dv}{dr}$$

where $\mu$ is the viscosity of the fluid.

The flow of blood in an artery is neither stationary nor fully developed. While this model therefore does not directly apply to blood circulation, it can offer some indication of why angioplasty is an effective means of increasing blood flow through a clogged artery: increasing the diameter of the artery has a large effect on the volume flow rate.

Goals:

- Find fluid velocity as a function of radius within a circular pipe.
- Explore how diameter changes affect fluid flow rates.

Problems:

a) Use the information provided to write down the force balance, and solve this balance for the velocity gradient $dv/dr$.

b) Integrate the result of part (a) under the boundary condition $v(R) = 0$ (this is called a “no slip” boundary condition) to find $v(r)$. Sketch $v(r)$ in the figure.

c) Find the volume flow rate $Q$ by integrating $v(r)$ over the cross section of the tube. By which factor is $Q$ changed if the diameter of the tube is increased by i) a factor of 1.5 and ii) a factor of 2, without changing the pressure gradient $\Delta p/L$?
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